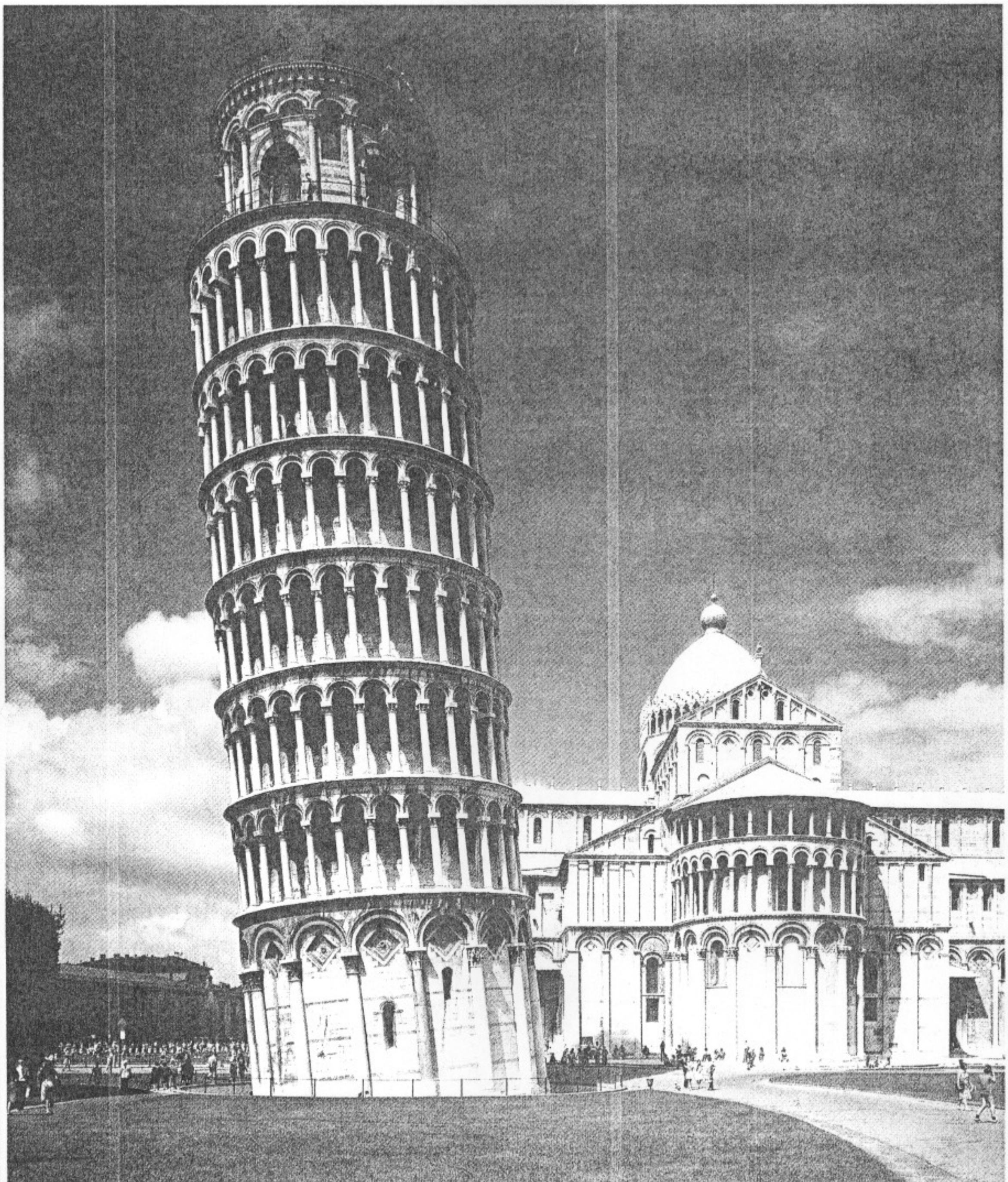
SECOND EDITION

FOUNDATION DESIGN

Principles and Practices



DONALD P. CODUTO

Foundation Design

Principles and Practices

Second Edition

Donald P. Coduto

Professor of Civil Engineering
California State Polytechnic University, Pomona

Prentice Hall
Upper Saddle River, New Jersey 07458
http://www.prenhall.com

Library of Congress Cataloging-in-Publication Data
Coduto, Donald P.
Foundation design : principles and practices / Donald P. Coduto.—2nd ed.
p. cm.
ISBN 0-13-589706-8
1. Foundations. I. Title.

TA775.C63 2001
624.1'5—dc21
00-038553

Vice president and editorial director: *Marcia Horton*
Acquisitions editor: *Laura Curless*
Editorial assistant: *Dolores Mars*
Senior marketing manager: *Danny Hoyt*
Production editor: *Pine Tree Composition*
Executive managing editor: *Vince O'Brien*
Managing editor: *David A. George*
Art director: *Jayne Conte*
Art editor: *Adam Velthaus*
Cover designer: *Bruce Kenselaar*
Manufacturing manager: *Trudy Pisciotti*
Manufacturing buyer: *Dawn Murrin*
Vice president and director of production and manufacturing: *David W. Riccardi*

Reprinted with corrections January, 2001

Printed in the United States of America
10 9 8 7 6 5 4

ISBN 0-13-589706-8

Prentice-Hall International (UK) Limited, *London*
Prentice Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Pearson Education Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

Contents

Preface

Foundation Design: Principles and Practices is primarily intended for use as a textbook in undergraduate and graduate-level foundation engineering courses. It also serves well as a reference book for practicing engineers. As the title infers, this book covers both "principles" (the fundamentals of foundation engineering) and "practices" (the application of these principles to practical engineering problems). Readers should have already completed at least one university-level course in soil mechanics, and should have had at least an introduction to structural engineering.

This second edition contains many improvements and enhancements. These have been the result of comments and suggestions from those who used the first edition, my own experience using it at Cal Poly Pomona, and recent advances in the state-of-the-art. The chapters on deep foundations have been completely reorganized and rewritten, and new chapters on reliability-based design and sheet pile walls have been added. Extraneous material has been eliminated, and certain analysis methods have been clarified and simplified. The manuscript was extensively tested in the classroom before going to press. This classroom testing allowed me to evaluate and refine the text, the example problems, the homework problems, and the software.

Key features of this book include:

- Integration with *Geotechnical Engineering: Principles and Practices* (Coduto, 1999), including consistent notation, terminology, analysis methods, and coordinated development of topics. However, readers who were introduced to geotechnical engineering using another text can easily transition to this book by reviewing the material in Chapters 3 and 4.
- Consideration of the geotechnical, structural, and construction engineering aspects of the design process, including emphasis on the roles of each discipline and the interrelationships between them.
- Frequent discussions of the sources and approximate magnitudes of uncertainties, along with comparisons of predicted and actual behavior.
- Use of both English and SI units, because engineers in North America and many other parts of the world need to be conversant in both systems.

- Integration of newly-developed Excel spreadsheets for foundation analysis and design. These spreadsheet files may be downloaded from the Prentice Hall website (www.prenhall.com/coduto). They are introduced only after the reader learns how to perform the analyses by manual computations.
- Extensive use of example problems, many of which are new to this edition.
- Inclusion of carefully developed homework problems distributed throughout the chapters, with comprehensive problems at the end of each chapter. Many of these problems are new or revised.
- Discussions of recent advances in foundation engineering, including Statnamic testing, load and resistance factor design (LRFD), and applications of the cone penetration test (CPT).
- Inclusion of extensive bibliographic references for those wishing to study certain topics in more detail.
- An instructor's manual is available to faculty. It may be obtained from your Prentice Hall campus representative.

ACKNOWLEDGMENTS

Many friends, colleagues, and other professionals contributed to this work. Much of the book is the product of their stimulating discussions, constructive reviews, and support. Professor Joseph Caliendo of Utah State University was especially helpful. Stanley Vitton (Michigan Technological University), Samuel Clemence (Syracuse University), Richard Handy (Iowa State University), Raymond Moore (University of Nebraska), John Horvath (Manhattan College), José Pires (University of California, Irvine), Paul Chan (New Jersey Institute of Technology), William and Sandra Houston (Arizona State University), and others reviewed part or all of the manuscripts for the first or second editions and provided many useful suggestions and comments. Iraj Noorany (San Diego State University), Michael O'Neill (University of Houston), Major William Kitch (U.S. Air Force Academy), James Olson (University of Vermont), Samuel Paikowsky (University of Massachusetts, Lowell), Richard W. Stephenson (University of Missouri, Rolla), William Kovacs (University of Rhode Island), Dan Burgess, Rick Drake (Fluor Daniel), Bengt Fellenius (Urkkada Technology), Mohamad Hussein (Goble, Rausche, Likins), and others also provided useful suggestions and advice.

A special note of thanks goes to the foundation engineering students at Cal Poly University who used various draft manuscripts of this book as a makeshift text. Their constructive comments and suggestions have made this book much more useful, and their proofreading has helped eliminate mistakes.

I welcome any constructive comments and suggestions from those who use this book. Please mail them to me at the Civil Engineering Department, Cal Poly University, Pomona, CA 91768.

Donald P. Coduto

Notation and Units of Measurement

There is no universally accepted notation in foundation engineering. However, the notation used in this book, as described in the following table, is generally consistent with popular usage.

Symbol	Description	Typical Units English	Typical Units SI	Defined on Page
A	Cross-sectional area	ft^2	m^2	438
A	Base area of foundation	ft^2	m^2	155
A_0	Initial cross-sectional area	in^2	mm^2	95
A_1	Cross-sectional area of column	in^2	mm^2	337
A_2	Base area of frustum	in^2	mm^2	337
A_b	Area of bottom of enlarged base	ft^2	m^2	528
A_f	Cross-sectional area at failure	in^2	mm^2	95
A_s	Steel area	in^2	mm^2	323
a_0	Factor in N_q equation	Unitless	Unitless	178
B	Width of foundation	ft-in	mm	146
B'	Effective foundation width	ft-in	m	275
B_b	Diameter at base of foundation	ft	m	548
B_s	Diameter of shaft	ft	m	547
b	Unit length	ft	m	156
b_c, b_q, b_γ	Base inclination factors	Unitless	Unitless	186
b_0	Length of critical shear surface	in	mm	310
C_1	Depth factor	Unitless	Unitless	235
C_2	Secondary creep factor	Unitless	Unitless	235
C_3	Shape factor	Unitless	Unitless	235
C_A	Aging factor	Unitless	Unitless	122
C_B	SPT borehold diameter correction	Unitless	Unitless	119
C_C	Compression index	Unitless	Unitless	66

C_{OCR}	Overconsolidation correction factor	Unitless	Unitless	122
C_P	Grain size correction factor	Unitless	Unitless	122
$C_{p\phi}$	Passive pressure factor	Unitless	Unitless	602
C_R	SPT rod length correction	Unitless	Unitless	119
C_r	Recompression index	Unitless	Unitless	67
C_s	SPT sampler correction	Unitless	Unitless	119
C_s	Side friction coefficient	Unitless	Unitless	535
C_t	Toe coefficient	Unitless	Unitless	534
C_w	Hydroconsolidation coefficient	Unitless	Unitless	709
c	Wave velocity in pile	ft/s	m/s	571
c	Column or wall width	in	mm	302
c'	Effective cohesion	lb/ft^2	kPa	84
c'_{adj}	Adjusted effective cohesion	lb/ft^2	kPa	198
c_T	Total cohesion	lb/ft^2	kPa	85
D	Depth of foundation	ft-in	mm or m	146
D_{50}	Grain size at which 50% is finer	Unitless	mm	122
D_{min}	Minimun required embedment depth	ft	m	593
D_r	Relative density	percent	percent	51
D_w	Depth from ground surface to groundwater table	ft	m	188
d	Effective depth	in	mm	306
d	Bolt diameter	in	mm	344
d	Vane diameter	in	mm	131
d_b	Reinforcing bar diameter	in	mm	306
d_c, d_q, d_γ	Depth factors	Unitless	Unitless	184
E	Portion of steel in center section	Unitless	Unitless	333
E	Modulus of elasticity	lb/in^2	MPa	231
E_m	SPT hammer efficiency	Unitless	Unitless	119
E_s	Equivalent modulus of elasticity	lb/ft^2	kPa	231
E_u	Undrained modulus of elasticity	lb/ft^2	kPa	226
EI	Expansion index	Unitless	Unitless	673
e	Eccentricity	ft	m	159
e	Void ratio	Unitless	Unitless	49
e	Base of natural logarithms	2.7183	2.7183	XXX
e_0	Initial void ratio	Unitless	Unitless	66
e_B	Eccentricity in the B direction	ft	m	165
e_L	Eccentricity in the L direction	ft	m	165
e_{max}	Maximum void ratio	Unitless	Unitless	51
e_{min}	Minimum void ratio	Unitless	Unitless	51
F	Factor of Safety	Unitless	Unitless	190
F_a	Allowable axial stress	lb/in^2	MPa	439

F_b	Allowable flexural stress	lb/in^2	MPa	439
F_v	Allowable shear stress	lb/in^2	MPa	439
f_a	Average normal stress due to axial load	lb/in^2	MPa	438
f_b	Normal stress in extreme fiber due to flexural load	lb/in^2	MPa	438
f_c'	28-day compressive strength of concrete	lb/in^2	MPa	303
f_{pe}	Effective prestress on gross section	lb/in^2	MPa	448
f_s	Unit side friction resistance	lb/ft^2	kPa	513
$(f_s)_m$	Mobilized unit side-friction resistance	lb/ft^2	kPa	544
f_{sc}	CPT cone side friction	T/ft^2	MPa or kg/cm^2	124
f_v	Shear stress	lb/in^2	MPa	439
f_y	Yield strength of steel	lb/in^2	MPa	303
G_h	Horizontal equivalent fluid density	lb/ft^3	kN/m^3	770
G_s	Specific gravity of solids	Unitless	Unitless	49
G_v	Vertical equivalent fluid density	lb/ft^3	kN/m^3	771
g_c, g_q, g_γ	Ground inclination factors	Unitless	Unitless	186
H	Thickness of soil stratum	ft	m	60
H	Wall height	ft	m	759
H_c	Critical height	ft	m	767
H_{fill}	Thickness of fill	ft	m	64
I_1, I_2	Influence factors	Unitless	Unitless	226
I_ε	Strain influence factor	Unitless	Unitless	234
I_p	Plasticity index	Unitless	Unitless	56
I_r	Rigidity index	Unitless	Unitless	501
i_c, i_q, i_γ	Load inclination factors	Unitless	Unitless	185
I_σ	Stress influence factor	Unitless	Unitless	210
K	Coefficient of lateral earth pressure	Unitless	Unitless	61
K_a	Coefficient of active earth pressure	Unitless	Unitless	760
K_p	Coefficient of passive earth pressure	Unitless	Unitless	762
k	Factor in computing depth factors	Unitless	Unitless	184
k_s	Coefficient of subgrade reaction	lb/in^3	kN/m^3	356
L	Length of foundation	ft-in	mm	146
L'	Effective foundation length	ft-in	m	275
LL	Liquid limit (see w_L)	Unitless	Unitless	54
l	Cantilever distance	in	mm	322
l_d	Development length	in	mm	318
l_{dh}	Development length for hook	in	mm	337
M	Moment load	ft-k	kN-m	15
M_c	Characteristic moment load	ft-lb	kN-m	601
M_D	Driving moment	ft-lb	kN-m	796

M_g	Applied moment to pile group	ft-lb	kN-m	616
M_{max}	Maximum moment	ft-lb	kN-m	603
M_n	Nominal moment load capacity	ft-k	kN-m	21
M_R	Resisting moment	ft-lb	kN-m	796
N	Number of piles in a group	Unitless	Unitless	538
N	SPT blow count recorded in field	Blows/ft	Blows/300 mm	116
$(N_1)_{60}$	SPT blow count corrected for field procedures and overburden stress	Blows/ft	Blows/300 mm	120
N_σ	Bearing capacity factor	Unitless	Unitless	502
N_{60}	SPT blow count corrected for field procedures	Blows/ft	Blows/300 mm	119
N_c, N_q, N_γ	Bearing capacity factors	Unitless	Unitless	178
N_c^*, N_q^*, N_γ^*	Bearing capacity factors	Unitless	Unitless	501
N_u	Uplift bearing capacity factor	Unitless	Unitless	527
OCR	Overconsolidation ratio	Unitless	Unitless	69
P	Normal load	k	kN	15
P_a	Allowable downward load capacity	k	kN	467
P_a	Normal force acting on a wall under active conditions	lb	kN	759
P_{ag}	Allowable load capacity of pile group	k	kN	538
P_{upward}	Upward load	k	kN	470
$(P_a)_{upward}$	Allowable upward load capacity	k	kN	470
P_D	Driving force	lb	kN	791
P_f	Axial load at failure	lb	N	95
PI	Plasticity index (see I_P)	Unitless	Unitless	54
P_0	Normal force acting on a wall under at-rest conditions	lb	kN	751
PL	Plastic limit (see w_P)	Unitless	Unitless	54
P_n	Nominal normal load capacity	k	kN	21
P_{nb}	Nominal bearing capacity	k	kN	336
P_p	Normal force acting on a wall under passive conditions	lb	kN	759
P_R	Resisting force	lb	kN	791
P_s	Side-friction resistance	k	kN	466
P_t	Toe-bearing resistance	k	kN	466
P_t'	Net toe-bearing resistance	k	kN	407
P_u	Factored normal load	k	kN	21
P_{ult}	Ultimate downward load capacity	k	kN	481
p	Lateral soil resistance per unit length of foundation	lb	kN	587
Q_c	Compressibility factor	Unitless	Unitless	128
q	Bearing pressure	lb/ft^2	kPa	154

q'	Net bearing pressure	lb/ft^2	kPa	158
q	Quake	in.	mm	566
q_a	Allowable bearing capacity	lb/ft^2	kPa	190
q_A	Allowable bearing pressure	lb/ft^2	kPa	262
q_c	CPT cone resistance	T/ft^2	MPa or kg/cm^2	124
q_E	Effective cone resistance	T/ft^2 or MPa	kg/cm^2	533
q_{EG}	Factor in Eslami and Fellenius method	T/ft^2	MPa	534
q_{equiv}	Equivalent bearing pressure	lb/ft^2	kPa	275
q_{max}	Maximum bearing pressure	lb/ft^2	kPa	162
q_{min}	Minimum bearing pressure	lb/ft^2	kPa	162
q_t'	Net unit toe-bearing resistance	lb/ft^2	kPa	500
$(q_t')_m$	Mobilized unit toe-bearing resistance	lb/ft^2	kPa	544
q_{tr}'	Reduced net unit toe-bearing resistance	lb/ft^2	kPa	506
q_u	Unconfined compressive strength	lb/ft^2	kPa	511
q_{ult}	Ultimate bearing capacity	lb/ft^2	kPa	176
r	Distance from centerline of cap	in	mm	615
r	Rigidity factor	Unitless	Unitless	219
R_f	Friction ratio	Unitless	Unitless	124
R_I	Moment of inertia ratio	Unitless	Unitless	601
RQD	Rock quality designation	Unitless	Unitless	511
R_u	Ultimate resistance	k	kN	566
S	Slope of foundation	radians	radians	587
S	Elastic section modulus	in^3	mm^3	438
S	Number of stories	Unitless	Unitless	109
S	Degree of saturation	percent	percent	49
S	Column spacing	ft	m	33
S_0	Degree of saturation before wetting	percent	percent	678
S_1, S_3	Allowable lateral soil pressure	lb/ft^2	kPa	593
s	Shear strength	lb/ft^2	kPa	84
s	Center-to-center spacing of piles	in	mm	540
s	Pile set	in	mm	560
s_c, s_q, s_γ	Shape factors	Unitless	Unitless	184
s_u	Undrained shear strength	lb/ft^2	kPa	89
T	Torsion load	k-ft	m-kN	15
T	Thickness of foundation	ft-in	mm	146
T_f	Torque at failure	in-lb	N-m	131
TMI	Thornthwaite moisture index	Unitless	Unitless	666
t	Time	yr	yr	235
t	Age of soil (since time of deposition)	yr	yr	122

u	Displacement of pile	in	mm	564
u	Pore water pressure	lb/ft^2	kPa	59
u_2	Pore water pressure behind cone point	lb/ft^2	kPa	533
u_D	Pore water pressure at bottom of foundation	lb/ft^2	kPa	155
u_e	Excess pore water pressure	lb/ft^2	kPa	59
u_h	Hydrostatic pore water pressure	lb/ft^2	kPa	58
V	Shear load	k	kN	15
V_a	Shear force under active condition	k	kN	759
V_c	Nominal shear capacity of concrete	lb	kN	309
V_c	Characteristic shear load	lb	kN	601
V_{fa}	Allowable footing shear load capacity	k	kN	276
V_n	Nominal shear load capacity	k	kN	21
V_{nc}	Nominal shear capacity on critical surface	lb	kN	309
V_p	Shear force under passive condition	k	kN	159
V_s	Nominal shear capacity of reinforcing steel	lb	kN	309
V_u	Factored shear load	k	kN	21
V_{uc}	Factored shear force on critical surface	lb	kN	309
W_f	Weight of foundation	lb	kN	154
W_r	Hammer ram weight	lb	kN	560
w	Moisture content	percent	percent	49
w_L	Liquid limit	Unitless	Unitless	54
w_P	Plastic limit	Unitless	Unitless	54
w_s	Shrinkage limit	Unitless	Unitless	54
y	Lateral deflection	in	mm	598
z	Depth below ground surface	ft	m	564
z_c	Depth to centroid of soil resistance	ft	m	544
z_f	Depth below to bottom of foundation	ft	m	210
z_f	Depth to imaginary footing	ft	m	552
z_w	Depth below to groundwater table	ft	m	58
α	Wetting coefficient	Unitless	Unitless	678
α	Adhesion factor	Unitless	Unitless	522
α	Slope of footing bottom	deg	deg	183
α	Inclination of wall from vertical	deg	deg	764
β	Side friction factor in β method	Unitless	Unitless	516
β	Slope of ground surface	deg	deg	759
β	Reliability index	Unitless	Unitless	724
β_0, β_1	Correlation factors	Unitless	Unitless	233
γ	Ratio of steel cage diameter to drilled shaft diameter	Unitless	Unitless	455
γ	Unit weight	lb/ft^3	kN/m^3	49
γ	Load Factor	Unitless	Unitless	21

