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Power Electronics

Exercise: Space Vector Modulation

2012

1 Theory

1.1 Introduction

Considering an electric drive system as shown in Figure 1, the controller generates a reference voltage, $\underline{u}_s$, represented with voltage space vector, as equation (1).

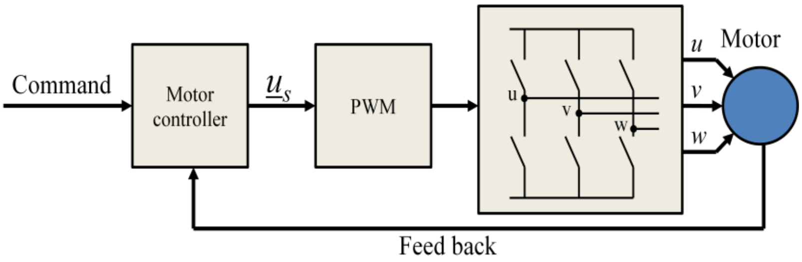


Figure 1. Structure of an electric drive system

$$\underline{u}_s = u_s e^{j\theta}. \quad (1)$$

In order to apply this voltage on the motor, it is required to convert this reference voltage to the switching signals for the inverter. To do this, several PWM strategies are available. In the sub-oscillation methods, the three phase voltages are firstly calculated and they are compared with a high frequency carrier signal to generate the pulses to control the inverter switches.

Besides such methods, it is possible to generate the switching signals directly using the space vector of the reference voltage, without having to convert the space vector to the three phase values at first. This method is called space vector modulation (SVM).

1.2 Principle

It is known that the three switching arms in the converter have eight base states as shown in Figure 2. Six vectors of them have non-zero magnitudes, while the other two are zero length vectors.

Referring to Figure 2, suppose a reference voltage $\underline{u}_s$ is to be applied to the motor. If $\underline{u}_s$ is not identical to one of the base vectors, it must be approximated using these eight vectors. In the case shown in Figure 2, $\underline{u}_s$ can be approximated based on timely switching among $\underline{u}_{100}$, $\underline{u}_{110}$ and the two zero vectors. In this case, vector $\underline{u}_{100}$ should be applied for a longer time than $\underline{u}_{110}$ since $\underline{u}_s$ is nearer to $\underline{u}_{100}$; and a time of zero vectors should also be applied in order to reduce the magnitude.

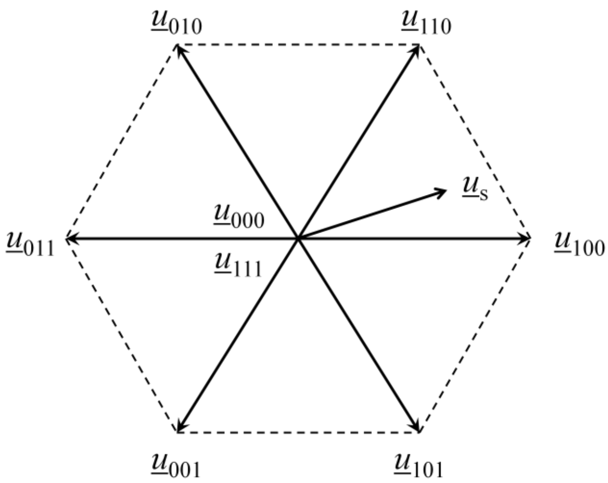


Figure 2. Voltage space vectors available using a three phase inverter

1.3 Switching Timing

The calculation of the durations of the base vectors is described based on Figure 3.

