

Introduction to the Finite Element Method (FEM): Lecture 1: Introduction

Abolfazl Hasani Baferani

Department of Mechanical Engineering
Tafresh University

Sep 2019

Outline:

- 1 Introduction
 - Overview
 - Structural Analysis
- 2 What's FEM
- 3 Discretization
- 4 Relationship to the FDM
- 5 Advantages and Disadvantages of the FEM

Overview:

- **Main Principle in FEM:** Divide a complex-shaped problem into smaller, simple shaped problems where a solution is known and easy.
- By using FEM, complex partial differential equations that describe *the behavior of structures* can be reduced to a set of linear equations that can easily be solved using the standard techniques of matrix algebra.
- FEM is very used for analysis:
 - 1 Structural Dynamic (Static and Dynamic)
 - 2 Thermal Analysis
 - 3 Fluid Mechanics
 - 4 Electro Dynamics
 - 5 Fluid-Structure Iteraction
 - 6 Sensitivity Analysis → optimization
 - 7 ...

Structural Analysis:

The finite element method can be used to analyze both structural and nonstructural problems. Typical structural areas include:

- Linear Stress Analysis: Stress & Deformation
- Normal Mode Analysis: Frequencies & Mode Shape
- Harmonic Analysis: Frequency response (Force Vibration in Harmonics)
- Transient Analysis: Frequency response (Dynamic Non-harmonic Force)
- Random Analysis: Frequency response (Dynamic Non-periodic Force)
- Buckling Analysis: Critical Force for instability
- Nonlinear Analysis: Large Deformation
- Contact Analysis
- Fatigue Analysis
- Die Design & Metal Forming

What's FEM:

- The field of Mechanics can be subdivided into four major areas:
 - 1 **Theoretical Mechanics** deals with fundamental laws and principles of mechanics studied for their intrinsic value.
 - 2 **Applied Mechanics** transfers this theoretical knowledge to scientific and engineering applications
 - 3 **Computational Mechanics** solves specific problems by combining mathematical models with numerical methods
 - 4 **Experimental Mechanics** puts physical laws, mathematical models and numerical simulations to the ultimate test of observation.
- The basic idea behind the FEM is to divide the structure and being analyzed into a large number of Finite elements.

What's FEM: (cont'd)

- Computational Mechanics is strongly interdisciplinary.

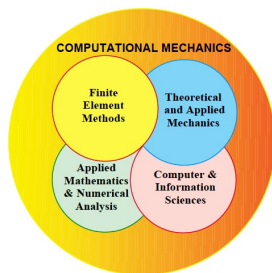


Figure 2.1: Computational Mechanics integrates aspects of four disciplines

- Many problems in engineering and applied science are governed by differential or integral equations.
- Complexities in the geometry, properties and in the boundary conditions that are seen in most real-world problems usually means that an exact solution cannot be obtained.

Discretization:

- Two key ideas of the finite element method are:
 - 1 Discretization of the Region
 - 2 Interpolating Polynomial

Definition

Discretization concerns the process of transferring continuous models and equations into discrete counterparts.

Definition

Interpolation is a method of constructing new data points within the range of a discrete set of known data points.

Example of Discretization:

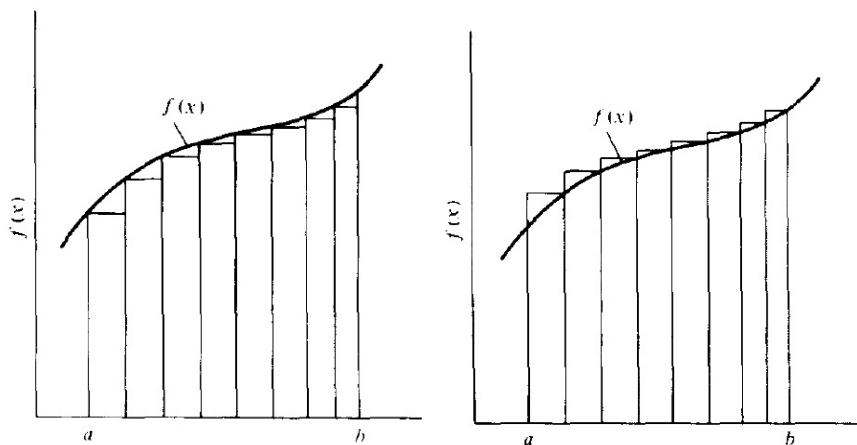


Figure 3.1: (a) Lower bound on estimate by using inscribed rectangles (b) Right
Picture: Upper bound on estimate by using circumscribed rectangles