γ_b	Buoyant unit weight	lb/ft^3	kN/m^3	49
γ_d	Dry unit weight	lb/ft^3	kN/m^3	49
γ_{fill}	Unit weight of fill	lb/ft^3	kN/m^3	64
γ_w	Unit weight of water	lb/ft^3	kN/m^3	49
γ'	Effective unit weight	lb/ft^3	kN/m^3	188
$\Delta\sigma_z$	Change in vertical stress	lb/ft^2	kPa	64
δ	Total settlement	in	mm	29
δ_a	Allowable total settlement	in	mm	29
δ_c	Consolidation settlement	in	mm	72
δ_D	Differential settlement	in	mm	31
δ_{Da}	Allowable differential settlement	in	mm	31
δ_d	Distortion settlement	in	mm	224
δ_e	Settlement due to elastic compression	in	mm	544
δ_u	Settlement required to mobilize ultimate resistance	in	mm	544
δ_w	Heave or settlement due to wetting	in	mm	680
ϵ_{50}	Axial strain at which 50 percent of the soil strength is mobilized	Unitless	Unitless	603
ϵ_f	Strain at failure	Unitless	Unitless	95
ϵ_w	Strain due to wetting	Unitless	Unitless	676
η	Factor in Shields' chart	Unitless	Unitless	286
η	Group efficiency factor	Unitless	Unitless	538
θ	Factor in Converse-Labarre formula	Unitless	Unitless	539
θ_a	Allowable angular distortion	radians	radians	33
λ	Lightweight concrete factor	Unitless	Unitless	319
λ	Factor in Shields' chart	Unitless	Unitless	286
λ	Vane shear correction factor	Unitless	Unitless	131
λ	Equivalent passive fluid density	lb/ft^3	kN/m^3	276
λ	Factor in Evans and Duncan's charts	Unitless	Unitless	602
λ_a	Allowable equivalent passive fluid density	lb/ft^3	kN/m^3	276
μ	Coefficient of friction	Unitless	Unitless	276
μ_a	Allowable coefficient of friction	Unitless	Unitless	276
μ_C	Mean ultimate capacity	k	kN	723
μ_L	Mean load	k	kN	723
υ	Poisson's ratio	Unitless	Unitless	502
ρ	Mass density	lb_m/ft^3	kg/m^3	564
ρ	Steel ratio	Unitless	Unitless	317
ρ_s	Ratio of volume of spiral reinforcement to total volume of core	Unitless	Unitless	459
σ	Total stress	lb/ft^2	kPa	60
σ	Normal pressure imparted on wall from soil	lb/ft^2	kPa	760

σ'	Effective stress	lb/ft^2	kPa	60
σ_C	Standard deviation of ultimate capacity	k	kN	724
σ_c'	Preconsolidation stress	lb/ft^2	kPa	67
σ_{hs}	Horizontal swelling pressure	lb/ft^2	kPa	691
σ_L	Standard deviation of load	k	kN	727
σ_m'	Preconsolidation margin	lb/ft^2	kPa	69
σ_p	Representative passive pressure	lb/ft^2	kPa	602
σ_t	Threshold collapse stress	lb/ft^2	kPa	708
σ_x	Horizontal total stress	lb/ft^2	kPa	61
σ_x'	Horizontal effective stress	lb/ft^2	kPa	61
σ_z	Vertical total stress	lb/ft^2	kPa	60
σ_z'	Vertical effective stress	lb/ft^2	kPa	60
σ_{z0}'	Initial vertical effective stress	lb/ft^2	kPa	64
σ_{zD}'	Effective stress at depth *D* below the ground surface	lb/ft^2	kPa	178
σ_{zD}	Total stress at depth *D* below the ground surface	lb/ft^2	kPa	175
σ_{zf}'	Final effective stress	lb/ft^2	kPa	64
σ_{zp}'	Initial vertical effective stress at depth of peak strain influence factor	lb/ft^2	kPa	234
τ	Shear stress imparted on wall from soil	lb/ft^2	kPa	760
ϕ	Resistance factor	Unitless	Unitless	21
ϕ'	Effective friction angle	deg	deg	82
ϕ'_{adj}	Adjusted effective friction angle	deg	deg	198
ϕ_T	Total friction angle	deg	deg	85
ϕ_w	Wall-soil interface friction angle	deg	deg	763
Ψ	Three dimensional adjustment factor	Unitless	Unitless	225
Ψ	Factor in Shields' chart	Unitless	Unitless	286

Foundation Design

1

Foundations in Civil Engineering

THE PARABLE OF THE WISE AND FOOLISH BUILDERS

I will show you what he is like who comes to me and hears my words and puts them into practice. He is like a man building a house, who dug down deep and laid the foundation on rock. When a flood came, the torrent struck that house but could not shake it, because it was well built. But the one who hears my words and does not put them into practice is like a man who built a house on the ground without a foundation. The moment the torrent struck that house, it collapsed and its destruction was complete.

Luke 6:47–49 NIV (circa AD 60)

A wise engineer once said "A structure is no stronger than its connections." Although this statement usually invokes images of connections between individual structural members, it also applies to those between a structure and the ground that supports it. These connections are known as its *foundations*. Even the ancient builders knew that the most carefully designed structures can fail if they are not supported by suitable foundations. The Tower of Pisa in Italy (perhaps the world's most successful foundation "failure") reminds us of this truth.

Although builders have recognized the importance of firm foundations for countless generations, and the history of foundation construction extends for thousands of years, the discipline of *foundation engineering* as we know it today did not begin to develop until the late nineteenth century.

1.1 THE EMERGENCE OF MODERN FOUNDATION ENGINEERING

Early foundation designs were based solely on precedent, intuition, and common sense. Through trial-and-error, builders developed rules for sizing and constructing foundations. For example, load-bearing masonry walls built on compact gravel in New York City during the nineteenth century were supported on spread footings that had a width 1.5 times that of the wall. Those built on sand or stiff clay were three times the width of the wall (Powell, 1884).

These empirical rules usually produced acceptable results as long as they were applied to structures and soil conditions similar to those encountered in the past. However, the results were often disastrous when builders extrapolated the rules to new conditions. This problem became especially troublesome when new methods of building construction began to appear during the late nineteenth century. The introduction of steel and reinforced concrete led to a transition away from rigid masonry structures to more flexible frame structures. These new materials also permitted buildings to be taller and heavier than before. In addition, as good sites became occupied, builders were forced to consider sites with poorer soil conditions, and these sites made foundation design and construction much more difficult. Thus, the old rules for foundation design no longer applied.

The Eiffel Tower

The Eiffel Tower in Paris is an excellent example of a new type of structure in which the old rules for foundations no longer applied. It was originally built for the Paris Universal Exposition of 1889 and was the tallest structure in the world. Alexandre Gustave Eiffel, the designer and builder, was very conscious of the need for adequate foundations, and clearly did not want to create another Leaning Tower of Pisa (Kerisel, 1987).

The Eiffel Tower is adjacent to the Seine River, and is underlain by difficult soil conditions, including uncompacted fill and soft alluvial soils. Piers for the nearby Alma bridge, which were founded in this alluvium, had already settled nearly 1 m. The tower could not tolerate such settlements.

Eiffel began exploring the subsurface conditions using the crude drilling equipment of the time, but was not satisfied with the results. He wrote: "What conclusions could one reasonably base on the examination of a few cubic decimeters of excavated soil, more often than not diluted by water, and brought to the surface by the scoop?" (Kerisel, 1987). Therefore, he devised a new means of exploring the soils, which consisted of driving a 200-mm diameter pipe filled with compressed air. The air kept groundwater from entering the tube, and thus permitted recovery of higher quality samples.

Eiffel's studies revealed that the two legs of the tower closest to the Seine were underlain by deeper and softer alluvium, and were immediately adjacent to an old river channel that had filled with soft silt. The foundation design had to accommodate these soil conditions, or else the two legs on the softer soils would settle more than the other two, causing the tower to tilt toward the river.

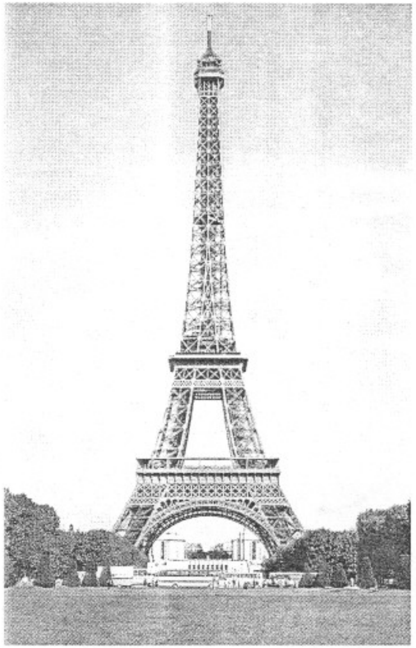


Figure 1.1 Two legs of the Eiffel Tower in Paris are underlain by softer soils, and thus could have settled more than the other two. Fortunately, Eiffel carefully explored the soil conditions, recognized this problem, and designed the foundations to accommodate these soil conditions. His foresight and diligence resulted in a well-designed foundation system that has not settled excessively.

Based on his study of the soil conditions, Eiffel placed the foundations for the two legs furthest from the river on the shallow but firm alluvial soils. The bottom of these foundations were above the groundwater table, so their construction proceeded easily. However, he made the foundations for the other two legs much deeper so they too were founded on firm soils. This required excavating about 12 m below the ground surface (6 m below the groundwater table). As a result of Eiffel's diligence, the foundations have safely supported the tower for over one hundred years, and have not experienced excessive differential settlements.

Further Developments

As structures continued to become larger and heavier, engineers continued to learn more about foundation design and construction. Instead of simply developing new empirical rules, they began to investigate the behavior of foundations and develop more rational methods of design, thus establishing the discipline of foundation engineering.

A significant advance came in 1873 when Frederick Baumann, a Chicago architect, published the pamphlet *The Art of Preparing Foundations, with Particular Illustration of* *the "Method of Isolated Piers" as Followed in Chicago* (Baumann, 1873). He appears to be the first to recommend that the base area of a foundation should be proportional to the applied load, and that the loads should act concentrically upon the foundation. He also gave allowable bearing pressures for Chicago soils and specified tolerable limits for total and differential settlements.

The growth of geotechnical engineering, which began in earnest during the 1920s, provided a better theoretical base for foundation engineering. It also provided improved methods of exploring and testing soil and rock. These developments continued throughout the twentieth century. Many new methods of foundation construction also have been developed, making it possible to build foundations at sites where construction had previously been impossible or impractical.

Today, our knowledge of foundation design and construction is much better than it was one hundred years ago. It is now possible to build reliable, cost-effective, high-capacity foundations for all types of modern structures.

1.2 THE FOUNDATION ENGINEER

Foundation engineering does not fit completely within any of the traditional civil engineering subdisciplines. Instead, the foundation engineer must be multidisciplinary and possess a working knowledge in each of the following areas:

- **Structural engineering**—A foundation is a structural member that must be capable of transmitting the applied loads, so we must also understand the principles and practices of structural engineering. In addition, the foundation supports a structure, so we must understand the sources and nature of structural loads and the structure's tolerance of foundation movements.
- **Geotechnical engineering**—All foundations interact with the ground, so the design must reflect the engineering properties and behavior of the adjacent soil and rock. Thus, the foundation engineer must understand geotechnical engineering. Most foundation engineers also consider themselves to be geotechnical engineers.
- **Construction engineering**—Finally, foundations must be built. Although the actual construction is performed by contractors and construction engineers, it is very important for the design engineer to have a thorough understanding of construction methods and equipment to develop a design that can be economically built. A knowledge of construction engineering is also necessary when dealing with problems that develop during construction.

This book focuses primarily on the design of foundations, and thus emphasizes the geotechnical and structural engineering aspects. Discussions of construction methods and equipment are generally limited to those aspects that are most important to design engineers. Other aspects which are primarily of interest to contractors, such as scheduling, detailed equipment selection procedures, construction safety, and cost estimating, are beyond the scope of this book.

1.3 UNCERTAINTIES

An unknown structural engineer suggested the following definition for structural engineering:

> Structural engineering is the art and science of molding materials we do not fully understand into shapes we cannot precisely analyze to resist forces we cannot accurately predict, all in such a way that the society at large is given no reason to suspect the extent of our ignorance.

We could apply the same definition, even more emphatically, to foundation engineering. In spite of the many advances in foundation engineering theory, there are still many gaps in our understanding. In general, the greatest uncertainties are the result of our limited knowledge of the soil conditions. Although foundation engineers use various investigation and testing techniques in an attempt to define the soil conditions beneath the site of a proposed foundation, even the most thorough investigation program encounters only a small portion of the soils and relies heavily on interpolation and extrapolation.

Limitations in our understanding of the interaction between a foundation and the soil also introduce uncertainties. For example, how does side friction resistance develop along the surface of a pile? How does the installation of a pile affect the engineering properties of the adjacent soils? These and other questions are the subjects of continued research.

It also is difficult to predict the actual service loads that will act on a foundation, especially live loads. Design values, such as those that appear in building codes, are usually conservative.

Because of these and other uncertainties, the wise engineer does not blindly follow the results of tests or analyses. These tests and analyses must be tempered with precedent, common sense, and engineering judgment. Foundation engineering is still both an art and a science. It is dangerous to view foundation engineering, or any other type of engineering, as simply a collection of formulas and charts to be followed using some "recipe" for design. This is why it is essential to understand the *behavior* of foundations and the basis and limitations of the analysis methods.

Rationalism and Empiricism

Since we do not fully understand the behavior of foundations, most of our analysis and design methods include a mixture of rational and empirical techniques. Rational techniques are those developed from the principles of physics and engineering science, and are useful ways to describe mechanisms we understand and are able to quantify. Conversely, empirical techniques are based primarily on experimental data and thus are especially helpful when we have a limited understanding of the physical mechanisms.

Methods of analyzing foundation problems often begin as simple rational models with little or no experimental data to validate them, or highly empirical techniques that reflect only the most basic insight into the mechanisms that control the observed behavior. Then, as engineers use these methods we search for experimental data to calibrate the rational methods and discernment to understand the empirical data. These efforts are intended to improve the accuracy of the predictions.

One of the keys to successful foundation engineering is to understand this mix of rationalism and empiricism, the strengths and limitations of each, and how to apply them to practical design problems.

Factors of Safety

In spite of the many uncertainties in foundation analysis and design, the public expects engineers to develop reliable and economical designs in a timely and efficient manner. Therefore, we compensate for these uncertainties by using factors of safety in our designs.

Although it's tempting to think of designs that have a factor of safety greater than some standard value as being "safe" and those with a lower factor of safety as "unsafe," it's better to view these two conditions as having different degrees of reliability or different probabilities of failure. All foundations can fail, but some are more likely to fail than others. The design factor of safety defines the engineer's estimate of the best compromise between cost and reliability. It is based on many factors, including the following:

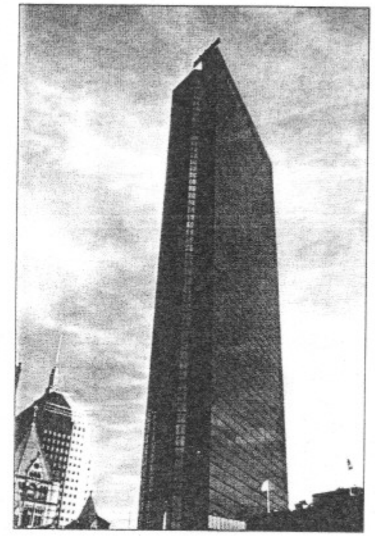
- Required reliability (i.e., the acceptable probability of failure).
- Consequences of a failure.
- Uncertainties in soil properties and applied loads.
- Construction tolerances (i.e., the potential differences between design and as-built dimensions).
- Ignorance of the true behavior of foundations.
- Cost-benefit ratio of additional conservatism in the design.

Factors of safety in foundations are typically greater than those in the superstructure because of the following:

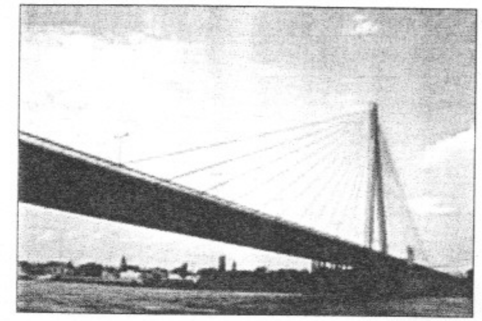
- Extra weight (another consequence of conservatism) in the superstructure increases the loads on members below, thus compounding the cost increases. However, the foundation is the lowest member in a structure, so additional weight does not affect other members. In fact, additional weight may be a benefit because it increases the foundation's uplift capacity.
- Construction tolerances in foundations are wider than those in the superstructure, so the as-built dimensions are often significantly different than the design dimensions.
- Uncertainties in soil properties introduce significantly more risk.
- Foundation failures can be more costly than failure in the superstructure.

Typical design values are presented throughout this book. These values are based primarily on precedent and professional judgement, and have generally provided suitable foundation designs.

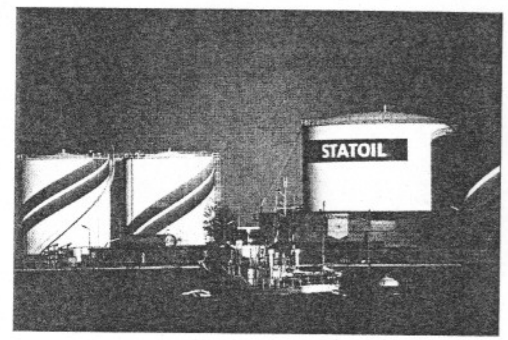
As further research helps us better understand the behavior of foundations and the reliability of our analysis and design methods, design values of the factor of safety will



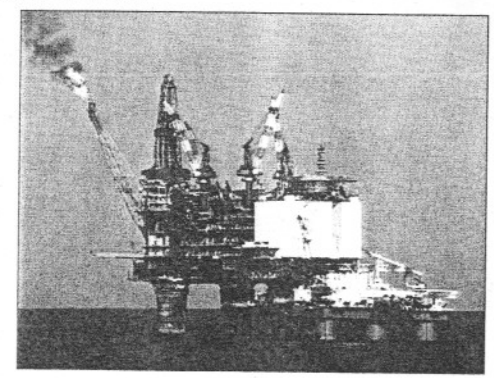
◀ Plate A
The John Hancock Building in Boston



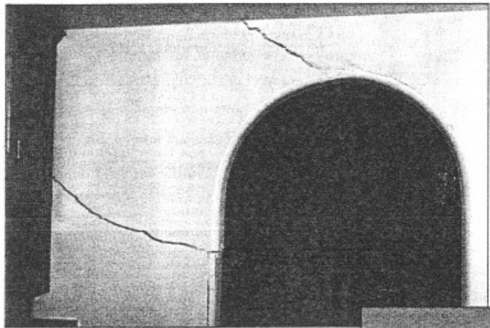
▼ Plate B
A Cable-stayed bridge



▲ Plate C
Petroleum storage tanks

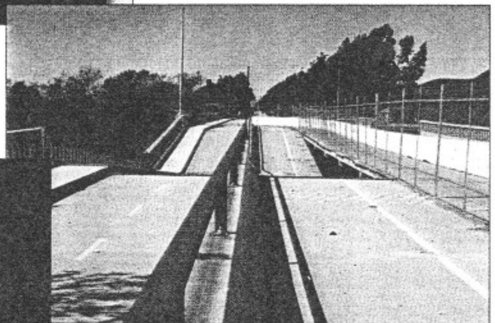


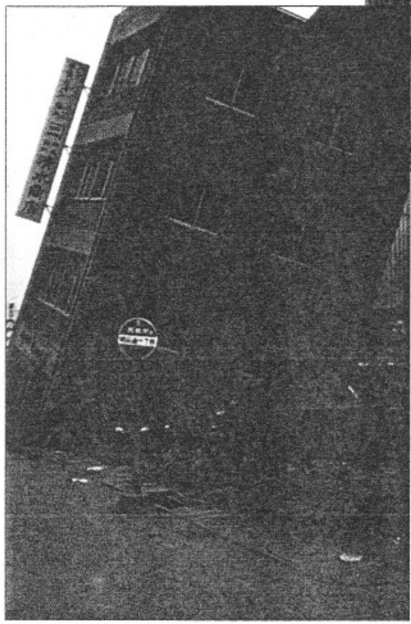
▼ Plate D
An offshore drilling platform



◀ Plate E
Cracks in the interior walls of a house that experienced excessive differential settlements