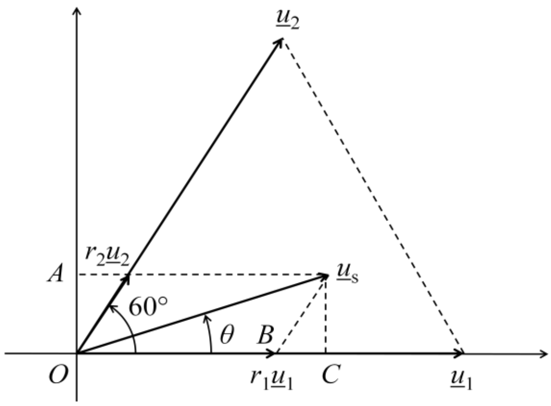


Figure 3. Approximation of an arbitrary voltage space vector using base vectors

Supposing the reference voltage space vector $\underline{u}_s$ falls between two adjacent base vectors, $\underline{u}_1$ and $\underline{u}_2$, the reference vector $\underline{u}_s$ can be represented with the combination of the two vectors, $\underline{u}_1$ and $\underline{u}_2$:

$$\underline{u_s} = r_1\underline{u}_1 + r_2\underline{u}_2, \tag{2}$$

where r_1 and r_2 are coefficients.

Using the basic trigonometric relations we get

$$\begin{cases} r_1 u_1 = u_s\cos\theta - BC = u_s\cos\theta - DC \cdot \tan 30° = u_s\cos\theta - u_s\sin\theta \cdot \tan 30° \\ = u_s\left(\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\right) = \frac{2}{\sqrt{3}}u_s(\sin 60° \cos\theta - \cos 60° \sin\theta) = \frac{2}{\sqrt{3}}u_s \sin(60° - \theta), \\ r_2 u_2 = \frac{AO}{\cos 30°} = \frac{u_s \sin\theta}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}u_s \sin\theta . \end{cases} \tag{3}$$

where u_1 and u_2 are the length of the two vectors with the value $\frac{2}{3}U_{dc}$, and u_s is the length of vector $\underline{u}_s$. To summarize we get

$$\begin{cases} r_1 = \sqrt{3}\frac{u_s}{U_{dc}}\sin(60° - \theta) \\ r_2 = \sqrt{3}\frac{u_s}{U_{dc}}\sin\theta \end{cases}. \tag{4}$$

For θ between 0 and 60°, r_1 and r_2 are within the range [0, 1]. They take the maximal value 1 when $\underline{u}_s$ coincides with $\underline{u}_1$ or $\underline{u}_2$, respectively. $r_1+r_2 = 1$ when $\underline{u}_s$ reaches the dashed line between the arrow peaks of $\underline{u}_1$ and $\underline{u}_2$.

This equation means that one should combine r_1 part of $\underline{u}_1$ and r_2 part of $\underline{u}_2$ to obtain the reference voltage $\underline{u}_s$. Since it is not possible to change the magnitude of the base vectors, $\underline{u}_1$ and $\underline{u}_2$, the combination is realized (approximated) using time division, as shown in Figure 4. If $\underline{u}_s$ doesn't touch the dashed edge of the triangle, the sum of r_1 and r_2 is less than 1. Therefore, the rest time is filled with zero vectors.

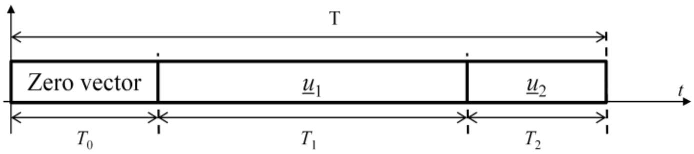


Figure 4. Combination of vectors using time division

The three time durations are defined as

$$\begin{cases} T_0 = (1 - r_1 - r_2)T \\ T_1 = r_1 T \\ T_2 = r_2 T \end{cases}, \tag{5}$$

where T is the PWM period, T_0 is the duration for zero vector, and T_1 and T_2 the durations for vector $\underline{u}_1$ and $\underline{u}_2$, respectively.

These equations mean that an arbitrary space vector within the triangle defined by the two adjacent base vectors, between which the expected vector is located, can be represented by the sum of these two vectors. This is realized by timely activating the two vectors combined with zero vectors sequentially. If the switching process is fast enough, meaning the period T is short, the approximation can precisely represent the reference vector.

1.4 Switching Sequence

From the above section it is clear that the inverter base vectors can be switched consequentially to generate other vectors. It is not yet clear how the switching process should be realized. This is explained here with an example. Referring to Figure 3, if $\underline{u}_1$ and $\underline{u}_2$ are $\underline{u}_{100}$ and $\underline{u}_{110}$, respectively, and the expected voltage space vector is located between these two vectors, a simplest switching sequence could be done as Figure 5, Figure 6 or Figure 7.