Relationship to the FDM:

- In FDM, a basic law is written for each node.
- Both methods require the analyst to discretize the structure but the discretization is done is fundamentally different.
- In FDM, the object or region being analyzed is divided into a finite number of lumps.
- In FEM, for structural and stress analysis problems, **the principle of virtual work** will be used and for nonstructural applications, **the weighted residuals method** will be used.

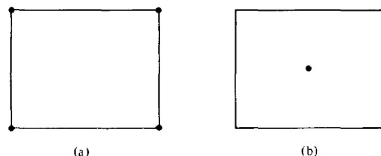
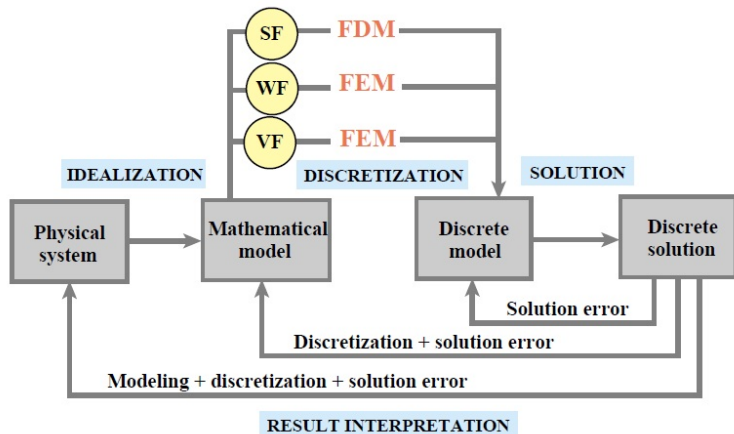


Figure 4.1: (a) An element used in finite element analysis (b) A lump used in finite-difference analysis

- **Note:** The four nodes on the element in (a) and the one node on the lump in (b).

The main stages of computer-based simulation:



SF \Leftrightarrow Strong Form

WF \Leftrightarrow Weak Form

VF \Leftrightarrow Variational Form

Advantages and Disadvantages of the FEM:

Advantages

- 1 FEM can handle irregular geometries routinely.
- 2 FEM routinely handled variable spacing of the nodes.
- 3 FEM may be handled nonhomogeneous and anisotropic materials.
- 4 In a typical FEM application may encounter all the various types of boundary conditions.
- 5 In FEM with relative ease the higher-order elements may be implemented.

Disadvantages

- 1 FEM is the necessity for a digital computer and fairly extensive software.

Higher Order Elements:

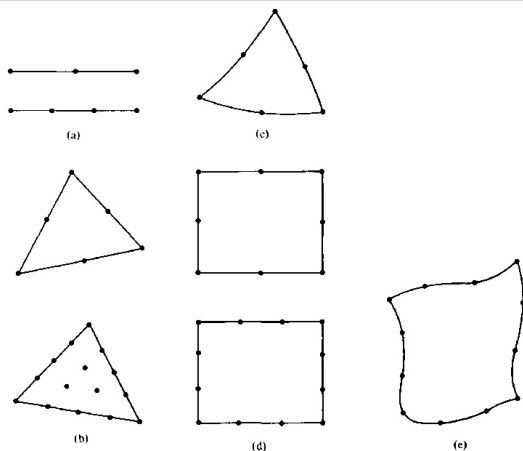


Figure 5.1: a) One-dimensional elements. b) Two-dimensional triangular elements with straight sides c) Two-dimensional triangular element with curved sides d) Two-dimensional rectangular elements with straight sides e) Two-dimensional quadrilateral element with curved sides