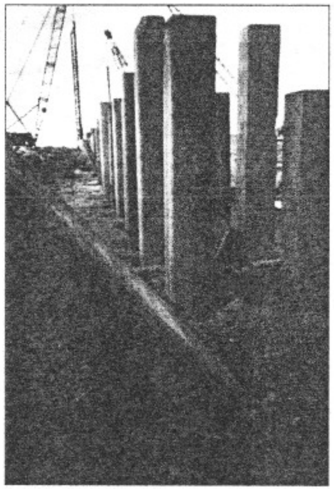
▼ Plate F
A bridge that failed because of scour beneath the foundations

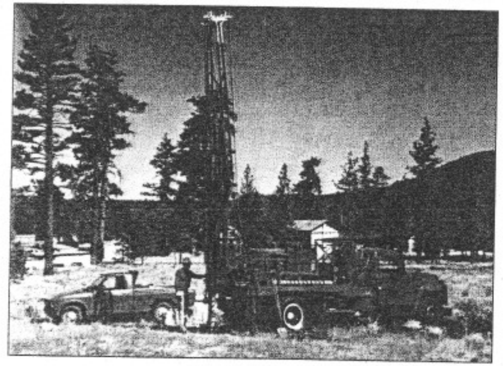




▲ Plate G
Failure of a building due to liquifaction of the underlying soils (*Earthquake Engineering Research Center Library, University of California Berkeley, Steinbrugee Collection*)

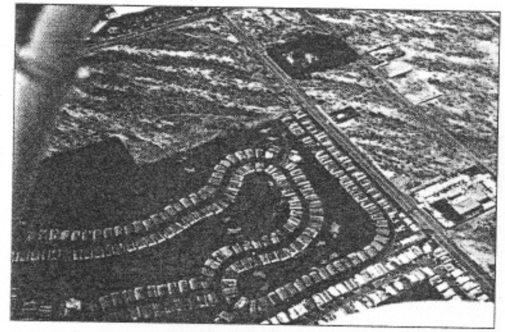
▼ Plate H
Breakage of a pile foundation during construction (*Goble Rausche Likins and Associates, Inc.*)



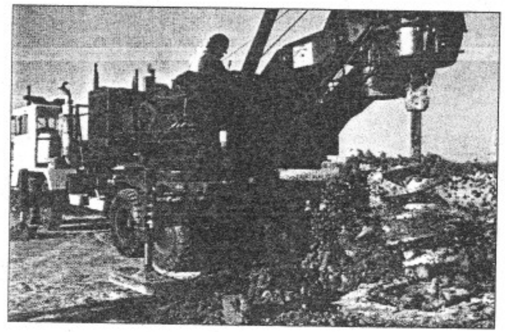


◀ Plate I
Drilling an exploratory boring

▼ Plate J
A golf course community in Palm Springs, California



▼ Plate K
Building a drilled shaft foundation (*ADSC: The International Association of Foundation Drilling*)



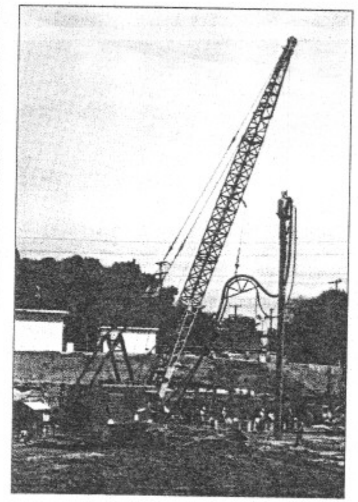
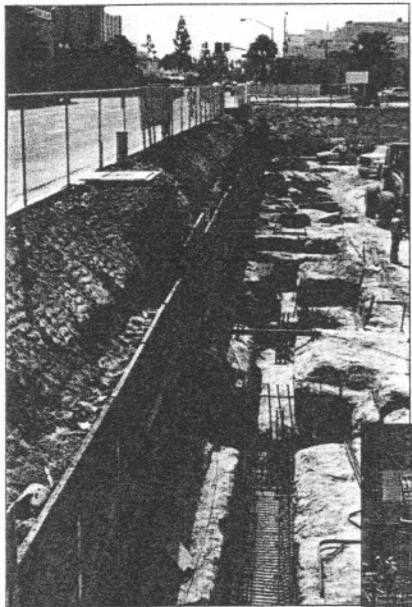
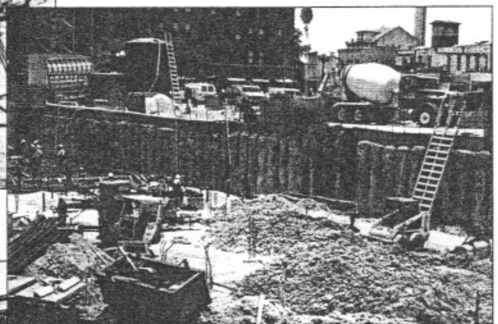


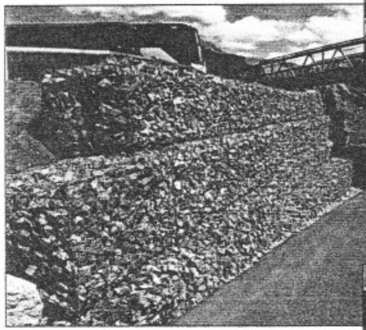
Plate L ▶
Using a vibrofloat to install a stone column



◄ Plate M
A soldier pile wall and a foundation system under construction

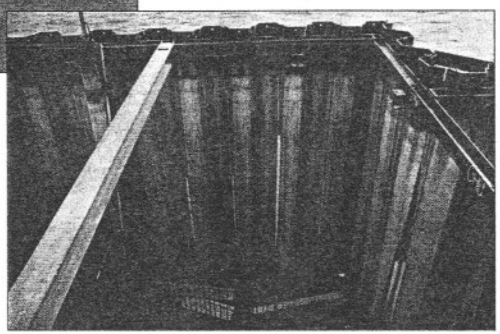
▼ Plate N
A retaining wall made from a series of closely-spaced drilled shafts





▲ Plate P
A gabion wall

▼ Plate O
A cellular cofferdam made of sheet piles



probably be modified accordingly. The introduction of load and resistance factor design (LRFD) into foundation design, which are discussed in Chapters 2 and 21, may provide a vehicle for these reliability assessments.

Accuracy of Computations

Those who are new to the field of foundation engineering often make the mistake of expressing the results of computations using too many significant figures. For example, to claim that the predicted settlement of a spread footing is 12.214 mm suggests an accuracy that is well beyond that which is possible using normal exploration and testing methods. Such practices give a false sense of security.

As a general rule, perform most foundation engineering calculations to three significant figures and express the final results and designs to two significant figures. For example, the settlement figure just quoted would best be stated as 12 mm, keeping in mind that the true precision may be on the order of ±50%.

1.4 BUILDING CODES

Building codes govern the design and construction of nearly all foundations. Although foundation engineering is not as codified as some other areas of civil engineering, we must be familiar with the applicable regulations for a particular project.

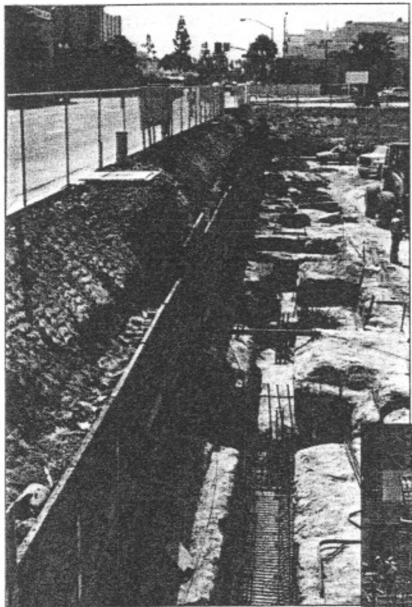
Most foundation construction in the United States, other than highway and railroad bridges, has been governed by one of the three "model" codes:

- The *Uniform Building Code* (ICBO, 1997), which is used primarily in states west of the Mississippi River
- The *National Building Code* (BOCA, 1996), which is used primarily in the midwestern and northeastern states
- The *Standard Building Code* (SBCCI, 1997), which is used primarily in the southeastern states

Some states and cities have adopted their own codes, or use modifications of these model codes to accommodate their own needs and circumstances. For example, the city of New Orleans generally follows the Standard Building Code, but has its own foundation design requirements that reflect the exceptionally poor soil conditions there.

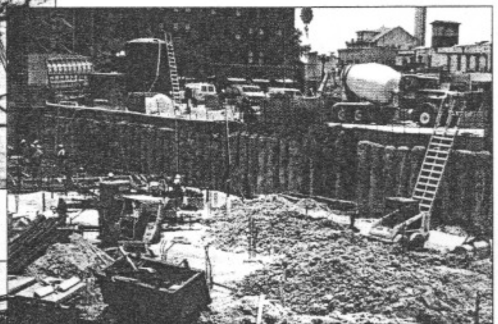
These three model codes have been merged into a single code, the *International Building Code* or *IBC* (ICC, 2000), which is the first true "national" building code in the United States. The ICBO, BOCA, and SBCCI codes will no longer be updated, and will eventually become obsolete as local jurisdictions adopt the IBC.

The model codes and the IBC focus primarily on buildings, towers, tanks, and other similar structures. However, these codes do not address the design of highway bridges because they have substantially different requirements. Most highway bridges in the United States and Canada are designed under the provisions of the *Standard Specifications for*



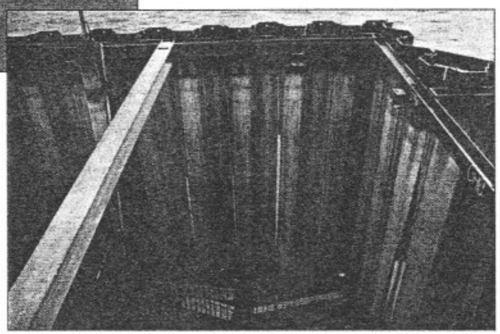
◀ Plate M
A soldier pile wall and a foundation system under construction

▼ Plate N
A retaining wall made from a series of closely-spaced drilled shafts



▲ Plate P
A gabion wall

▼ Plate O
A cellular cofferdam made of sheet piles



probably be modified accordingly. The introduction of load and resistance factor design (LRFD) into foundation design, which are discussed in Chapters 2 and 21, may provide a vehicle for these reliability assessments.

Accuracy of Computations

Those who are new to the field of foundation engineering often make the mistake of expressing the results of computations using too many significant figures. For example, to claim that the predicted settlement of a spread footing is 12.214 mm suggests an accuracy that is well beyond that which is possible using normal exploration and testing methods. Such practices give a false sense of security.

As a general rule, perform most foundation engineering calculations to three significant figures and express the final results and designs to two significant figures. For example, the settlement figure just quoted would best be stated as 12 mm, keeping in mind that the true precision may be on the order of ±50%.

1.4 BUILDING CODES

Building codes govern the design and construction of nearly all foundations. Although foundation engineering is not as codified as some other areas of civil engineering, we must be familiar with the applicable regulations for a particular project.

Most foundation construction in the United States, other than highway and railroad bridges, has been governed by one of the three "model" codes:

- The *Uniform Building Code* (ICBO, 1997), which is used primarily in states west of the Mississippi River
- The *National Building Code* (BOCA, 1996), which is used primarily in the midwestern and northeastern states
- The *Standard Building Code* (SBCCI, 1997), which is used primarily in the southeastern states

Some states and cities have adopted their own codes, or use modifications of these model codes to accommodate their own needs and circumstances. For example, the city of New Orleans generally follows the Standard Building Code, but has its own foundation design requirements that reflect the exceptionally poor soil conditions there.

These three model codes have been merged into a single code, the *International Building Code* or *IBC* (ICC, 2000), which is the first true "national" building code in the United States. The ICBO, BOCA, and SBCCI codes will no longer be updated, and will eventually become obsolete as local jurisdictions adopt the IBC.

The model codes and the IBC focus primarily on buildings, towers, tanks, and other similar structures. However, these codes do not address the design of highway bridges because they have substantially different requirements. Most highway bridges in the United States and Canada are designed under the provisions of the *Standard Specifications for*

Foundation Design

Principles and Practices

Second Edition

Donald P. Coduto

Professor of Civil Engineering
California State Polytechnic University, Pomona

Prentice Hall
Upper Saddle River, New Jersey 07458
http://www.prenhall.com

Library of Congress Cataloging-in-Publication Data
Coduto, Donald P.
Foundation design : principles and practices / Donald P. Coduto.—2nd ed.
p. cm.
ISBN 0-13-589706-8
1. Foundations. I. Title.

TA775.C63 2001
624.1'5—dc21
00-038553

Vice president and editorial director: *Marcia Horton*
Acquisitions editor: *Laura Curless*
Editorial assistant: *Dolores Mars*
Senior marketing manager: *Danny Hoyt*
Production editor: *Pine Tree Composition*
Executive managing editor: *Vince O'Brien*
Managing editor: *David A. George*
Art director: *Jayne Conte*
Art editor: *Adam Velthaus*
Cover designer: *Bruce Kenselaar*
Manufacturing manager: *Trudy Pisciotti*
Manufacturing buyer: *Dawn Murrin*
Vice president and director of production and manufacturing: *David W. Riccardi*

Reprinted with corrections January, 2001

Printed in the United States of America
10 9 8 7 6 5 4

ISBN 0-13-589706-8

Prentice-Hall International (UK) Limited, *London*
Prentice Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Pearson Education Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

Contents

Preface

Foundation Design: Principles and Practices is primarily intended for use as a textbook in undergraduate and graduate-level foundation engineering courses. It also serves well as a reference book for practicing engineers. As the title infers, this book covers both "principles" (the fundamentals of foundation engineering) and "practices" (the application of these principles to practical engineering problems). Readers should have already completed at least one university-level course in soil mechanics, and should have had at least an introduction to structural engineering.

This second edition contains many improvements and enhancements. These have been the result of comments and suggestions from those who used the first edition, my own experience using it at Cal Poly Pomona, and recent advances in the state-of-the-art. The chapters on deep foundations have been completely reorganized and rewritten, and new chapters on reliability-based design and sheet pile walls have been added. Extraneous material has been eliminated, and certain analysis methods have been clarified and simplified. The manuscript was extensively tested in the classroom before going to press. This classroom testing allowed me to evaluate and refine the text, the example problems, the homework problems, and the software.

Key features of this book include:

- Integration with *Geotechnical Engineering: Principles and Practices* (Coduto, 1999), including consistent notation, terminology, analysis methods, and coordinated development of topics. However, readers who were introduced to geotechnical engineering using another text can easily transition to this book by reviewing the material in Chapters 3 and 4.
- Consideration of the geotechnical, structural, and construction engineering aspects of the design process, including emphasis on the roles of each discipline and the interrelationships between them.
- Frequent discussions of the sources and approximate magnitudes of uncertainties, along with comparisons of predicted and actual behavior.
- Use of both English and SI units, because engineers in North America and many other parts of the world need to be conversant in both systems.

- Integration of newly-developed Excel spreadsheets for foundation analysis and design. These spreadsheet files may be downloaded from the Prentice Hall website (www.prenhall.com/coduto). They are introduced only after the reader learns how to perform the analyses by manual computations.
- Extensive use of example problems, many of which are new to this edition.
- Inclusion of carefully developed homework problems distributed throughout the chapters, with comprehensive problems at the end of each chapter. Many of these problems are new or revised.
- Discussions of recent advances in foundation engineering, including Statnamic testing, load and resistance factor design (LRFD), and applications of the cone penetration test (CPT).
- Inclusion of extensive bibliographic references for those wishing to study certain topics in more detail.
- An instructor's manual is available to faculty. It may be obtained from your Prentice Hall campus representative.

ACKNOWLEDGMENTS

Many friends, colleagues, and other professionals contributed to this work. Much of the book is the product of their stimulating discussions, constructive reviews, and support. Professor Joseph Caliendo of Utah State University was especially helpful. Stanley Vitton (Michigan Technological University), Samuel Clemence (Syracuse University), Richard Handy (Iowa State University), Raymond Moore (University of Nebraska), John Horvath (Manhattan College), José Pires (University of California, Irvine), Paul Chan (New Jersey Institute of Technology), William and Sandra Houston (Arizona State University), and others reviewed part or all of the manuscripts for the first or second editions and provided many useful suggestions and comments. Iraj Noorany (San Diego State University), Michael O'Neill (University of Houston), Major William Kitch (U.S. Air Force Academy), James Olson (University of Vermont), Samuel Paikowsky (University of Massachusetts, Lowell), Richard W. Stephenson (University of Missouri, Rolla), William Kovacs (University of Rhode Island), Dan Burgess, Rick Drake (Fluor Daniel), Bengt Fellenius (Urkkada Technology), Mohamad Hussein (Goble, Rausche, Likins), and others also provided useful suggestions and advice.

A special note of thanks goes to the foundation engineering students at Cal Poly University who used various draft manuscripts of this book as a makeshift text. Their constructive comments and suggestions have made this book much more useful, and their proofreading has helped eliminate mistakes.

I welcome any constructive comments and suggestions from those who use this book. Please mail them to me at the Civil Engineering Department, Cal Poly University, Pomona, CA 91768.

Donald P. Coduto

Notation and Units of Measurement

There is no universally accepted notation in foundation engineering. However, the notation used in this book, as described in the following table, is generally consistent with popular usage.

Symbol	Description	Typical Units English	Typical Units SI	Defined on Page
A	Cross-sectional area	ft^2	m^2	438
A	Base area of foundation	ft^2	m^2	155
A_0	Initial cross-sectional area	in^2	mm^2	95
A_1	Cross-sectional area of column	in^2	mm^2	337
A_2	Base area of frustum	in^2	mm^2	337
A_b	Area of bottom of enlarged base	ft^2	m^2	528
A_f	Cross-sectional area at failure	in^2	mm^2	95
A_s	Steel area	in^2	mm^2	323
a_0	Factor in N_q equation	Unitless	Unitless	178
B	Width of foundation	ft-in	mm	146
B'	Effective foundation width	ft-in	m	275
B_b	Diameter at base of foundation	ft	m	548
B_s	Diameter of shaft	ft	m	547
b	Unit length	ft	m	156
b_c, b_q, b_γ	Base inclination factors	Unitless	Unitless	186
b_0	Length of critical shear surface	in	mm	310
C_1	Depth factor	Unitless	Unitless	235
C_2	Secondary creep factor	Unitless	Unitless	235
C_3	Shape factor	Unitless	Unitless	235
C_A	Aging factor	Unitless	Unitless	122
C_B	SPT borehold diameter correction	Unitless	Unitless	119
C_c	Compression index	Unitless	Unitless	66

C_{OCR}	Overconsolidation correction factor	Unitless	Unitless	122
C_P	Grain size correction factor	Unitless	Unitless	122
$C_{p\phi}$	Passive pressure factor	Unitless	Unitless	602
C_R	SPT rod length correction	Unitless	Unitless	119
C_r	Recompression index	Unitless	Unitless	67
C_s	SPT sampler correction	Unitless	Unitless	119
C_s	Side friction coefficient	Unitless	Unitless	535
C_t	Toe coefficient	Unitless	Unitless	534
C_w	Hydroconsolidation coefficient	Unitless	Unitless	709
c	Wave velocity in pile	ft/s	m/s	571
c	Column or wall width	in	mm	302
c'	Effective cohesion	lb/ft^2	kPa	84
c'_{adj}	Adjusted effective cohesion	lb/ft^2	kPa	198
c_T	Total cohesion	lb/ft^2	kPa	85
D	Depth of foundation	ft-in	mm or m	146
D_{50}	Grain size at which 50% is finer	Unitless	mm	122
D_{min}	Minimun required embedment depth	ft	m	593
D_r	Relative density	percent	percent	51
D_w	Depth from ground surface to groundwater table	ft	m	188
d	Effective depth	in	mm	306
d	Bolt diameter	in	mm	344
d	Vane diameter	in	mm	131
d_b	Reinforcing bar diameter	in	mm	306
d_c, d_q, d_γ	Depth factors	Unitless	Unitless	184
E	Portion of steel in center section	Unitless	Unitless	333
E	Modulus of elasticity	lb/in^2	MPa	231
E_m	SPT hammer efficiency	Unitless	Unitless	119
E_s	Equivalent modulus of elasticity	lb/ft^2	kPa	231
E_u	Undrained modulus of elasticity	lb/ft^2	kPa	226
EI	Expansion index	Unitless	Unitless	673
e	Eccentricity	ft	m	159
e	Void ratio	Unitless	Unitless	49
e	Base of natural logarithms	2.7183	2.7183	XXX
e_0	Initial void ratio	Unitless	Unitless	66
e_B	Eccentricity in the B direction	ft	m	165
e_L	Eccentricity in the L direction	ft	m	165
e_{max}	Maximum void ratio	Unitless	Unitless	51
e_{min}	Minimum void ratio	Unitless	Unitless	51
F	Factor of Safety	Unitless	Unitless	190
F_a	Allowable axial stress	lb/in^2	MPa	439