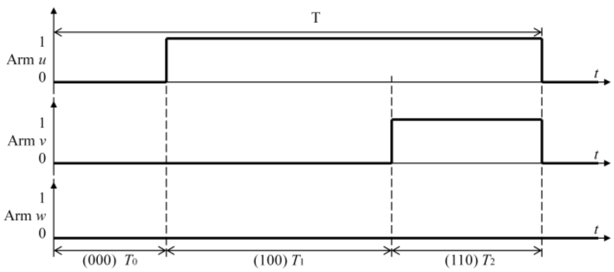


Figure 5. PWM switching sequence using asymmetric pulsation and $\underline{u}_{000}$ as zero vector

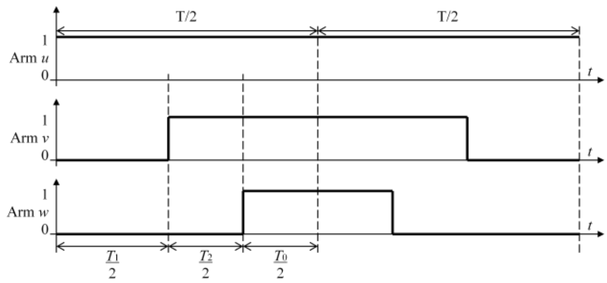


Figure 6. PWM switching sequence using symmetric pulsation and $\underline{u}_{111}$ as zero vector

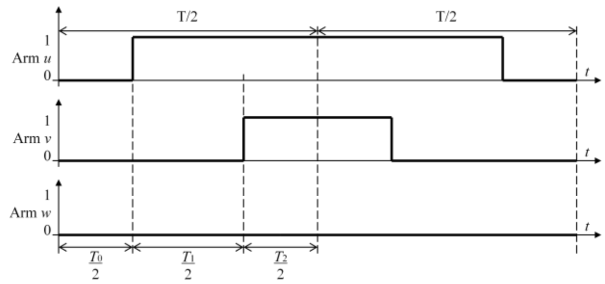


Figure 7. PWM switching sequence using symmetric pulsation and $\underline{u}_{000}$ as zero vector

The method shown in Figure 5 is not good because two bridge arms have to be switched at a certain time. This would cause much harmonic noises. The other two methods are named flattop modulation. A dominant property of such methods is that within one 60° sector of the hexagonal space vector graph of the inverter, only two bridge arms are being switched. This is advantageous to reduce switching loss. However the long time off-state of a bridge arm causes asymmetric load among the arms. And furthermore, this is also a source of more harmonic noises.

A better but more complex pulsation method is shown in Figure 8. It is symmetric pulsation and both zeros vectors are applied.

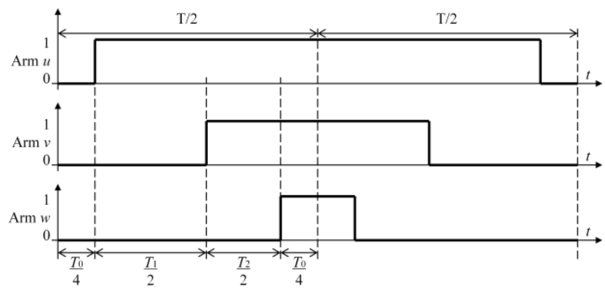


Figure 8. PWM switching sequence using both $\underline{u}_{000}$ and $\underline{u}_{111}$ as zero vectors

1.5 Over-Modulation

To generate a rotating space vector with constant magnitude, the reference vector must be limited within the inscribed circle of the hexagon, as shown in Figure 9.

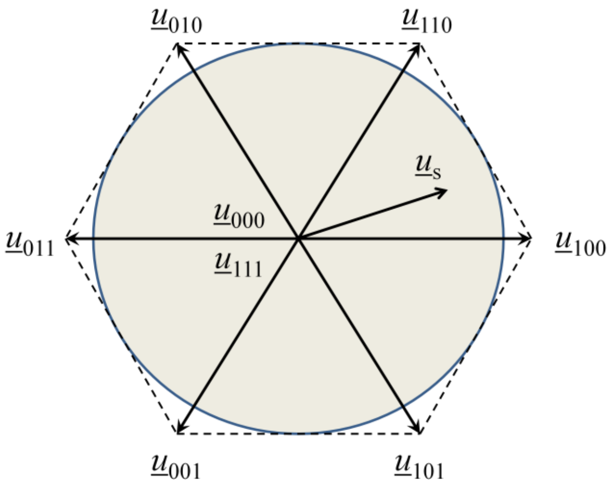


Figure 9. The inscribed circle of the base vector hexagon

However, it is possible to run the SVM in over-modulation mode. In this mode, the reference vector follows a circular trajectory that can extend the boundary of the hexagon (Figure 10). For the circle portions inside the hexagon utilize, the same SVM equations for determining the state times are used. However, during the circle portions outside the hexagon, the magnitude of the reference voltage has to be limited by the hexagon (as the case in Figure 10) and the calculations for the timing are still based on equation (4). In this case, there is no zero state time.

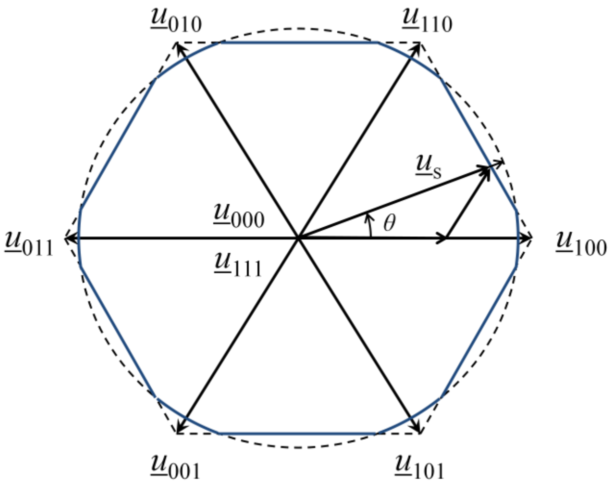


Figure 10. Over-modulation of SVM

Over-modulation allows more bus utilization. However, it results in non-sinusoidal output. It is thus only used as a transitioning step form the SVM into six-step operation.

2 Exercises

2.1 Exercise 1

2.1.1 Questions

What parameters are required for the implementation of SVM method?

What values are obtained from the calculation of SVM method?

2.1.2 Answer

The required parameters according to equation (4) are:

- The reference voltage represented with a space vector (angle and magnitude).
- The voltage value on the DC bus.
- The PWM period.

After the calculation we get:

- Two base vectors.
- The time durations of these two base vectors.
- The time duration of the zero vectors.

2.2 Exercise 2

2.2.1 Question

If SVM method is applied to an inverter for motor control, is it necessary to know the values of the three phase voltages for driving the motor? Why?

2.2.2 Answer

To use SVM method, it is not necessary to know the three phase voltages. The reason is that the input parameter for SVM method is the voltage space vector for driving the motor. The switching timing for the inverter bridge arms is directly obtained from the angle and magnitude of this vector.

2.3 Exercise 3

2.3.1 Problem

It is expected to output a three-phase voltage that is represented with a space vector $\underline{u}_s = 100e^{j165°}$ (V) using SVM method. The known parameters are:

- DC link voltage, U_{dc} = 600 V.
- PWM frequency, f = 8 kHz

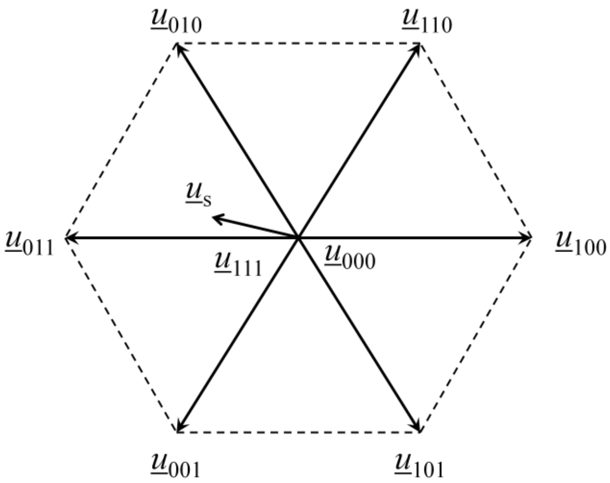
Questions:

1. Please determine which base vectors are needed for generating this voltage.
2. Please calculate the time durations of these base vectors.