F_b	Allowable flexural stress	lb/in^2	MPa	439
F_v	Allowable shear stress	lb/in^2	MPa	439
f_a	Average normal stress due to axial load	lb/in^2	MPa	438
f_b	Normal stress in extreme fiber due to flexural load	lb/in^2	MPa	438
f_c'	28-day compressive strength of concrete	lb/in^2	MPa	303
f_{pe}	Effective prestress on gross section	lb/in^2	MPa	448
f_s	Unit side friction resistance	lb/ft^2	kPa	513
$(f_s)_m$	Mobilized unit side-friction resistance	lb/ft^2	kPa	544
f_{sc}	CPT cone side friction	T/ft^2	MPa or kg/cm^2	124
f_v	Shear stress	lb/in^2	MPa	439
f_y	Yield strength of steel	lb/in^2	MPa	303
G_h	Horizontal equivalent fluid density	lb/ft^3	kN/m^3	770
G_s	Specific gravity of solids	Unitless	Unitless	49
G_v	Vertical equivalent fluid density	lb/ft^3	kN/m^3	771
g_c, g_q, g_γ	Ground inclination factors	Unitless	Unitless	186
H	Thickness of soil stratum	ft	m	60
H	Wall height	ft	m	759
H_c	Critical height	ft	m	767
H_{fill}	Thickness of fill	ft	m	64
I_1, I_2	Influence factors	Unitless	Unitless	226
I_ϵ	Strain influence factor	Unitless	Unitless	234
I_p	Plasticity index	Unitless	Unitless	56
I_r	Rigidity index	Unitless	Unitless	501
i_c, i_q, i_γ	Load inclination factors	Unitless	Unitless	185
I_σ	Stress influence factor	Unitless	Unitless	210
K	Coefficient of lateral earth pressure	Unitless	Unitless	61
K_a	Coefficient of active earth pressure	Unitless	Unitless	760
K_p	Coefficient of passive earth pressure	Unitless	Unitless	762
k	Factor in computing depth factors	Unitless	Unitless	184
k_s	Coefficient of subgrade reaction	lb/in^3	kN/m^3	356
L	Length of foundation	ft-in	mm	146
L'	Effective foundation length	ft-in	m	275
LL	Liquid limit (see w_L)	Unitless	Unitless	54
l	Cantilever distance	in	mm	322
l_d	Development length	in	mm	318
l_{dh}	Development length for hook	in	mm	337
M	Moment load	ft-k	kN-m	15
M_c	Characteristic moment load	ft-lb	kN-m	601
M_D	Driving moment	ft-lb	kN-m	796

u	Displacement of pile	in	mm	564
u	Pore water pressure	lb/ft^2	kPa	59
u_2	Pore water pressure behind cone point	lb/ft^2	kPa	533
u_D	Pore water pressure at bottom of foundation	lb/ft^2	kPa	155
u_e	Excess pore water pressure	lb/ft^2	kPa	59
u_h	Hydrostatic pore water pressure	lb/ft^2	kPa	58
V	Shear load	k	kN	15
V_a	Shear force under active condition	k	kN	759
V_c	Nominal shear capacity of concrete	lb	kN	309
V_c	Characteristic shear load	lb	kN	601
V_{fa}	Allowable footing shear load capacity	k	kN	276
V_n	Nominal shear load capacity	k	kN	21
V_{nc}	Nominal shear capacity on critical surface	lb	kN	309
V_p	Shear force under passive condition	k	kN	159
V_s	Nominal shear capacity of reinforcing steel	lb	kN	309
V_u	Factored shear load	k	kN	21
V_{uc}	Factored shear force on critical surface	lb	kN	309
W_f	Weight of foundation	lb	kN	154
W_r	Hammer ram weight	lb	kN	560
w	Moisture content	percent	percent	49
w_L	Liquid limit	Unitless	Unitless	54
w_P	Plastic limit	Unitless	Unitless	54
w_s	Shrinkage limit	Unitless	Unitless	54
y	Lateral deflection	in	mm	598
z	Depth below ground surface	ft	m	564
z_c	Depth to centroid of soil resistance	ft	m	544
z_f	Depth below to bottom of foundation	ft	m	210
z_I	Depth to imaginary footing	ft	m	552
z_w	Depth below to groundwater table	ft	m	58
α	Wetting coefficient	Unitless	Unitless	678
α	Adhesion factor	Unitless	Unitless	522
α	Slope of footing bottom	deg	deg	183
α	Inclination of wall from vertical	deg	deg	764
β	Side friction factor in β method	Unitless	Unitless	516
β	Slope of ground surface	deg	deg	759
β	Reliability index	Unitless	Unitless	724
β_0, β_1	Correlation factors	Unitless	Unitless	233
γ	Ratio of steel cage diameter to drilled shaft diameter	Unitless	Unitless	455
γ	Unit weight	lb/ft^3	kN/m^3	49
γ	Load Factor	Unitless	Unitless	21

γ_b	Buoyant unit weight	lb/ft^3	kN/m^3	49
γ_d	Dry unit weight	lb/ft^3	kN/m^3	49
γ_{fill}	Unit weight of fill	lb/ft^3	kN/m^3	64
γ_w	Unit weight of water	lb/ft^3	kN/m^3	49
γ'	Effective unit weight	lb/ft^3	kN/m^3	188
$\Delta\sigma_z$	Change in vertical stress	lb/ft^2	kPa	64
δ	Total settlement	in	mm	29
δ_a	Allowable total settlement	in	mm	29
δ_c	Consolidation settlement	in	mm	72
δ_D	Differential settlement	in	mm	31
δ_{Da}	Allowable differential settlement	in	mm	31
δ_d	Distortion settlement	in	mm	224
δ_e	Settlement due to elastic compression	in	mm	544
δ_u	Settlement required to mobilize ultimate resistance	in	mm	544
δ_w	Heave or settlement due to wetting	in	mm	680
ϵ_{50}	Axial strain at which 50 percent of the soil strength is mobilized	Unitless	Unitless	603
ϵ_f	Strain at failure	Unitless	Unitless	95
ϵ_w	Strain due to wetting	Unitless	Unitless	676
η	Factor in Shields' chart	Unitless	Unitless	286
η	Group efficiency factor	Unitless	Unitless	538
θ	Factor in Converse-Labarre formula	Unitless	Unitless	539
θ_a	Allowable angular distortion	radians	radians	33
λ	Lightweight concrete factor	Unitless	Unitless	319
λ	Factor in Shields' chart	Unitless	Unitless	286
λ	Vane shear correction factor	Unitless	Unitless	131
λ	Equivalent passive fluid density	lb/ft^3	kN/m^3	276
λ	Factor in Evans and Duncan's charts	Unitless	Unitless	602
λ_a	Allowable equivalent passive fluid density	lb/ft^3	kN/m^3	276
μ	Coefficient of friction	Unitless	Unitless	276
μ_a	Allowable coefficient of friction	Unitless	Unitless	276
μ_C	Mean ultimate capacity	k	kN	723
μ_L	Mean load	k	kN	723
υ	Poisson's ratio	Unitless	Unitless	502
ρ	Mass density	lb_m/ft^3	kg/m^3	564
ρ	Steel ratio	Unitless	Unitless	317
ρ_s	Ratio of volume of spiral reinforcement to total volume of core	Unitless	Unitless	459
σ	Total stress	lb/ft^2	kPa	60
σ	Normal pressure imparted on wall from soil	lb/ft^2	kPa	760

σ'	Effective stress	lb/ft^2	kPa	60
σ_C	Standard deviation of ultimate capacity	k	kN	724
σ_c'	Preconsolidation stress	lb/ft^2	kPa	67
σ_{hs}	Horizontal swelling pressure	lb/ft^2	kPa	691
σ_L	Standard deviation of load	k	kN	727
σ_m'	Preconsolidation margin	lb/ft^2	kPa	69
σ_p	Representative passive pressure	lb/ft^2	kPa	602
σ_t	Threshold collapse stress	lb/ft^2	kPa	708
σ_x	Horizontal total stress	lb/ft^2	kPa	61
σ_x'	Horizontal effective stress	lb/ft^2	kPa	61
σ_z	Vertical total stress	lb/ft^2	kPa	60
σ_z'	Vertical effective stress	lb/ft^2	kPa	60
σ_{z0}'	Initial vertical effective stress	lb/ft^2	kPa	64
σ_{zD}'	Effective stress at depth D below the ground surface	lb/ft^2	kPa	178
σ_{zD}	Total stress at depth D below the ground surface	lb/ft^2	kPa	175
σ_{zf}'	Final effective stress	lb/ft^2	kPa	64
σ_{zp}'	Initial vertical effective stress at depth of peak strain influence factor	lb/ft^2	kPa	234
τ	Shear stress imparted on wall from soil	lb/ft^2	kPa	760
ϕ	Resistance factor	Unitless	Unitless	21
ϕ'	Effective friction angle	deg	deg	82
ϕ'_{adj}	Adjusted effective friction angle	deg	deg	198
ϕ_T	Total friction angle	deg	deg	85
ϕ_w	Wall-soil interface friction angle	deg	deg	763
Ψ	Three dimensional adjustment factor	Unitless	Unitless	225
Ψ	Factor in Shields' chart	Unitless	Unitless	286

Foundation Design

1

Foundations in Civil Engineering

THE PARABLE OF THE WISE AND FOOLISH BUILDERS

I will show you what he is like who comes to me and hears my words and puts them into practice. He is like a man building a house, who dug down deep and laid the foundation on rock. When a flood came, the torrent struck that house but could not shake it, because it was well built. But the one who hears my words and does not put them into practice is like a man who built a house on the ground without a foundation. The moment the torrent struck that house, it collapsed and its destruction was complete.

Luke 6:47–49 NIV (circa AD 60)

A wise engineer once said "A structure is no stronger than its connections." Although this statement usually invokes images of connections between individual structural members, it also applies to those between a structure and the ground that supports it. These connections are known as its *foundations*. Even the ancient builders knew that the most carefully designed structures can fail if they are not supported by suitable foundations. The Tower of Pisa in Italy (perhaps the world's most successful foundation "failure") reminds us of this truth.

Although builders have recognized the importance of firm foundations for countless generations, and the history of foundation construction extends for thousands of years, the discipline of *foundation engineering* as we know it today did not begin to develop until the late nineteenth century.

1.1 THE EMERGENCE OF MODERN FOUNDATION ENGINEERING

Early foundation designs were based solely on precedent, intuition, and common sense. Through trial-and-error, builders developed rules for sizing and constructing foundations. For example, load-bearing masonry walls built on compact gravel in New York City during the nineteenth century were supported on spread footings that had a width 1.5 times that of the wall. Those built on sand or stiff clay were three times the width of the wall (Powell, 1884).

These empirical rules usually produced acceptable results as long as they were applied to structures and soil conditions similar to those encountered in the past. However, the results were often disastrous when builders extrapolated the rules to new conditions. This problem became especially troublesome when new methods of building construction began to appear during the late nineteenth century. The introduction of steel and reinforced concrete led to a transition away from rigid masonry structures to more flexible frame structures. These new materials also permitted buildings to be taller and heavier than before. In addition, as good sites became occupied, builders were forced to consider sites with poorer soil conditions, and these sites made foundation design and construction much more difficult. Thus, the old rules for foundation design no longer applied.

The Eiffel Tower

The Eiffel Tower in Paris is an excellent example of a new type of structure in which the old rules for foundations no longer applied. It was originally built for the Paris Universal Exposition of 1889 and was the tallest structure in the world. Alexandre Gustave Eiffel, the designer and builder, was very conscious of the need for adequate foundations, and clearly did not want to create another Leaning Tower of Pisa (Kerisel, 1987).

The Eiffel Tower is adjacent to the Seine River, and is underlain by difficult soil conditions, including uncompacted fill and soft alluvial soils. Piers for the nearby Alma bridge, which were founded in this alluvium, had already settled nearly 1 m. The tower could not tolerate such settlements.

Eiffel began exploring the subsurface conditions using the crude drilling equipment of the time, but was not satisfied with the results. He wrote: "What conclusions could one reasonably base on the examination of a few cubic decimeters of excavated soil, more often than not diluted by water, and brought to the surface by the scoop?" (Kerisel, 1987). Therefore, he devised a new means of exploring the soils, which consisted of driving a 200-mm diameter pipe filled with compressed air. The air kept groundwater from entering the tube, and thus permitted recovery of higher quality samples.

Eiffel's studies revealed that the two legs of the tower closest to the Seine were underlain by deeper and softer alluvium, and were immediately adjacent to an old river channel that had filled with soft silt. The foundation design had to accommodate these soil conditions, or else the two legs on the softer soils would settle more than the other two, causing the tower to tilt toward the river.

Figure 1.1 Two legs of the Eiffel Tower in Paris are underlain by softer soils, and thus could have settled more than the other two. Fortunately, Eiffel carefully explored the soil conditions, recognized this problem, and designed the foundations to accommodate these soil conditions. His foresight and diligence resulted in a well-designed foundation system that has not settled excessively.

Based on his study of the soil conditions, Eiffel placed the foundations for the two legs furthest from the river on the shallow but firm alluvial soils. The bottom of these foundations were above the groundwater table, so their construction proceeded easily. However, he made the foundations for the other two legs much deeper so they too were founded on firm soils. This required excavating about 12 m below the ground surface (6 m below the groundwater table). As a result of Eiffel's diligence, the foundations have safely supported the tower for over one hundred years, and have not experienced excessive differential settlements.

Further Developments

As structures continued to become larger and heavier, engineers continued to learn more about foundation design and construction. Instead of simply developing new empirical rules, they began to investigate the behavior of foundations and develop more rational methods of design, thus establishing the discipline of foundation engineering.

A significant advance came in 1873 when Frederick Baumann, a Chicago architect, published the pamphlet *The Art of Preparing Foundations, with Particular Illustration of* *the "Method of Isolated Piers" as Followed in Chicago* (Baumann, 1873). He appears to be the first to recommend that the base area of a foundation should be proportional to the applied load, and that the loads should act concentrically upon the foundation. He also gave allowable bearing pressures for Chicago soils and specified tolerable limits for total and differential settlements.

The growth of geotechnical engineering, which began in earnest during the 1920s, provided a better theoretical base for foundation engineering. It also provided improved methods of exploring and testing soil and rock. These developments continued throughout the twentieth century. Many new methods of foundation construction also have been developed, making it possible to build foundations at sites where construction had previously been impossible or impractical.

Today, our knowledge of foundation design and construction is much better than it was one hundred years ago. It is now possible to build reliable, cost-effective, high-capacity foundations for all types of modern structures.

1.2 THE FOUNDATION ENGINEER

Foundation engineering does not fit completely within any of the traditional civil engineering subdisciplines. Instead, the foundation engineer must be multidisciplinary and possess a working knowledge in each of the following areas:

- **Structural engineering**—A foundation is a structural member that must be capable of transmitting the applied loads, so we must also understand the principles and practices of structural engineering. In addition, the foundation supports a structure, so we must understand the sources and nature of structural loads and the structure's tolerance of foundation movements.
- **Geotechnical engineering**—All foundations interact with the ground, so the design must reflect the engineering properties and behavior of the adjacent soil and rock. Thus, the foundation engineer must understand geotechnical engineering. Most foundation engineers also consider themselves to be geotechnical engineers.
- **Construction engineering**—Finally, foundations must be built. Although the actual construction is performed by contractors and construction engineers, it is very important for the design engineer to have a thorough understanding of construction methods and equipment to develop a design that can be economically built. A knowledge of construction engineering is also necessary when dealing with problems that develop during construction.

This book focuses primarily on the design of foundations, and thus emphasizes the geotechnical and structural engineering aspects. Discussions of construction methods and equipment are generally limited to those aspects that are most important to design engineers. Other aspects which are primarily of interest to contractors, such as scheduling, detailed equipment selection procedures, construction safety, and cost estimating, are beyond the scope of this book.

1.3 UNCERTAINTIES

An unknown structural engineer suggested the following definition for structural engineering:

> Structural engineering is the art and science of molding materials we do not fully understand into shapes we cannot precisely analyze to resist forces we cannot accurately predict, all in such a way that the society at large is given no reason to suspect the extent of our ignorance.

We could apply the same definition, even more emphatically, to foundation engineering. In spite of the many advances in foundation engineering theory, there are still many gaps in our understanding. In general, the greatest uncertainties are the result of our limited knowledge of the soil conditions. Although foundation engineers use various investigation and testing techniques in an attempt to define the soil conditions beneath the site of a proposed foundation, even the most thorough investigation program encounters only a small portion of the soils and relies heavily on interpolation and extrapolation.

Limitations in our understanding of the interaction between a foundation and the soil also introduce uncertainties. For example, how does side friction resistance develop along the surface of a pile? How does the installation of a pile affect the engineering properties of the adjacent soils? These and other questions are the subjects of continued research.

It also is difficult to predict the actual service loads that will act on a foundation, especially live loads. Design values, such as those that appear in building codes, are usually conservative.

Because of these and other uncertainties, the wise engineer does not blindly follow the results of tests or analyses. These tests and analyses must be tempered with precedent, common sense, and engineering judgment. Foundation engineering is still both an art and a science. It is dangerous to view foundation engineering, or any other type of engineering, as simply a collection of formulas and charts to be followed using some "recipe" for design. This is why it is essential to understand the *behavior* of foundations and the basis and limitations of the analysis methods.

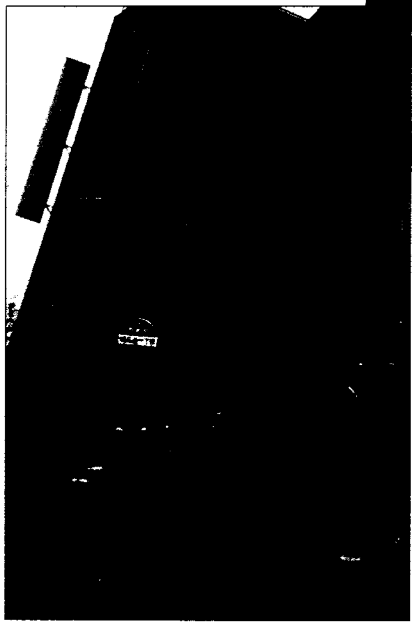
Rationalism and Empiricism

Since we do not fully understand the behavior of foundations, most of our analysis and design methods include a mixture of rational and empirical techniques. Rational techniques are those developed from the principles of physics and engineering science, and are useful ways to describe mechanisms we understand and are able to quantify. Conversely, empirical techniques are based primarily on experimental data and thus are especially helpful when we have a limited understanding of the physical mechanisms.

Methods of analyzing foundation problems often begin as simple rational models with little or no experimental data to validate them, or highly empirical techniques that reflect only the most basic insight into the mechanisms that control the observed behavior. Then, as engineers use these methods we search for experimental data to calibrate the ra-

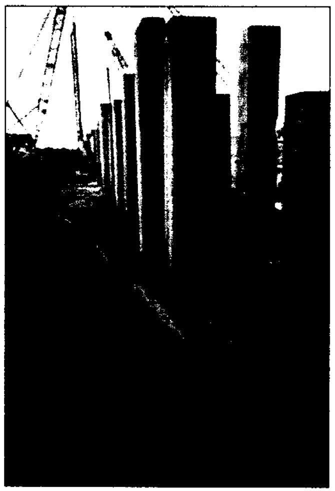
◀ Plate E
Cracks in the interior walls of a house that experienced excessive differential settlements

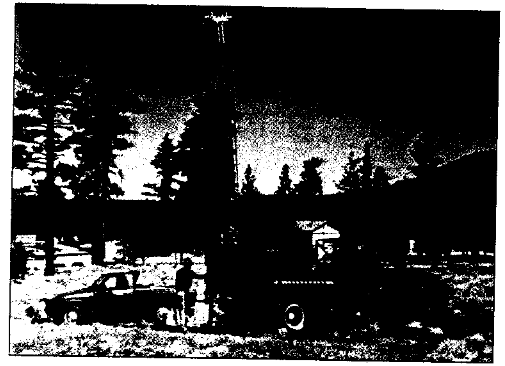
▼ Plate F
A bridge that failed because of scour beneath the foundations



▲ Plate G
Failure of a building due to liquifaction of the underlying soils (*Earthquake Engineering Research Center Library, University of California Berkeley, Steinbrugee Collection*)

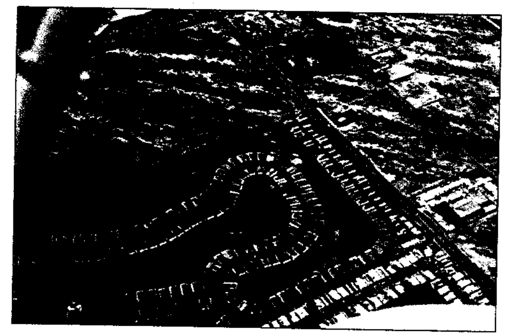
▼ Plate H
Breakage of a pile foundation during construction (*Goble Rausche Likins and Associates, Inc.*)





◀ Plate I
Drilling an exploratory boring

▼ Plate J
A golf course community in Palm Springs, California



▼ Plate K
Building a drilled shaft foundation (*ADSC: The International Association of Foundation Drilling*)

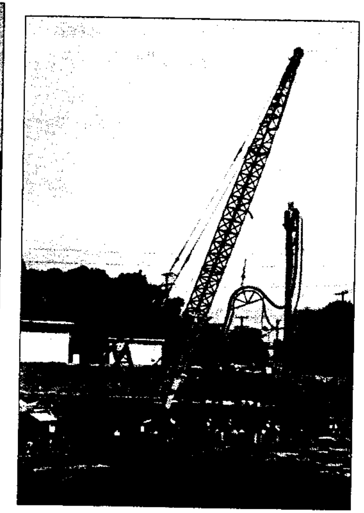


Plate L ▶
Using a vibrofloat to install a stone column

tional methods and discernment to understand the empirical data. These efforts are intended to improve the accuracy of the predictions.

One of the keys to successful foundation engineering is to understand this mix of rationalism and empiricism, the strengths and limitations of each, and how to apply them to practical design problems.

Factors of Safety

In spite of the many uncertainties in foundation analysis and design, the public expects engineers to develop reliable and economical designs in a timely and efficient manner. Therefore, we compensate for these uncertainties by using factors of safety in our designs.

Although it's tempting to think of designs that have a factor of safety greater than some standard value as being "safe" and those with a lower factor of safety as "unsafe," it's better to view these two conditions as having different degrees of reliability or different probabilities of failure. All foundations can fail, but some are more likely to fail than others. The design factor of safety defines the engineer's estimate of the best compromise between cost and reliability. It is based on many factors, including the following:

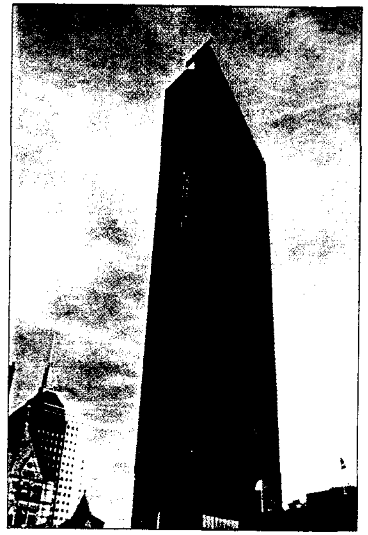
- Required reliability (i.e., the acceptable probability of failure).
- Consequences of a failure.
- Uncertainties in soil properties and applied loads.
- Construction tolerances (i.e., the potential differences between design and as-built dimensions).
- Ignorance of the true behavior of foundations.
- Cost-benefit ratio of additional conservatism in the design.

Factors of safety in foundations are typically greater than those in the superstructure because of the following:

- Extra weight (another consequence of conservatism) in the superstructure increases the loads on members below, thus compounding the cost increases. However, the foundation is the lowest member in a structure, so additional weight does not affect other members. In fact, additional weight may be a benefit because it increases the foundation's uplift capacity.
- Construction tolerances in foundations are wider than those in the superstructure, so the as-built dimensions are often significantly different than the design dimensions.
- Uncertainties in soil properties introduce significantly more risk.
- Foundation failures can be more costly than failure in the superstructure.

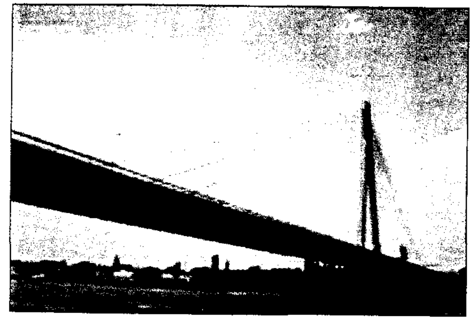
Typical design values are presented throughout this book. These values are based primarily on precedent and professional judgement, and have generally provided suitable foundation designs.

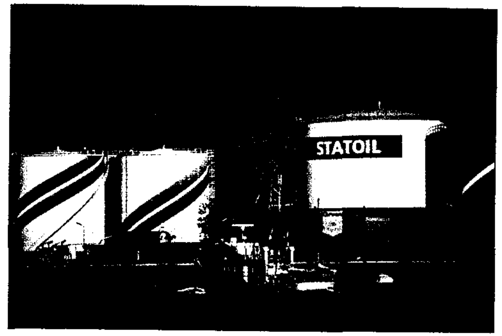
As further research helps us better understand the behavior of foundations and the reliability of our analysis and design methods, design values of the factor of safety will



◀ Plate A
The John Hancock Building in Boston

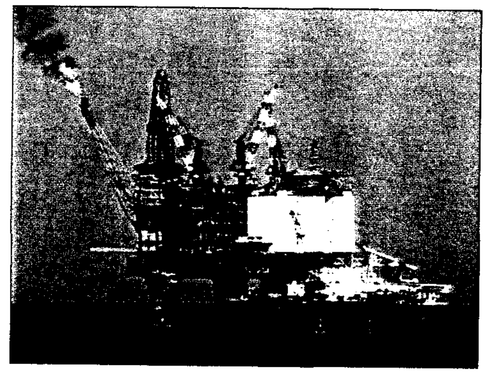
▼ Plate B
A Cable-stayed bridge





▲ Plate C
Petroleum storage tanks

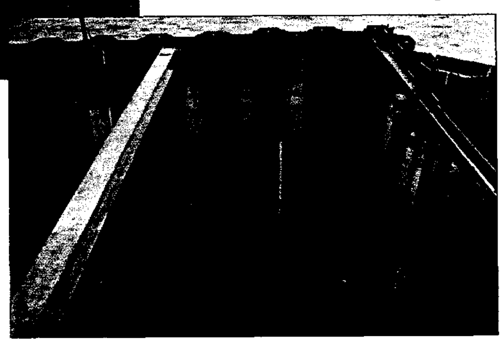
▼ Plate D
An offshore drilling platform



◄ Plate M
A soldier pile wall and a foundation system under construction

▼ Plate N
A retaining wall made from a series of closely-spaced drilled shafts

▲ Plate P
A gabion wall



▼ Plate O
A cellular cofferdam made of sheet piles

probably be modified accordingly. The introduction of load and resistance factor design (LRFD) into foundation design, which are discussed in Chapters 2 and 21, may provide a vehicle for these reliability assessments.

Accuracy of Computations

Those who are new to the field of foundation engineering often make the mistake of expressing the results of computations using too many significant figures. For example, to claim that the predicted settlement of a spread footing is 12.214 mm suggests an accuracy that is well beyond that which is possible using normal exploration and testing methods. Such practices give a false sense of security.

As a general rule, perform most foundation engineering calculations to three significant figures and express the final results and designs to two significant figures. For example, the settlement figure just quoted would best be stated as 12 mm, keeping in mind that the true precision may be on the order of ±50%.

1.4 BUILDING CODES

Building codes govern the design and construction of nearly all foundations. Although foundation engineering is not as codified as some other areas of civil engineering, we must be familiar with the applicable regulations for a particular project.

Most foundation construction in the United States, other than highway and railroad bridges, has been governed by one of the three "model" codes:

- The *Uniform Building Code* (ICBO, 1997), which is used primarily in states west of the Mississippi River
- The *National Building Code* (BOCA, 1996), which is used primarily in the midwestern and northeastern states
- The *Standard Building Code* (SBCCI, 1997), which is used primarily in the southeastern states

Some states and cities have adopted their own codes, or use modifications of these model codes to accommodate their own needs and circumstances. For example, the city of New Orleans generally follows the Standard Building Code, but has its own foundation design requirements that reflect the exceptionally poor soil conditions there.

These three model codes have been merged into a single code, the *International Building Code* or *IBC* (ICC, 2000), which is the first true "national" building code in the United States. The ICBO, BOCA, and SBCCI codes will no longer be updated, and will eventually become obsolete as local jurisdictions adopt the IBC.

The model codes and the IBC focus primarily on buildings, towers, tanks, and other similar structures. However, these codes do not address the design of highway bridges because they have substantially different requirements. Most highway bridges in the United States and Canada are designed under the provisions of the *Standard Specifications for*

Highway Bridges published by the American Association of State Highway and Transportation Officials (AASHTO, 1996).

Other codes also govern the design of foundations for certain projects in certain localities. These include:

- *The National Building Code of Canada* (CCC, 1995), which governs the design of buildings and other structures in Canada.
- The *Ontario Highway Bridge Design Code* (MTO, 1991), which is particularly noteworthy in its use of LRFD design for foundations.
- *Planning, Designing and Constructing Fixed Offshore Platforms* (API, 1996, 1997) governs the design of offshore drilling platforms.

Virtually all of these codes rely on other specialty codes. The two most important ones for foundation engineering are:

- *Building Code Requirements for Structural Concrete* (ACI 318-99 and 318M-99), published by the American Concrete Institute (ACI, 1999).
- *Manual of Steel Construction*, published by the American Institute of Steel Construction (AISC, 1989, 1995).

The various codes are sometimes conflicting and contradictory, and one could write an entire book devoted exclusively to code provisions and their interpretation. Although this book does refer to various code requirements as appropriate, it is not a comprehensive commentary and is not a substitute for the code books. These code references are identified in brackets. For example, [IBC 1801.1] refers to section 1801.1 of the International Building Code.

Building codes represent *minimum* design requirements. Simply meeting code requirements does not necessarily produce a satisfactory design, especially in foundation engineering. Often, these requirements must be exceeded and, on occasion, it is appropriate to seek exceptions from certain requirements. In addition, many important aspects of foundation engineering are not even addressed in the codes. Therefore, think of codes as guides, not dictators, and certainly not as a substitute for engineering knowledge, judgment, or common sense.

1.5 CLASSIFICATION OF FOUNDATIONS

As indicated on the first page of this chapter, we will be using the term *foundation* to describe the structural elements that connect a structure to the ground. These elements are made of concrete, steel, wood, or perhaps other materials. We will divide foundations into two broad categories: *shallow foundations* and *deep foundations*, as shown in Figure 1.2. Shallow foundations transmit the structural loads to the near-surface soils; deep foundations transmit some or all of the loads to deeper soils. These two categories are discussed in Chapters 5 to 10 and 11 to 17, respectively.

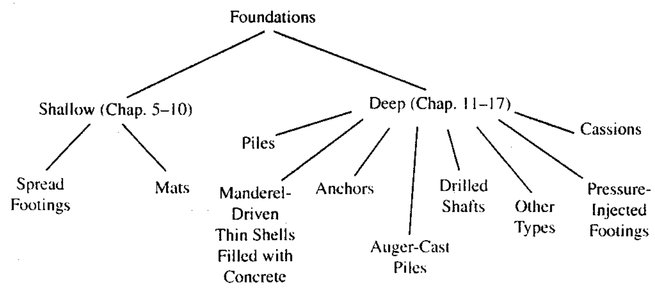


Figure 1.2 Classification of foundations.

Some engineers, especially those involved in the design and construction of dams, use the term *foundation* to describe the underlying soil or rock. However, we will not use this definition.

KEY TO COLOR PHOTOGRAPHS

The following pages contain color photographs showing foundations and earth-retaining structures.

First Page: Foundations in Civil Engineering

All civil engineering structures require foundations. Here are some examples:

A. Buildings impart their weight and other loads, such as wind or seismic forces, onto their foundations. These loads must be safely and economically transmitted into the ground. Large buildings, such as this one, require extensive foundation systems that represent a significant part of the total construction cost.

B. Bridges also generate large structural loads, but unlike buildings they focus these loads on very few supports. For example, this cable-stayed bridge concentrates nearly all of its structural loads on the tower foundations. In addition, many bridge foundations must be built in difficult conditions, such as in the middle of a river, and thus require special design and construction methods.

C. These petroleum storage tanks are part of a refinery complex, and are typical heavy industrial structures. When filled with oil, they are much heavier than a building of comparable height and thus require a high-capacity foundation system.

D. Offshore drilling platforms are subjected to structural loads similar to those in buildings and bridges, as well as significant environmental loads. For example, those in the Gulf of Mexico must survive hurricane-level ocean currents and winds. Many of these loads are especially difficult for the foundation engineer because they act horizontally. The construction of such foundations also is difficult because the water is often very deep and because these platforms are located in the open sea.

Second Page: Foundation Failures

Foundation-related problems can be very costly. Here are some examples of foundation failures and their consequences:

E. The foundations for this single family residence experienced excessive differential settlement, which produced 15-mm wide cracks in the interior drywall. Such problems can be very expensive to repair, but can normally be avoided by careful design and construction practices.

F. This highway bridge crosses a river that normally carries very little water, and the center of the bridge was supported on foundations built in the middle of the river. Unfortunately, during a period of heavy rain, the flow rate in the river increased dramatically, which washed away some of the soils in the river bed (a process called scour) and undermined the foundations. This caused large settlements in the center of the bridge, as shown in the photograph. This problem could have been avoided by building the foundation at a depth below the potential scour zone.

G. The soils beneath this building in Niigata, Japan, liquefied during a magnitude 7.5 earthquake. Since the building was supported on shallow foundations, it sank into the liquefied ground. This failure could have been avoided by first recognizing the presence of liquefiable soil, and then improving the soil or using a different type of foundation.

H. Foundation problems also can develop during construction. This pile was seriously damaged while it was being driven, and is an example of a foundation design that did not properly consider constructibility issues. Such problems can usually be avoided by conducting pile driveability analyses before construction, and using the results of such analyses to develop designs that satisfy both the structural and constructibility requirements.

Third Page: Design and Construction Methods

Engineers and contractors have developed methods of designing and building foundations and earth-retaining structures that are both safe and economical. These photographs illustrate some of these methods.

I. Foundation engineers use drill rigs such as this to explore the soil and rock conditions beneath a construction site and to obtain samples that are later brought to a laboratory for evaluation and testing. Information gained from these efforts is then used to design the foundations.

J. Sometimes the process of developing a site introduces important changes in the underlying soils. This golf course community in Palm Springs, California, is an example because it includes extensive turf areas that must be irrigated, which is a dramatic change from the natural arid conditions shown in the upper portion of the photograph. Some of this irrigation water soaks into the underlying soil and may cause it to compress, possibly producing large settlements. This potential problem must be considered during the design and construction of foundations at such sites.

K. Drilled shafts are one type of deep foundation. In this photograph, a drill rig is creating a cylindrical hole in the ground. The contractor will then insert a reinforcing steel cage and fill the hole with concrete.

L. Structures built on sites underlain by poor soils are usually supported on foundations that accommodate the existing conditions. However, sometimes it is more cost-effective to first improve the soils, then support the structure on a more modest foundation system. Many soil improvement methods are available, each of which is best suited to particular soil conditions and projects. In this case, a series of stone columns are being installed using a vibrofloat. These cylindrical columns of gravel improve the load-bearing ability of the soil beneath the site of a new water tank.

Fourth Page: Earth-Retaining Structures

Many civil engineering projects require earth-retaining structures to maintain a difference in elevation between adjacent ground surfaces. The kind of earth-retaining structure to use in a particular circumstance depends on the required height, the soil conditions, and many other factors. Here are some examples:

M. This soldier pile wall is being used to provide temporary support for a building construction site. It consists of horizontal timber lagging that spans across a series of vertical steel beams embedded into the ground, and permits construction of the building foundations in their proper location. These foundations will support the building and the basement wall, which will then act as the permanent earth-retaining structure.

N. The design for this construction site used a series of closely-spaced drilled shaft foundations to form an earth-retaining structure. Once the shafts had been constructed, the soil inside the construction site was excavated to the basement level and building construction began. In this case, the shafts also will serve as the permanent earth-retaining structure.

O. This sheet pile cofferdam has been constructed in the middle of a river to enable construction of a bridge foundation. It restrains the lateral force from 30 feet of water. The foundation construction is visible in the bottom of the photo.

P. Gabions consist of wire baskets filled with gravel and assembled to form an earth-retaining structure such as the one shown in this photograph. This method requires no specialized equipment, and can be very cost-effective in certain situations.

2

Performance Requirements

If a builder builds a house for a man and does not make its construction firm, and the house which he has built collapses and causes the death of the owner of the house, that builder shall be put to death.

From *The Code of Hammurabi*, Babylon, circa 2000 B.C.

One of the first steps in any design process is to define the performance requirements. What functions do we expect the final product to accomplish? What are the appropriate design criteria? What constitutes acceptable performance, and what would be unacceptable?

A common misconception, even among some engineers, is that foundations are either perfectly rigid and unyielding, or they are completely incapable of supporting the necessary loads and fail catastrophically. This "it's either black or white" perspective is easy to comprehend, but it is not correct. All engineering products, including foundations, have varying *degrees* of performance that we might think of as various shades of gray. The engineer must determine which shades are acceptable and which are not. Leonards (1982) defined *failure* as "an unacceptable difference between expected and observed performance." For example, consider an engineer who designs a foundation such that it is not expected to settle more than 1 inch when loaded. If it actually settles 1.1 inch, the engineer will probably not consider it to have failed because the difference between the expected and observed performance is small and well within the design factor of safety. However, a settlement of 10 inches would probably be unacceptable and therefore classified as a failure.

Foundation performance standards are not the same for all structures or in all locations. For example, foundation settlements that produce 3-mm wide cracks in walls would probably be unacceptable in an expensive house, and may generate a lawsuit, whereas the

same settlements and cracks in a heavy industrial building would probably not even be noticed.

This chapter examines performance requirements for structural foundations. It begins by discussing design loads, then progresses to discussions of performance requirements in each of the following categories:

- Strength requirements
- Serviceability requirements
- Constructibility requirements
- Economic requirements

2.1 DESIGN LOADS

The foundation design process cannot begin until the *design loads* have been defined. These are the loads imparted from the superstructure to the foundation.

Types and Sources

There are four different types of design loads:

- ***Normal loads***, designated by the variable P
- ***Shear loads***, designated by the variable V
- ***Moment loads***, designated by the variable M
- ***Torsion loads***, designated by the variable T

Each of these is shown in Figure 2.1.

Normal loads are those that act parallel to the foundation axis. Usually this axis is vertical, so the normal load becomes the vertical component of the applied load. It may act either downward (compression) or upward (tension).

Shear loads act perpendicular to the foundation axis. They may be expressed as two perpendicular components, V_x and V_y. Moment loads also may be expressed using two perpendicular components, M_x and M_y. Sometimes torsion loads, T, also are important, such as with cantilever highway signs. However, in most designs the torsion loads are small and may be ignored.

Most foundations, especially those that support buildings or bridges, are designed primarily to support downward normal loads, so this type of loading receives the most attention in this book. However, other types of loads also can be important, and in some cases can control the design. For example, the design of foundations for electrical transmission towers is often controlled by upward normal loads induced by overturning moments on the tower.

Design loads also are classified according to their source:

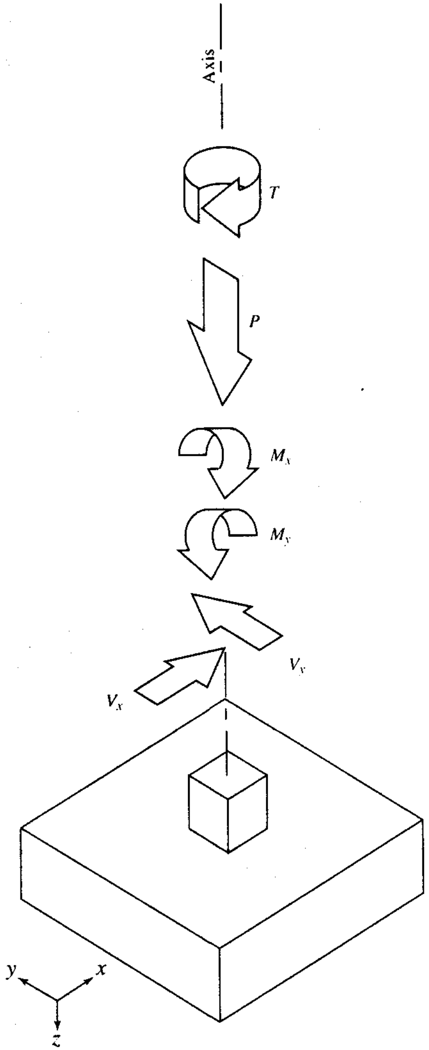


Figure 2.1 Types of structural loads acting on a foundation.

- ***Dead loads*** (*D*) are those caused by the weight of the structure, including permanently installed equipment.
- ***Live loads*** (*L*) are those caused by the intended use and occupancy. These include loads from people, furniture, inventory, maintenance activities, moveable partitions, moveable equipment, vehicles, and other similar sources.
- ***Snow loads*** (*S*) and ***rain loads*** (*R*) are a special type of live load caused by the accumulation of snow or rain. Sometimes rain loads caused by ponding (the static accumulation of water on the roof) are considered separately.
- ***Earth pressure loads*** (*H*) are caused by the weight and lateral pressures from soil or rock, such as those acting on a retaining wall.
- ***Fluid loads*** (*F*) are those caused by fluids with well-defined pressures and maximum heights, such as water in a storage tank.
- ***Earthquake loads*** (*E*) are the result of accelerations from earthquakes.
- ***Wind loads*** (*W*) are imparted by wind onto the structure.
- ***Self-straining loads*** (*T*) are those caused by temperature changes, shrinkage, moisture changes, creep, differential settlement, and other similar processes.
- ***Impact loads*** (*I*) are the result of vibratory, dynamic, and impact effects. Impact loads from vessels are especially important in some bridge and port facilities.
- ***Stream flow loads*** (*SF*) and ***ice loads*** (*ICE*) are caused by the action of water and ice in bodies of water, and are especially important in bridges, offshore drilling platforms, and port facilities.
- ***Centrifugal*** (*CF*) and ***braking loads*** (*BF*) are caused by the motion of vehicles moving on the structure. Centrifugal forces occur when the vehicle is turning, such as on a curved bridge, while braking forces are those transmitted to the structure when a vehicle brakes.