3. Please draw one period of the PWM switching sequence to generate the expected voltage using symmetric pulsation and both zero vectors, and indicate the duration of every switching state in the diagram.
4. Please draw one period of the PWM switching sequence to generate the expected voltage using flattop method, and indicate the duration of every switching state.

2.3.2 Solution to Question 1

The expected space vector in the base vector hexagon is shown in the following figure. It is obvious that the expected vector $\underline{u}_s$ should be represented using $\underline{u}_{010}$, $\underline{u}_{011}$ and the zero vectors.



2.3.3 Solution to Question 2

Using equation (4),

$$\begin{cases} r_1 = \sqrt{3} \times \frac{100}{600} \times \sin(60° - (165° - 120°)) \approx 0.0747 \\ r_2 = \sqrt{3} \times \frac{100}{600} \times \sin(165° - 120°) \approx 0.204 \end{cases}.$$

PWM period,

$$T = \frac{1}{f} = 125\ (\mu s)$$

Therefore, the durations for zero vectors is

$$T_0 = (1 - 0.0747 - 0.204) \times 125 = 90.2\ (\mu s),$$

for vector $\underline{u}_{010}$ is

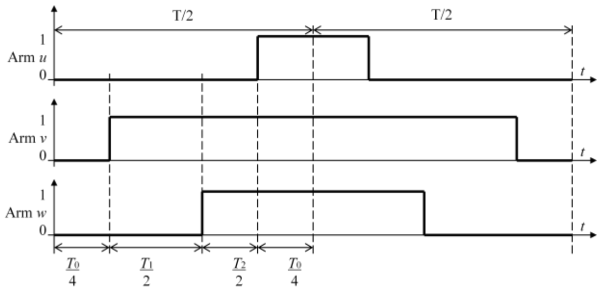
$$T_1 = 0.0747 \times 125 = 9.3\ (\mu s),$$

and for vector $\underline{u}_{011}$ is

$$T_2 = 0.204 \times 125 = 25.5\ (\mu s).$$

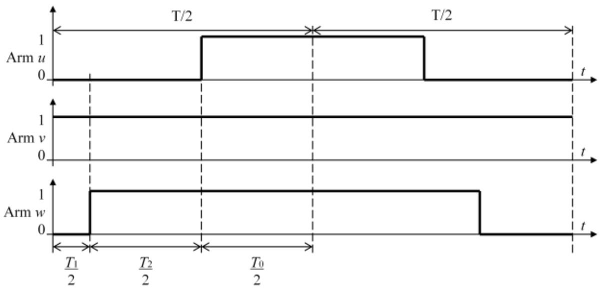
2.3.4 Solution to Question 3

SVM using both zero vectors:

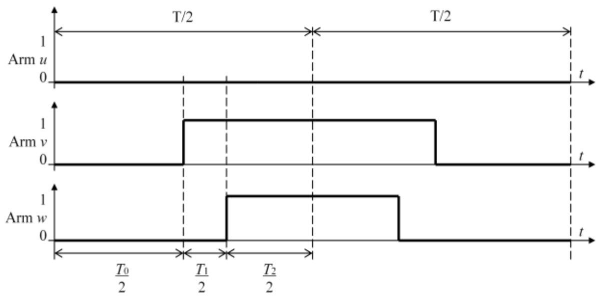


2.3.5 Solution to Question 4

SVM flattop pulsation using zero vector, $\underline{u}_{111}$:



or using another zero vector, $\underline{u}_{000}$:



3 References

Valentine, Richard (1998). Motor control electronics handbook. New York: McGraw-Hill (pages from 254)