We will identify each load source using subscripts. For example, P_D is a dead normal load, M_E is a moment load caused by an earthquake, etc.

Structural engineers compute the dead loads by simply summing the weights of the structural members. These weights can be accurately predicted and remain essentially constant throughout the life of the structure, so the design dead load values should be very close to the actual dead loads. Incorrect design dead loads are usually due to miscommunication between the structural engineer and others. For example, remodeling may result in new walls, or HVAC (heating, ventilation, and air conditioning) equipment may be heavier than anticipated. Dead loads also can differ if the as-built dimensions are significantly different from those shown on the design drawings.

Some design loads must be computed from the principles of mechanics. For example, fluid loads in a tank are based on fluid statics. The remaining design loads are usually dictated by codes. For example, the BOCA National Building Code (BOCA, 1996) specifies a floor live load of 40 lb/ft^2 for design of classrooms (which is the same as the live load used for prisons!). Most of these code-based design loads are conservative, which is appropriate. This means the real service loads acting on a foundation are probably less than the design loads.

Methods of Expression

There are two methods of expressing and working with design loads: The *allowable stress design* (ASD) method, and the *load and resistance factor design* (LRFD) method. We will be using both of them in this book.

Allowable Stress Design (ASD)

When using the *allowable stress design (ASD) method* (also known as the *working stress design method*), the design loads reflect conservative estimates of the actual service loads. All of the geotechnical analyses in this book, except for those in Chapter 21, use the ASD method. Some of the structural analyses also use ASD.

Evaluating the Design Load

The design load is the most critical combination of the various load sources, as defined by codes. For example, the ANSI/ASCE *Minimum Design Loads for Buildings and Other Structures* (ASCE, 1996a), defines the ASD design load as the greatest of the following four load combinations [ANSI/ASCE 2.4.1]:

$$\boxed{D} \tag{2.1}$$

$$\boxed{D + L + F + H + T + (L_r \text{ or } S \text{ or } R)} \tag{2.2}$$

$$\boxed{D + L + (L_r \text{ or } S \text{ or } R) + (W \text{ or } E)} \tag{2.3}$$

$$\boxed{D + (W \text{ or } E)} \tag{2.4}$$

Other codes use different equations for computing the ASD design load, so it is important to check the applicable code for each project.

Equations 2.1 to 2.4 apply to all types of loads (normal, shear, and moment). For example, when evaluating the normal load, Equation 2.4 produces $P = P_D + P_W$ or $P = P_D + P_E$, whichever is greater. When adding these loads, be sure to consider their direction. For example, when evaluating the normal load, some components may induce compression, while others induce tension.

Equation 2.1 governs only when some of the loads act in opposite directions. For example, if a certain column is subjected to a 500 kN compressive dead load and a 100 kN tensile live load, the design load would be 500 kN per Equation 2.1.

Alternative Method of Evaluating Wind and Seismic Loads

Some codes (e.g., IBC 1605.3.2 and Table 1804.2; UBC 1612.3.2 and Table 18-I-A; BOCA 1805.2) permit greater allowable load capacities in structural materials and soil when considering load combinations that include wind or seismic components. Usually this increased capacity is one-third greater than the static load capacity. For example, if a particular foundation has an allowable load capacity of 600 kN when subjected to static loads (Equations 2.1 and 2.2), then the allowable load capacity under wind or seismic loads (Equations 2.3 or 2.4) would be 600 × 1.33 = 800 kN. Even if this increase is not specifically authorized by the prevailing code, foundation engineers normally have the authority to use it in foundation design. Section 8.4 discusses this topic in more detail.

One way to implement this criterion is to size the foundation twice: first using the design load from Equations 2.1 and 2.2 with the static allowable load capacity, and second using the design load from Equations 2.3 and 2.4 with the wind/seismic load capacity, then using the larger of the two designs. However, this method is tedious and time-consuming. An alternative approach is to divide the loads computed in Equations 2.3 and 2.4 by 1.33 (or multiply them by 1/1.33 = 0.75) instead of increasing the capacity by a factor of 1.33. Then the loads from all four equations can be compared to the same allowable capacity and the foundation needs to be sized only once. Thus, for ASD design of foundations, we rewrite Equations 2.3 and 2.4 as follows:

$$\boxed{0.75[D + L + (L_r \text{ or } S \text{ or } R) + (W \text{ or } E)]} \tag{2.3a}$$

$$\boxed{0.75[D + (W \text{ or } E)]} \tag{2.4a}$$

Therefore, when designing foundations using ASD, we will normally compute the design load as the largest of Equations 2.1, 2.2, 2.3a, or 2.4a, then size the foundation using an allowable bearing pressure that does not include the 33 percent increase.

Example 2.1

A column carries the following vertical compressive loads: P_D = 2100 kN downward, P_L = 1400 kN downward, and P_W = 600 kN upward. Using the ASD load combinations, compute the design normal load for use in foundation design.

Solution

Using Equations 2.1, 2.2, 2.3a, and 2.4a:

$$P = P_D = 2100 \text{ kN}$$

$$P = P_D + P_L + P_F + P_H + P_T + (P_{Lr} \text{ or } P_S \text{ or } P_R)$$
$$= 2100 \text{ kN} + 1400 \text{ kN} + 0 + 0 + 0 + 0$$
$$= 3500 \text{ kN} \quad \Leftarrow \textit{Governs}$$

$$P = 0.75[P_D + P_L + (P_{Lr} \text{ or } P_S \text{ or } P_R) + (P_W \text{ or } P_E)]$$
$$= 0.75[2100 \text{ kN} + 1400 \text{ kN} + 0 - 600 \text{ kN}]$$
$$= 2175 \text{ kN}$$

$$P = 0.75[P_D + (P_W \text{ or } P_E)] = 0.75[2100 \text{ kN} - 600 \text{ kN}] = 1125 \text{ kN}$$

Therefore, the design normal load is **3500 kN downward** $\Leftarrow$ *Answer*

Commentary

Notice how this solution considers downward (compressive) loads to be positive and upward (tensile) loads negative. This is the sign convention customarily used by geotechnical engineers. However, structural engineers normally use the opposite sign convention (tension is positive, compression is negative). This difference can be a source of confusion, so it is important to always be conscious of which sign convention is being used.

The ASD design process then compares the design load with the allowable load, which is the ultimate capacity divided by a factor of safety. For example, if the foundation that supports the column described in Example 2.1 has an ultimate downward capacity of 9000 kN, and is being designed for a factor of safety of 2, the allowable downward capacity is 9000/2 = 4500 kN. Since this is greater than the design load of 3500 kN, the design is satisfactory, or perhaps somewhat overdesigned.

Example 2.2

The column described in Example 2.1 will be supported by a group of four steel H-pile foundations. These H-piles are similar to wide flange beams, and are driven vertically into the ground. The piles will be made of A36 steel (F_y = 248 MPa) and the allowable compressive stress, F_a, is 0.50 F_y. Considering only the stresses in the steel, determine the required cross-sectional area of each pile.

Solution

$$P/\text{pile} = \frac{3500 \text{ kN}}{4} = 875 \text{ kN}$$

$$F_a = 0.50\, F_y = (0.50)(248 \text{ MPa}) = 124 \text{ MPa} = 124{,}000 \text{ kPa}$$

$$A = \frac{P}{F_a} = \left(\frac{875 \text{ kN}}{124{,}000 \text{ kPa}}\right)\left(\frac{1000 \text{ mm}}{\text{m}}\right)^2 = \mathbf{7056 \text{ mm}^2} \quad \Leftarrow \textit{Answer}$$

Commentary

The next step in the design process would be to select a standard H-pile section that has a cross-sectional area of at least 7056 mm^2. We will discuss this step in Chapter 17.

This analysis considers only the stress in the steel. A complete design would also need to consider the load transfer between the pile and the ground, as discussed later in this book.

Load and Resistance Factor Design (LRFD)

The *load and resistance factor design* (*LRFD*) method (also known as the *ultimate strength design* method) uses a different approach. It applies *load factors*, γ, most of which are greater than one, to the nominal loads to obtain the *factored load, U*. In the case of normal loads, the factored load P_u is:

$$\boxed{P_u = \gamma_1 P_D + \gamma_2 P_L + \ldots} \tag{2.5}$$

Design codes present a series of equations in the form of Equation 2.5, each with a different load combination, and define the factored load as the largest load computed from these equations. The LRFD method also applies a *resistance factor*, ϕ (also known as a *strength reduction factor*) to the ultimate capacity from a strength limit analysis. Nearly all resistance factors are less than one. Finally, the design must satisfy the following criteria:

$$\boxed{P_u \leq \phi\, P_n} \tag{2.6}$$

Where:

P_u = factored normal load
γ = load factor
P_D = normal dead load
P_L = normal live load
ϕ = resistance factor
P_n = nominal normal load capacity

Similar equations also are used for shear and moment loads. Chapter 21 discusses LRFD in more detail.

American Concrete Institute (ACI) Code

The first widely accepted LRFD code in North America was developed by the American Concrete Institute (ACI) for the design of reinforced concrete. In its current form (ACI, 1999), this code defines the factored load as the largest of those computed from the following equations [ACI 9.2]:

$$\boxed{U = 1.4D + 1.7L} \tag{2.7}$$

$$U = 0.75(1.4D + 1.4T + 1.7L) \quad (2.8)$$

$$U = 0.9D + 1.4F \quad (2.9)$$

$$U = 1.4D + 1.7L + 1.4F \quad (2.10)$$

$$U = 1.4D + 1.7L + 1.7H \quad (2.11)$$

$$U = 0.9D + 1.3W \quad (2.12)$$

$$U = 0.9D + 1.43E \quad (2.13)$$

$$U = 0.75(1.4D + 1.7L + 1.7W) \quad (2.14)$$

$$U = 0.75(1.4D + 1.7L + 1.87E) \quad (2.15)$$

$$U = 0.9D + 1.7H \quad (2.16)$$

$$U = 1.4(D + T) \quad (2.17)$$

Once again, these load combinations apply to all types of loads, and thus are expressed as P_U, M_U, and so forth. Note how the 0.75 reduction factor, as discussed under ASD, is already included in Equations 2.14 and 2.15.

ANSI/ASCE and AISC Codes

The American National Standards Institute (ANSI) and the American Society of Civil Engineers (ASCE) have developed a reliability-based LRFD standard for computing the factored load. It has been published as ANSI/ASCE 7-95 *Minimum Design Loads for Buildings and Other Structures* (ASCE, 1996a). This standard forms the basis for LRFD design of steel structures (AISC, 1995), and is accepted as an alternative method for concrete structures (ACI, 1999). It will probably become the universal standard in North America for structures other than bridges.

According to the ANSI/ASCE standard, the factored load is the largest load computed from the following formulas [ANSI/ASCE 2.3.2]:

$$U = 1.4D \quad (2.18)$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \quad (2.19)$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.8W) \quad (2.20)$$

$$U = 1.2D + 1.3W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R) \quad (2.21)$$

$$U = 1.2D + 1.0E + 0.5L + 0.2S \quad (2.22)$$

$$U = 0.9D + (1.3W \text{ or } 1.0E) \quad (2.23)$$

In certain special cases, these formulas are modified slightly (see ASCE, 1996). The International Building Code (ICC, 2000) uses a modified version of these formulas [IBC 1605].

AASHTO Code

The American Association of State Highway and Transportation Officials (AASHTO) have developed a separate standard for computing the factored load (Nowak, 1995; AASHTO, 1996). This standard is based on loadings for bridges, which are significantly different than those on buildings and other structures. In addition, the probabilistic evaluations of these loads are different from those for buildings. Therefore, the AASHTO standard produces factored loads that are different from both the ACI and ANSI/ASCE standards.

Example 2.3

Solve Examples 2.1 and 2.2 using LRFD with the ANSI/ASCE load factors and a resistance factor of 0.70.

Solution

The factored load is governed by Equation 2.19:

$$\begin{aligned} P_U &= 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \\ &= 1.2(2100) + 1.6(1400) + 0 + 0 \\ &= 4760 \text{ kN} \end{aligned}$$

$$P_u/\text{pile} = \frac{4760 \text{ kN}}{4} = 1190 \text{ kN}$$

$$P_n = F_a A = (248{,}000 \text{ kPa})\, A$$

$$P_u \leq \phi P_n$$

$$1190 \text{ kN} \leq (0.70)(248{,}000 \text{ kPa})\, A$$

$$A \geq \mathbf{6855\ mm^2} \qquad \Leftarrow \textit{Answer}$$

Commentary

In this case, the computed area is slightly less than that from the ASD analysis.

2.2 STRENGTH REQUIREMENTS

Once the design loads have been defined, we need to develop foundation designs that satisfy several performance requirements. The first category is *strength requirements*, which are intended to avoid catastrophic failures. There are two types: geotechnical strength requirements and structural strength requirements.

Geotechnical Strength Requirements

Geotechnical strength requirements are those that address the ability of the soil or rock to accept the loads imparted by the foundation without failing. The strength of soil is governed by its capacity to sustain shear stresses, so we satisfy geotechnical strength requirements by comparing shear stresses with shear strengths and designing accordingly.

In the case of spread footing foundations, geotechnical strength is expressed as the *bearing capacity* of the soil. If the load-bearing capacity of the soil is exceeded, the resulting shear failure is called a *bearing capacity failure*, as shown in Figure 2.2. Later in this book we will examine such failures in detail, and learn how to design foundations that have a sufficient factor of safety against such failures.

Geotechnical strength analysis are almost always performed using allowable stress design (ASD) methods. However, LRFD-based design is beginning to appear, as discussed in Chapter 21.

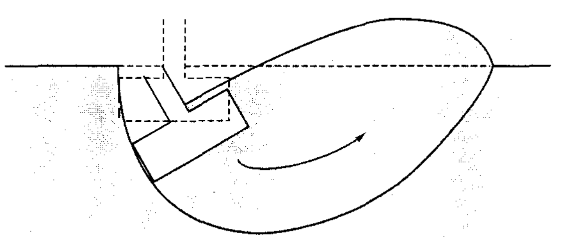


Figure 2.2 A bearing capacity failure beneath a spread footing foundation. The soil has failed in shear, causing the foundation to collapse.

Structural Strength Requirements

Structural strength requirements address the foundation's structural integrity and its ability to safely carry the applied loads. For example, pile foundations made from A36 steel are normally designed for a maximum allowable compressive stress of 12,600 lb/in^2. Thus, the thickness of the steel must be chosen such that the stresses induced by the design loads do not exceed this allowable value. Foundations that are loaded beyond their structural capacity will, in principle, fail catastrophically.

Structural strength analyses are conducted using either ASD or LRFD methods, depending on the type of foundation, the structural materials, and the governing code.

QUESTIONS AND PRACTICE PROBLEMS

Unless otherwise stated all ASD design loads should be computed using Equations 2.1, 2.2, 2.3a, and 2.4a.

2.1 A proposed column has the following design loads:

Axial load: $P_D = 200$ k, $P_L = 170$ k, $P_E = 50$ k, $P_W = 60$ k (all compression)

Shear load: $V_D = 0$, $V_L = 0$, $V_E = 40$ k, $V_W = 48$ k

Compute the design axial and shear loads for foundation design using ASD.

2.2 Repeat Problem 2.1 using LRFD with the ACI load factors.

2.3 A certain foundation will experience a bearing capacity failure when it is subjected to a downward load of 2200 kN. Using ASD with a factor of safety of 3, determine the maximum allowable load that will satisfy geotechnical strength requirements.

2.4 A steel pile foundation with a cross-sectional area of 15.5 in^2 and $F_y = 50$ k/in^2 is to carry axial compressive dead and live loads, of 300 and 200 k, respectively. Using LRFD with the ANSI/ASCE load factors and a resistance factor of 0.75, determine whether this pile satisfies structural strength requirements for axial compression.

2.3 SERVICEABILITY REQUIREMENTS

Foundations that satisfy strength requirements will not collapse, but they still may not have adequate performance. For example, they may experience excessive settlement. Therefore, we have the second category of performance requirements, which are known as *serviceability requirements*. These are intended to produce foundations that perform well when subjected to the service loads. These requirements include:

- **Settlement**—Most foundations experience some downward movement as a result of the applied loads. This movement is called *settlement*. Keeping settlements within tolerable limits is usually the most important foundation serviceability requirement.
- **Heave**—Sometimes foundations move upward instead of downward. We call this upward movement *heave*. The most common source of heave is the swelling of expansive soils.
- **Tilt**—When settlement or heave occurs only on one side of the structure, it may begin to *tilt*. The Leaning Tower of Pisa is an extreme example of tilt.
- **Lateral movement**—Foundations subjected to lateral loads (shear or moment) deform horizontally. This *lateral movement* also must remain within acceptable limits to avoid structural distress.
- **Vibration**—Some foundations, such as those supporting certain kinds of heavy machinery, are subjected to strong vibrations. Such foundations need to accommodate these vibrations without experiencing resonance or other problems.
- **Durability**—Foundations must be resistant to the various physical, chemical, and biological processes that cause deterioration. This is especially important in waterfront structures, such as docks and piers.

Failure to satisfy these requirements generally results in increased maintenance costs, aesthetic problems, diminished usefulness of the structure, and other similar effects.

Settlement

The vertical downward load is usually the greatest load acting on foundations, and the resulting vertical downward movement is usually the largest and most important movement. We call this vertical downward movement *settlement*. Sometimes settlement also occurs as a result of other causes unrelated to the presence of the foundation, such as consolidation due to the placement of a fill.

Although foundations with zero settlement would be ideal, this is not an attainable goal. Stress and strain always go together, so the imposition of loads from the foundation always cause some settlement in the underlying soils. Therefore, the question that faces the foundation engineer is not *if* the foundation will settle, but rather defining the *amount* of settlement that would be tolerable and designing the foundation to accommodate this requirement. This design process is analogous to that for beams where the deflection must not exceed some maximum tolerable value.

Structural Response to Settlement

Structures can settle in many different ways, as shown in Figure 2.3a. Sometimes the settlement is uniform, so the entire structure moves down as a unit. In this case, there is no damage to the structure itself, but there may be problems with its interface with the adjacent ground or with other structures. Another possibility is settlement that varies linearly

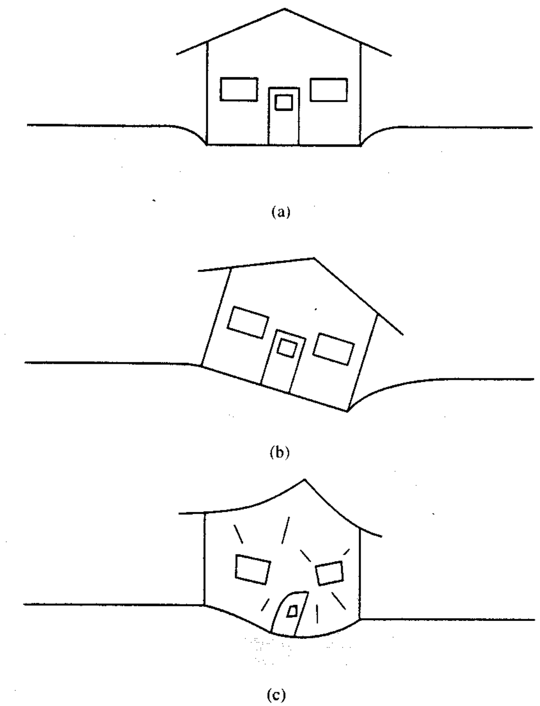


Figure 2.3 Modes of settlement: (a) uniform; (b) tilting with no distortion; (c) distortion.

across the structure as shown in Figure 2.3b. This causes the structure to tilt. Finally, Figure 2.3c shows a structure with irregular settlements. This mode distorts the structure and typically is the greatest source of problems.

The response of structures to foundation settlement is very complex, and a complete analysis would require consideration of many factors. Such analyses would be very time-consuming, are thus not practical for the vast majority of structures. Therefore, we simplify the problem by describing settlement using only two parameters: total settlement and differential settlement (Skempton and MacDonald, 1956; Polshin and Tokar, 1957; Burland and Wroth, 1974; Grant, et al., 1974; Wahls, 1981; Wahls, 1994; Frank, 1994).

Total Settlement

The *total settlement*, δ, is the change in foundation elevation from the original unloaded position to the final loaded position, as shown in Figure 2.4.

Structures that experience excessive total settlements might have some of the following problems:

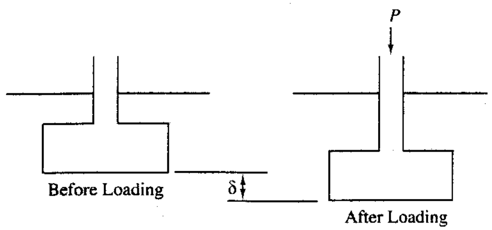


Figure 2.4 Total settlement in a spread footing foundation.

- **Connections with existing structures**—Sometimes buildings must join existing structures. In such cases, the floors in the new building must be at the same elevation as those in the existing building. However, if the new building settles excessively, the floors will no longer match, causing serious serviceability problems.
- **Utility lines**—Buildings, tanks, and many other kinds of structures are connected to various utilities, many of which are located underground. If the structure settles excessively, the utility connections can be sheared or distorted. This is especially troublesome with gravity flow lines, such as sewers.
- **Surface drainage**—The ground floor of buildings must be at a slightly higher elevation than the surrounding ground so rainwater does not enter. However, settlement might destroy these drainage patterns and cause rainwater to enter the structure.
- **Access**—Vehicles and pedestrians may need to access the structure, and excessive settlement might impede them.
- **Aesthetics**—Excessive settlement may cause aesthetic problems long before there is any threat to structural integrity or serviceability.

Some structures have sustained amazingly large total settlements, yet remain in service. For example, many buildings have had little or no ill effects even after settling as much as 250 mm (10 in). Others have experienced some distress, but continue to be used following even greater settlements. Some of the most dramatic examples are located in Mexico City, where buildings have settled more than 2 m (7 ft) and are still in use. Some bridges, tanks, and other structures also might tolerate very large settlements. However, these are extreme examples. Normally engineers have much stricter performance requirements.

Table 2.1 presents typical design values for the allowable total settlement, δ_a. These values already include a factor of safety, and thus may be compared directly to the pre-

TABLE 2.1 TYPICAL ALLOWABLE TOTAL SETTLEMENTS FOR FOUNDATION DESIGN

Type of Structure	Typical Allowable Total Settlement, δ_a (in)	(mm)
Office buildings	0.5–2.0 (1.0 is the most common value)	12–50 (25 is the most common value)
Heavy industrial buildings	1.0–3.0	25–75
Bridges	2.0	50

dicted settlement. The design meets total settlement requirements if the following condition is met:

$$\boxed{\delta \leq \delta_a} \tag{2.24}$$

Where:

δ = total settlement of foundation

δ_a = allowable total settlement

Methods of computing δ are covered later in this book.

When using Table 2.1, keep the following caveats in mind:

1. Customary engineering practice in this regard varies significantly between regions, which is part of the reason for the wide ranges of values in this table. For example, an office building in one state may need to be designed for a total settlement of 12 mm (0.5 in), while the same building in another state might be designed using 25 mm (1.0 in). It is important to be aware of local practices, since engineers are normally expected to conform to them. However, it also is important to recognize that local practices in some areas are overly conservative and produce foundations that are more expensive than necessary.
2. Foundations with large total settlements also tend to have large differential settlements. Therefore, limits on allowable differential settlement often indirectly place limits on total settlement that are stricter than those listed in Table 2.1. Chapter 7 discusses this aspect in more detail.

If the predicted settlement, δ, is greater than δ_a, we could consider any or all of the following measures:

- **Adjust the foundation design** — Adjustments in the foundation design often will solve problems with excessive settlement. For example, the settlement of spread footing foundations can be reduced by increasing their width.
- **Use a more elaborate foundation**—For example, we might use piles instead of spread footings, thus reducing the settlement.
- **Improve the properties of the soil**—Many techniques are available to do this; some of them are discussed in Chapter 18.
- **Redesign the structure so it is more tolerant of settlements**—For example, flexible joints could be installed on pipes, as shown in Figure 2.5.

Differential Settlement

Engineers normally design the foundations for a structure such that all of them have the same computed total settlement. Thus, in theory, the structure will settle uniformly. Unfortunately, the actual performance of the foundations will usually not be exactly as predicted, with some of them settling more than expected and other less. This discrepancy between predicted behavior and actual behavior has many causes, including the following:

- **The soil profile may not be uniform across the site**—This is nearly always true, no matter how uniform it might appear to be.
- **The ratio between the actual load and the design load may be different for each column**—Thus, the column with the lower ratio will settle less than that with the higher ratio.

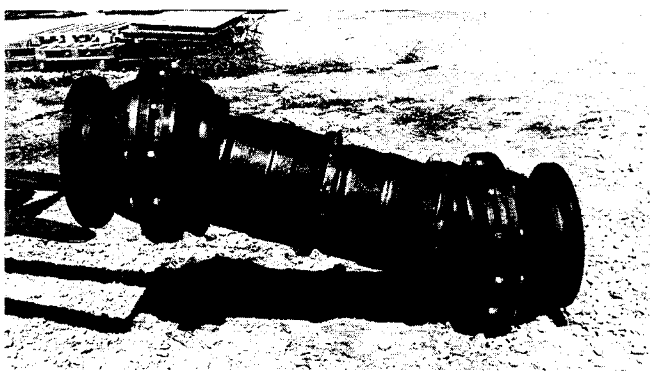


Figure 2.5 This flexible-extendible pipe coupler has ball joints at each end and a telescoping section in the middle. These couplers can be installed where utility lines enter structures, thus accommodating differential settlement, lateral movement, extension, and compression. For example, an 8-in (203 mm) diameter connector can accommodate differential settlements of up to 15-in (380 mm). (Photo courtesy of EBAA Iron Sales, Inc.).

- **The ratio of dead load to live load may be different for each column**— Settlement computations are usually based on dead-plus-live load, and the foundations are sized accordingly. However, in many structures much of the live load will rarely, if ever, occur, so foundations that have a large ratio of design live load to design dead load will probably settle less than those carrying predominantly dead loads.
- **The as-built foundation dimensions may differ from the plan dimensions**—This will cause the actual settlements to be correspondingly different.

The *differential settlement*, δ_D, is the difference in total settlement between two foundations or between two points on a single foundation. Differential settlements are generally more troublesome than total settlements because they distort the structure, as shown in Figure 2.3c. This causes cracking in walls and other members, jamming in doors and windows, poor aesthetics, and other problems. If allowed to progress to an extreme, differential settlements could threaten the integrity of the structure. Figures 2.6 and 2.7 show examples of structures that have suffered excessive differential settlement.

Therefore, we define a maximum *allowable differential settlement*, δ_{Da}, and design the foundations so that:

$$\boxed{\delta_D \leq \delta_{Da}} \qquad (2.25)$$

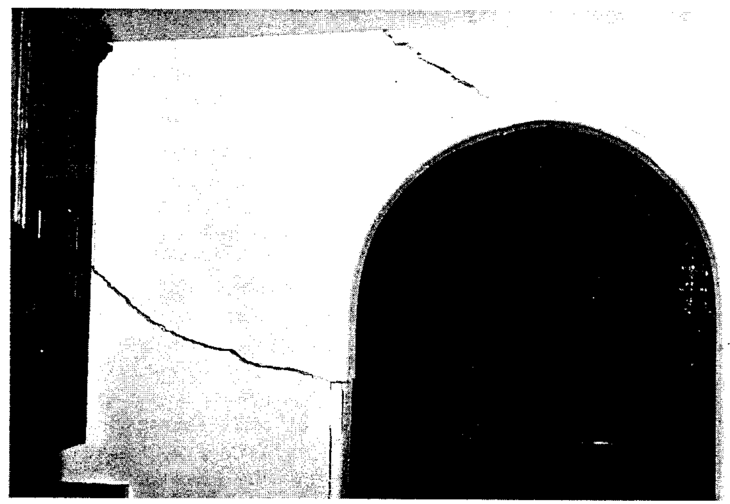


Figure 2.6 This wood-frame house was built on an improperly compacted fill, and thus experienced excessive differential settlements. The resulting distortions produced drywall cracks up to 15-mm wide, as shown in this photograph, along with additional cracks in the exterior walls and the floor slab.

Figure 2.7 This old brick building has experienced excessive differential settlements, as evidenced by the diagonal cracks in the exterior wall.

In buildings, δ_{Da} depends on the potential for jamming doors and windows, excessive cracking in walls and other structural elements, aesthetic concerns, and other similar issues. The physical processes that cause these serviceability problems are very complex, and depend on many factors, including the type and size of the structure, the properties of the building materials and the subsurface soils, and the rate and uniformity of the settlement (Wahls, 1994). These processes are much too complex to model using rational structural analyses, so engineers depend on empirical methods. These methods are based on measurements of the actual differential settlements in real buildings and assessments of their performance.

Comprehensive studies of differential settlements in buildings include Skempton and MacDonald (1956), Polshin and Tokar (1957), and Grant et al. (1974). Skempton and MacDonald's work is based on the observed performance of ninety eight buildings of various types, forty of which had evidence of damage due to excessive settlements. Polshin and Tokar reported the results of 25 years of observing the performance of structures in the Soviet Union and reflected Soviet building codes. The study by Grant et al. encompassed data from ninety five buildings, fifty six of which had damage.

Table 2.2 presents a synthesis of these studies, expressed in terms of the allowable angular distortion, θ_a. These values already include a factor of safety of at least 1.5, which is why they are called "allowable." We use them to compute δ_{Da} as follows:

$$\boxed{\delta_{Da} = \theta_a S} \tag{2.26}$$

Where:

δ_{Da} = allowable differential settlement
θ_a = allowable angular distortion (from Table 2.2)
S = column spacing (horizontal distance between columns)

Empirical data also suggests that typical buildings have architectural damage at $\theta \approx 1/300$ and structural damage at $\theta \approx 1/150$ (Stephenson, 1995). However, both these values and those in Table 2.2 especially depend on the type of exterior cladding, because cracks in the cladding can allow water to enter the structure and damage the interior. For example, corrugated metal siding tolerates much more differential settlement than stucco. In addition, it may be wise to use lower δ_a and δ_{Da} values for warehouses, tanks, and other structures in which the live load represents a large portion of the total load and does not occur until after the structure is complete.

Be sure to consider local practice and precedent when developing design values of δ_{Da}. Engineers in some areas routinely design structures to accommodate relatively large settlements, and may be willing to accept some long-term maintenance costs (i.e., repairing minor cracking, rebuilding entranceways, etc.) in exchange for reduced construction costs. However, in other areas, even small settlements induce lawsuits, so foundations are designed to meet stricter standards.

TABLE 2.2 ALLOWABLE ANGULAR DISTORTION, θ_a (COMPILED FROM WAHLS, 1994; AASHTO, 1996; AND OTHER SOURCES)

Type of Structure	θ_a
Steel tanks	1/25
Bridges with simply-supported spans	1/125
Bridges with continuous spans	1/250
Buildings that are very tolerant of differential settlements, such as industrial buildings with corrugated steel siding and no sensitive interior finishes.	1/250
Typical commercial and residential buildings.	1/500
Overhead traveling crane rails.	1/500
Buildings that are especially intolerant of differential settlement, such as those with sensitive wall or floor finishes.	1/1000
Machinery[a]	1/1500
Buildings with unreinforced masonry load-bearing walls	
Length/height ≤ 3	1/2500
Length/height ≥ 5	1/1250

[a] Large machines, such as turbines or large punch presses, often have their own foundation, separate from that of the building that houses them. It often is appropriate to discuss allowable differential settlement issues with the machine manufacturer.

Methods for computing δ_D are discussed later in this book within the chapters related to the type of foundation being considered.

Design Load

Serviceability requirements are dictated by the performance of the structure under the actual service loads. This is quite different from strength requirements, which are concerned with avoiding failure under extreme loading events. Therefore, settlement analyses are based on the unfactored static working loads (i.e., the larger of Equations 2.1 to 2.4). This is true regardless of whether ASD or LRFD methods are used to satisfy strength requirements.

Sometimes engineers perform settlement analyses using only the static loads (i.e., the larger of Equations 2.1 and 2.2). This is because wind or seismic loads represent extreme events. In addition, these loads generally have a very short duration (i.e., the peak wind gusts and the peak earthquake accelerations are very short), so the soil may not have enough time to respond in the same way it does for long-term loads. Even if there were no additional settlements, some nonstructural cracking will probably occur during anyway as a result of swaying in the superstructure. Very few building owners would object to patching a few cracks after a design-level earthquake or hurricane.

Example 2.4

A steel-frame office building has a column spacing of 20 ft. It is to be supported on spread footings founded on a clayey soil. What are the allowable total and differential settlements?

Solution

Per Table 2.1, use δ_a = **1.0 in** ⇐ ***Answer***
Per Table 2.2, $\theta_a = 1/500$

$$\delta_{Da} = \theta_a S$$
$$= (1/500)(20)$$
$$= 0.04 \text{ ft} = \mathbf{0.5 \text{ in}}$$ ⇐ ***Answer***

Rate of Settlement

It also is important to consider the rate of settlement and how it compares with the rate of construction. Foundations in sands settle about as rapidly as the loads are applied, whereas those in saturated clays move much more slowly.

In some structures, much of the load is applied to the foundation before the settlement-sensitive elements are in place. For example, settlements of bridge piers that occur before the deck is placed are far less important than those that occur after. Buildings may not be sensitive to differential settlements until after sensitive finishes, doors, and other architectural items are in place, yet if the foundation is in sand, most of the settlement may have already occurred by then.

However, other structures generate a large portion of their loads after they are completed. For example, the greatest load on a water storage tank is its contents.

Heave

Sometimes foundations move upward instead of downward. This kind of movement is called *heave*. It may be due to applied upward loads, but more often it is the result of external forces, especially those from expansive soils. The design criteria for heave are the same as those for settlement. However, if some foundations are heaving while others are settling, then the differential is the sum of the two.

Tilt

Excessive tilt is often a concern in tall, rigid structures, such as chimneys, silos, and water towers. To preserve aesthetics, the tilt, ω, from the vertical should be no more than 1/500 (7 min of arc). Greater tilts would be noticeable, especially in taller structures and those that are near other structures. In some cases, stricter limits on tilt are appropriate, especially for exceptionally tall structures. For comparison, the Leaning Tower of Pisa has a tilt of about 1/10.

Lateral Movement

Foundations subjected to lateral loads have corresponding lateral movements. These movement also have tolerable limits. For bridge foundations, Bozozuk (1978) recommended maximum lateral movements of 25 mm (1 in).

Vibration

Foundations that support large machinery are sometimes subjected to substantial vibratory loads. Such foundations must be designed to accommodate these vibratory loads without introducing problems, such as resonance.

Durability

Soil can be a very hostile environment to place engineering materials. Whether they are made of concrete, steel, or wood, structural foundations may be susceptible to chemical and/or biological attack that can adversely affect their integrity.

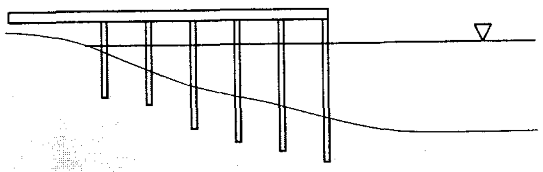
Corrosion of Steel

Under certain conditions, steel can be the object of extensive corrosion. This can be easily monitored when the steel is above ground, and routine maintenance, such as painting, will usually keep corrosion under control. However, it is impossible to inspect underground steel visually, so it is appropriate to be concerned about its potential for corrosion and long-term integrity.

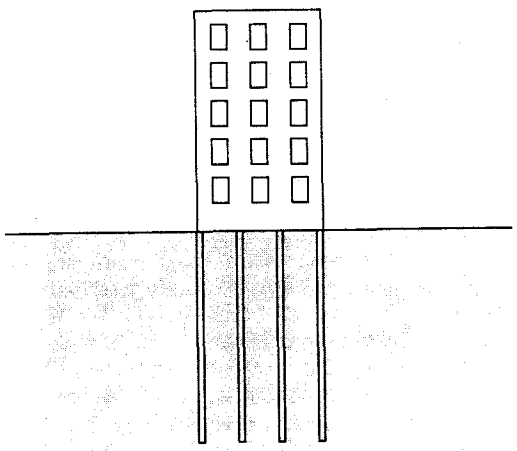
Owners of underground steel pipelines are especially conscious of corrosion problems. They often engage in extensive corrosion surveys and include appropriate preventive measures in their designs. These procedures are well established and effective, but should they also be used for steel foundations such as H-piles or steel pipe piles?

For corrosion assessment, steel foundations can be divided into two categories: those in marine environments and those in land environments. Both are shown in Figure 2.8.

Steel foundations in marine environments have a significant potential for corrosion, especially those exposed to salt water. Studies of waterfront structures have found that steel is lost at a rate of 0.075 to 0.175 mm/yr (Whitaker, 1976). This corrosion occurs most rapidly in the tidal and splash zones (Dismuke et. al., 1981) and can also be very extensive immediately above the sea floor; then it becomes almost negligible at depths more than about 0.5 m (2 ft) below the sea floor. Such structures may also be prone to abrasion from moving sand, ships, floating debris, and other sources. It is common to protect such foundations with coatings or jackets, at least through the water and splash zones.



(a)



(b)

Figure 2.8 (a) Marine environments include piers, docks, drilling platforms, and other similar structures where a portion of the foundation is exposed to open water. (b) Land environments include buildings and other structures that are built directly on the ground and the entire foundation is buried.

However, the situation in land environments is quite different. Based on extensive studies, Romanoff (1962, 1970) observed that no structural failures have been attributed to the corrosion of steel piles in land environments. One likely reason for this excellent performance record is that piles, unlike pipelines, can tolerate extensive corrosion, even to the point of occasionally penetrating through the pile, and remain serviceable.

Romanoff also observed that piles founded in natural soils (as opposed to fills) experienced little or no corrosion, even when the soil could be identified as potentially corrosive. The explanation for this behavior seems to be that natural soils contain very little free oxygen, an essential ingredient for the corrosion process.

However, fills do contain sufficient free oxygen and, under certain circumstances, can be very corrosive environments. Therefore, concern over corrosion of steel piles in land environments can normally be confined to sites where the pile penetrates through fill. Some fills have very little potential for corrosion, whereas others could corrode steel at rates of up to 0.08 mm/yr (Tomlinson, 1987), which means that a typical H-pile section could lose half of its thickness in about 50 years.

Schiff (1982) indicated that corrosion would be most likely in the following soil conditions:

- High moisture content
- Poorly aerated
- Fine grained
- Black or gray color
- Low electrical resistivity
- Low or negative redox potential
- Organic material present
- High chemical content
- Highly acidic
- Sulfides present
- Anaerobic microorganisms present

Areas where the elevation of the groundwater table fluctuates, such as tidal zones, are especially difficult because this scenario continually introduces both water and oxygen to the pile. Contaminated soils, such as sanitary landfills and shorelines near old sewer outfalls, are also more likely to have problems.

One of the most likely places for corrosion on land piles is immediately below a concrete pile cap. Local electrical currents can develop because of the change in materials, with the concrete acting as a cathode and the soil as an anode. Unfortunately, this is also the most critical part of the pile because the stresses are greatest there.

If the foundation engineer suspects that corrosion may be a problem, it is appropriate to retain the services of a corrosion engineer. Detailed assessments of corrosion and the development of preventive designs are beyond the expertise of most foundation engineers.

The corrosion engineer will typically conduct various tests to quantify the corrosion potential of the soil and consider the design life of the foundation to determine whether any preventive measures are necessary. Such measures could include the following:

- Use a different construction material (i.e., concrete, wood).
- Increase the thickness of steel sections by an amount equal to the anticipated deterioration.
- Cover the steel with a protective coating (such as coal tar epoxy) to protect it from the soil. This method is commonly used with underground tanks and pipes, and has also been successfully used with pile foundations. However, consider the possibility that some of the coating may be removed by abrasion when the pile is driven into the ground, especially when sands or gravels are present. Coatings can also be an effective means of combatting corrosion near pile caps, as discussed earlier. In this case, the coating is applied to the portion of the steel that will be encased in the concrete, thus providing the electrical insulation needed to stop or significantly slow the corrosion process.
- Provide a *cathodic protection system*. Such systems consist of applying a DC electrical potential between the foundation (the cathode) and a buried sacrificial metal (the anode). This system causes the corrosion to be concentrated at the anode and protects the cathode (see Figure 2.9). Rectifiers connected to a continuous power source provide the electricity. These systems consume only nominal amounts of electricity. In some cases, it is possible to install a self-energizing system that generates its own current.

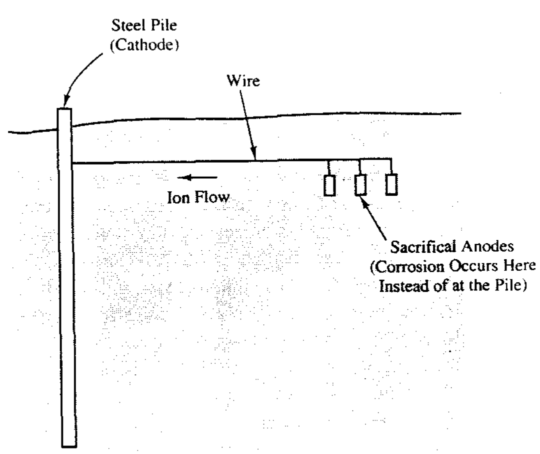


Figure 2.9 Use of a cathodic protection system to protect steel foundations from corrosion.

Sulfate Attack on Concrete

Buried concrete is usually very resistant to corrosion and will remain intact for many years. However, serious degradation can occur in concrete subjected to soils or groundwater that contain high concentrations of sulfates (SO_4). These sulfates can react with the cement to form calcium sulfoaluminate (ettringite) crystals. As these crystals grow and expand, the concrete cracks and disintegrates. In some cases, serious degradation has occurred within 5 to 30 years of construction. Although we do not yet fully understand this process (Mehta, 1983), engineers have developed methods of avoiding these problems.

We can evaluate a soil's potential for sulfate attack by measuring the concentration of sulfates in the soil and/or in the groundwater and comparing them with soils that have had problems with sulfate attack. Soils with some or all of the following properties are most likely to have high sulfate contents:

- Wet
- Fine-grained
- Black or gray color
- High organic content
- Highly acidic or highly alkaline

Some fertilizers contain a high concentration of sulfates that may cause problems when building in areas that were formerly used for agricultural purposes. The same is true for some industrial wastes. It is often wise to consult with corrosion experts in such cases. Seawater also has a high concentration: about 2300 ppm.

If the laboratory tests indicate that the soil or groundwater has a high sulfate content, design the buried concrete to resist attack by using one or more of the following methods:

- **Reduce the water:cement ratio**—This reduces the permeability of the concrete, thus retarding the chemical reactions. This is one of the most effective methods of resisting sulfate attack. Suggested maximum ratios are presented in Table 2.3.
- **Increase the cement content**—This also reduces the permeability. Therefore, concrete that will be exposed to problematic soils should have a cement content of at least 6 sacks/yd^3 (564 lb/yd^3 or 335 kg/m^3).
- **Use sulfate-resisting cement**—Type II low-alkali and type V portland cements are specially formulated for use in moderate and severe sulfate conditions, respectively. Pozzolan additives to a type V cement also help. Type II is easily obtained, but type V may not be readily available in some areas. Table 2.3 gives specific guidelines.
- **Coat the concrete with an asphalt emulsion**—This is an attractive alternative for retaining walls or buried concrete pipes, but not for foundations.

TABLE 2.3 USE OF SULFATE-RESISTING CEMENTS AND LOW WATER:CEMENT RATIOS TO AVOID SULFATE ATTACK OF CONCRETE (Adapted from Kosmatka and Panarese, 1988, and PCA, 1991). Used with permission of the Portland Cement Association.

Water-Soluble Sulfates in Soil (% by weight)	Sulfates in Water (ppm)	Sulfate Attack Hazard	Cement Type	Maximum Water:Cement Ratio
0.00–0.10	0–150	Negligible	—	—
0.10–0.20	150–1500	Moderate	II	0.50
0.20–2.00	1500–10,000	Severe	V	0.45
>2.00	>10,000	Very severe	V plus pozzolan	0.45

Decay of Timber

The most common use of wood in foundations is timber piles. The lifespan of these piles varies depending on their environment. Even untreated timber piles can have a very long life if they are continually submerged below the groundwater table. This was illustrated when a campanile in Venice fell in 1902. The submerged timber piles, which had been driven in A.D. 900, were found to be in good condition and were used to support the replacement structure (Chellis, 1962). However, when located above the groundwater table, timber can be subject to deterioration from several sources (Chellis, 1961), including:

- **Decay** caused by the growth of fungi. This process requires moisture, oxygen, and favorable temperatures. These conditions are often most prevalent in the uppermost 2 m (6 ft) of the soil. If the wood is continually very dry, then decay will be limited due to the lack of moisture.
- **Insect attack**, including termites, beetles, and marine borers.
- **Fire**, especially in marine structures.

The worst scenario is one in which the piles are subjected to repeated cycles of wetting and drying. Such conditions are likely to be found near the groundwater table because it usually rises and sinks with the seasons and near the water surface in marine applications where splashing and tides will cause cyclic wetting and drying.

To reduce problems of decay, insect attack, and fungi growth, timber piles are usually treated before they are installed. The most common treatment consists of placing them in a pressurized tank filled with creosote or some other preserving chemical. This *pressure treatment* forces some of the creosote into the wood and forms a thick coating on the outside, leaving a product that is almost identical to many telephone poles. When the piles are fully embedded into soil, creosote-treated piles normally have a life at least as long as the design life of the structure.

Timber piles also will lose part of their strength if they are subjected to prolonged high temperatures. Therefore, they should not be used under hot structures such as blast furnaces.

QUESTIONS AND PRACTICE PROBLEMS

2.5 A seven-story steel-frame office building will have columns spaced 7 m on center and will have typical interior and exterior finishes. Compute the allowable total and differential settlements for this building.

2.6 A two-story reinforced concrete art museum is to be built using an unusual architectural design. It will include many tile murals and other sensitive wall finishes. The column spacing will vary between 5 and 8 m. Compute the allowable total and differential settlements for this buiding.

2.7 A 40 ft × 60 ft one-story agricultural storage building will have corrugated steel siding and no interior finish or interior columns. However, it will have two roll-up doors. Compute the allowable total and differential settlement for this building.

2.8 A sandy soil has 0.03 percent sulfates. Evaluate the potential for sulfate attack of concrete exposed to this soil and recommend preventive design measures, if needed.

2.4 CONSTRUCTIBILITY REQUIREMENTS

The third category of performance requirements is *constructibility*. The foundation must be designed such that a contractor can build it without having to use extraordinary methods or equipment. There are many potential designs that might be quite satisfactory from a design perspective, but difficult or impossible to build.

For example, Chapter 11 of this book discusses different types of deep foundations. One of these, a *pile foundation*, consists of a prefabricated pole that is driven into the ground using a modified crane called a pile driver. The pile driver lifts the pile into the air, then drives it into the ground, as shown in Figure 2.10. Therefore, piles can be installed only at locations that have sufficient headroom. Fortunately, the vast majority of construction sites have plenty of headroom.

Nevertheless, a design engineer who is not familiar with this construction procedure might ask for piles to be installed at a location with minimal headroom. For example, the pile design shown in Figure 2.11 is unbuildable because it is impossible to fit the required installation equipment into such a small space.

Karl Terzaghi expressed this concept very succinctly when he said:

> Do not design on paper what you have to wish into the ground.

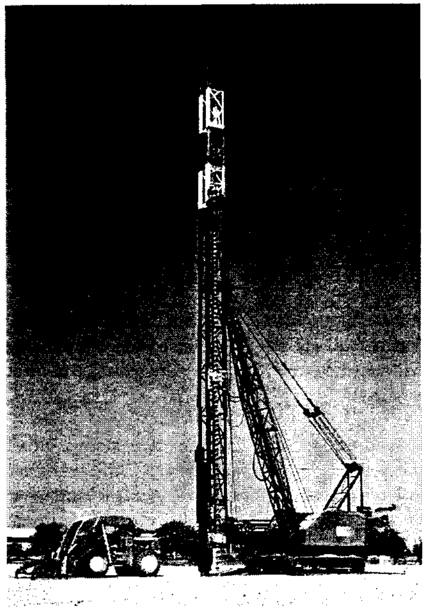


Figure 2.10 Pile foundations are installed using a pile driver, such as this one. The pile is lifted into the vertical section, which is called the leads, then driven into the ground with the pile hammer. Thus, the pile driver must be slightly taller than the pile to be installed.

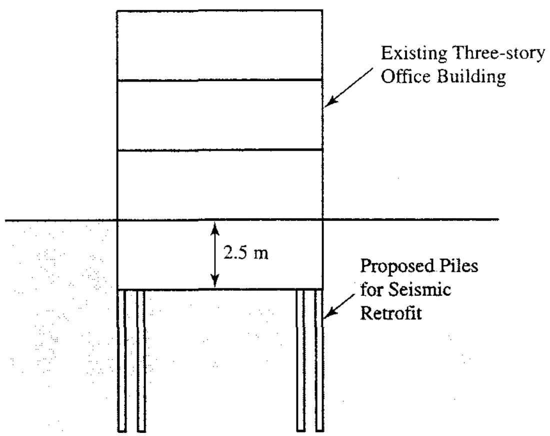


Figure 2.11 As part of a seismic retrofit project, a design engineer has called for installing 450-mm diameter, 9-m long prestressed concrete pile foundations to be installed beneath the basement of an existing building. This pile foundation design is unbuildable because the required pile-driving equipment would not fit in the basement, and because there is not enough room to set the pile upright.

This is why it is important for design engineers to have at least a rudimentary understanding of construction.

2.5 ECONOMIC REQUIREMENTS

Foundation designs are usually more conservative than those in the superstructure. This approach is justified for the following reasons:

- Foundation designs rely on our assessments of the soil and rock conditions. These assessments always include considerable uncertainty.
- Foundations are not built with the same degree of precision as the superstructure. For example, spread footings are typically excavated with a backhoe and the sides of the excavation becomes the "formwork" for the concrete, compared to concrete members in the superstructure that are carefully formed with plywood or other materials.
- The structural materials may be damaged when they are installed. For example, cracks and splits may develop in a timber pile during hard driving.
- There is some uncertainty in the nature and distribution of the load transfer between foundations and the ground, so the stresses at any point in a foundation are not always known with as much certainty as might be the case in much of the superstructure.
- The consequences of a catastrophic failure are much greater.
- The additional weight brought on by the conservative design is of no consequence, because the foundation is the lowest structural member and therefore does not affect the dead load on any other member. Additional weight in the foundation is actually beneficial in that it increases its uplift resistance.

However, gross overconservatism is not warranted. An overly conservative design can be very expensive to build, especially with large structures where the foundation is a greater portion of the total project cost. This also is a type of "failure": the failure to produce an economical design.

The nineteenth-century engineer Arthur Wellington once said that an engineer's job is that of "doing well with one dollar which any bungler can do with two." We must strive to produce designs that are both safe and cost-effective. Achieving the optimum balance between reliability (safety) and cost is part of good engineering.

Designs that minimize the required quantity of construction materials do not necessarily minimize the cost. In some cases, designs that use more materials may be easier to build, and thus have a lower overall cost. For example, spread footing foundations are usually made of low-strength concrete, even though it makes them thicker. In this case, the savings in materials and inspection costs are greater than the cost of buying more concrete.

SUMMARY

Major Points

1. The foundation engineer must determine the necessary performance requirements before designing a foundation.
2. Foundations must support various types of structural loads. These can include normal, shear, moment, and/or torsion loads. The magnitude and direction of these loads may vary during the life of the structure.
3. Loads also are classified according to their source. These include dead loads, live loads, wind loads, earthquake loads, and several others.
4. Design loads may be expressed using either the allowable stress design (ASD) or the load and resistance factor design (LRFD) method. It is important to know which method is being used, because the design computations must be performed accordingly.
5. Strength requirements are those that are intended to avoid catastrophic failure. There are two kinds: geotechnical strength requirements and structural strength requirements.
6. Serviceability requirements are those intended to produce foundations that perform well when subjected to the service loads. These requirements include settlement, heave, tile, lateral movement, vibration, and durability.
7. Settlement is often the most important serviceability requirement. The response of structures to settlements is complex, so we simplify the problem by considering two types of settlement: total settlement and differential settlement. We assign maximum allowable values for each, then design the foundations to satisfy these requirements.
8. Durability is another important serviceability requirement. Foundations must be able to resist the various corrosive and deteriorating agents in soil and water.
9. Foundations must be buildable, so design engineers need to have at least a rudimentary understanding of construction methods and equipment.
10. Foundation designs must be economical. Although conservatism is appropriate, excessively conservative designs can be too needlessly expensive to build.

Vocabulary

Allowable differential settlement
Allowable total settlement
Allowable angular distortion
Allowable stress design
Braking load
Cathodic protection
Centrifugal load
Column spacing
Constructibility requirement
Dead load
Design load
Differential settlement
Durability
Earth pressure load
Earthquake load
Economic requirement
Failure
Fluid load
Geotechnical strength requirement
Heave
Impact load
Lateral movement
Live load
Load factor
Load and resistance factor design
Moment load
Normal load
Performance requirement
Resistance factor
Self-straining load
Serviceability requirement
Settlement
Shear load
Snow load
Stream flow loads
Strength requirement
Structural strength requirement
Sulfate attack
Tilt
Torsion load
Total settlement
Vibration
Wind load

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

2.9 A certain clayey soil contains 0.30 percent sulfates. Would you anticipate a problem with concrete foundations in this soil? Are any preventive measures necessary? Explain.

2.10 A series of 50-ft long steel piles are to be driven into a natural sandy soil. The groundwater table is at a depth of 35 ft below the ground surface. Would you anticipate a problem with corrosion? What additional data could you gather to make a more informed decision?

2.11 A one-story steel warehouse building is to be built of structural steel. The roof is to be supported by steel trusses that will span the entire 70 ft width of the building and supported on columns adjacent to the exterior walls. These trusses will be placed 24 ft on center. No interior columns will be present. The walls will be made of corrugated steel. There will not be any roll-up doors. Compute the allowable total and differential settlements.

2.12 The grandstands for a minor league baseball stadium are to be built of structural steel. The structural engineer plans to use a very wide column spacing (25 m) to provide the best spectator visibility. Compute the allowable total and differential settlements.

2.13 The owner of a 100-story building purchased a plumb bob with a very long string. He selected a day with no wind, and then gently lowered the plumb bob from his penthouse office win-

Figure 2.12 Proposed department store for Problem 2.14.

dow. When it reached the sidewalk, it was 1.0 m from the side of the building. Is this building tilting excessively? Explain.

2.14 A two-story department store identical to the one in Figure 2.12 is to be built. This structure will have reinforced masonry exterior walls. The ground floor will be slab-on-grade. The reinforced concrete upper floor and roof will be supported on a steel-frame with columns 50 ft on-center. Compute the allowable total and differential settlements for this structure.

3

Soil Mechanics

Measure it with a micrometer,
mark it with chalk,
cut it with an axe.

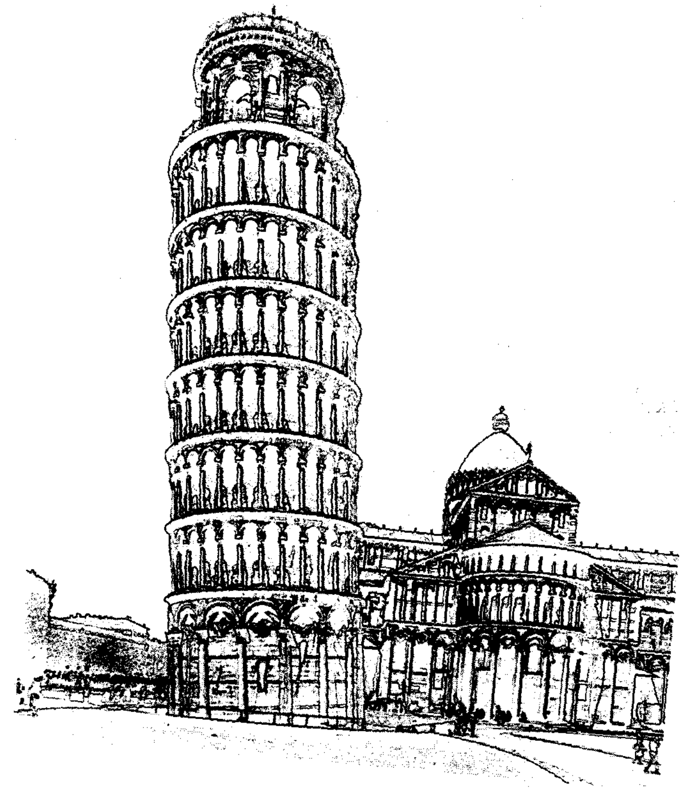
An admonition to maintain a consistent degree of precision throughout the analysis, design, and construction phases of a project.

Engineers classify earth materials into two broad categories: *rock* and *soil*. Although both materials play an important role in foundation engineering, most foundations are supported by soil. In addition, foundations on rock are often designed much more conservatively because of the rock's greater strength, whereas economics prevents overconservatism when building foundations on soil. Therefore, it is especially important for the foundation engineer to be familiar with soil mechanics.

Users of this book should already have acquired at least a fundamental understanding of the principles of soil mechanics. This chapter reviews these principles, emphasizing those that are most important in foundation analyses and design. Relevant principles of rock mechanics are included in later chapters within the context of specific applications. *Geotechnical Engineering: Principles and Practices* (Coduto, 1999), the companion volume to this book, explores all of these topics in much more detail.

3.1 SOIL COMPOSITION

One of the fundamental differences between soil and most other engineering materials is that it is a *particulate material*. This means that it is an assemblage of individual particles rather than being a *continuum* (a continuous solid mass). The engineering properties of



Part A

General Principles

Figure 2.12 Proposed department store for Problem 2.14.

dow. When it reached the sidewalk, it was 1.0 m from the side of the building. Is this building tilting excessively? Explain.

2.14 A two-story department store identical to the one in Figure 2.12 is to be built. This structure will have reinforced masonry exterior walls. The ground floor will be slab-on-grade. The reinforced concrete upper floor and roof will be supported on a steel-frame with columns 50 ft on-center. Compute the allowable total and differential settlements for this structure.

3

Soil Mechanics

Measure it with a micrometer,
mark it with chalk,
cut it with an axe.

An admonition to maintain a consistent degree of precision throughout the analysis, design, and construction phases of a project.

Engineers classify earth materials into two broad categories: *rock* and *soil*. Although both materials play an important role in foundation engineering, most foundations are supported by soil. In addition, foundations on rock are often designed much more conservatively because of the rock's greater strength, whereas economics prevents overconservatism when building foundations on soil. Therefore, it is especially important for the foundation engineer to be familiar with soil mechanics.

Users of this book should already have acquired at least a fundamental understanding of the principles of soil mechanics. This chapter reviews these principles, emphasizing those that are most important in foundation analyses and design. Relevant principles of rock mechanics are included in later chapters within the context of specific applications. *Geotechnical Engineering: Principles and Practices* (Coduto, 1999), the companion volume to this book, explores all of these topics in much more detail.

3.1 SOIL COMPOSITION

One of the fundamental differences between soil and most other engineering materials is that it is a *particulate material*. This means that it is an assemblage of individual particles rather than being a *continuum* (a continuous solid mass). The engineering properties of soil, such as strength and compressibility, are dictated primarily by the arrangement of these particles and the interactions between them, rather than by their internal properties.

Another important characteristic that differentiates soil from most other materials is that it can contain all three phases of matter (solid, liquid, and gas) simultaneously. The solid portion (the particles) includes one or more of the following materials:

- ***Rock fragments*** such as granite, limestone, and basalt.
- ***Rock minerals*** such as quartz, feldspar, mica, and gypsum.
- ***Clay minerals*** such as kaolinite, smectite, and illite.
- ***Organic matter*** such as decomposed plant materials.
- ***Cementing agents*** such as calcium carbonate.
- ***Miscellaneous materials*** such as man-made debris.

Liquids and/or gasses fill the voids between the solid particles. The liquid component is usually water, but it also could contain various chemicals in solution. The latter could come from natural sources, such as calcite leached from limestone, or artificial sources such as gasoline from leaking tanks or pipes. Likewise, the gas component is usually air, but also could consist of other materials, such as methane. For simplicity, we will refer to these components as "water" and "air."

A special exception to this three-phase structure is the case of a saturated soil at a temperature below the freezing point of water. These frozen soils are essentially a completely solid material and require special analysis and design techniques.

Weight-Volume Relationships

A knowledge of the relative proportions of solids, water, and air can give important insights into the engineering behavior of a particular soil. A *phase diagram*, as shown in Figure 3.1, describes these proportions.

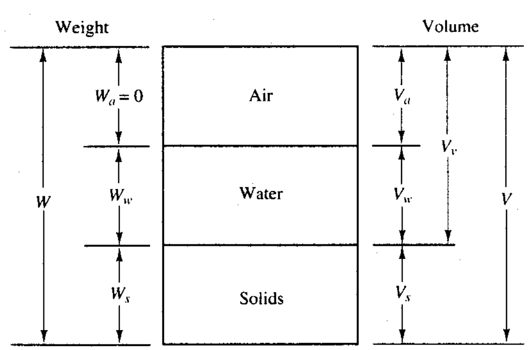


Figure 3.1 A phase diagram describes the relative proportions of solids, water, and air in a soil. The weight of the air, W_a, is negligible.

Geotechnical engineers have developed several standard parameters to define these proportions, and these parameters form much of the basic vocabulary of soil mechanics. Table 3.1 contains the definitions of weight-volume parameters commonly used in foundation engineering, along with typical numerical values. These values are not absolute limits and unusual soils may have properties outside these ranges.

These weight-volume parameters are related to each other, and many formulas are available to express these relationships. Some of the more useful formulas include the following:

$$\boxed{e = \frac{w\,G_s}{S}} \tag{3.1}$$

$$\boxed{e = \frac{G_s\,\gamma_w}{\gamma_d} - 1} \tag{3.2}$$

TABLE 3.1 DEFINITIONS AND TYPICAL VALUES OF COMMON SOIL WEIGHT-VOLUME PARAMETERS

Parameter	Symbol	Definition	Typical Range: English	Typical Range: SI
Unit weight	γ	$\frac{W}{V}$	90–130 lb/ft^3	14–20 kN/m^3
Dry unit weight	γ_d	$\frac{W_s}{V}$	60–125 lb/ft^3	9–19 kN/m^3
Unit weight of water	γ_w	$\frac{W_w}{V_w}$	62.4 lb/ft^3	9.8 kN/m^3
Buoyant unit weight	γ_b	$\gamma_{sat} - \gamma_w$	28–68 lb/ft^3	4–10 kN/m^3
Degree of saturation	S	$\frac{V_w}{V_v} \times 100\%$	2–100%	2–100%
Moisture content	w	$\frac{W_w}{W_s} \times 100\%$	3–70%	3–70%
Void ratio	e	$\frac{V_v}{V_s}$	0.1–1.5	0.1–1.5
Porosity	n	$\frac{V_v}{V} \times 100\%$	9–60 %	9–60 %
Specific gravity of solids	G_s	$\frac{W_s}{V_s \gamma_w}$	2.6–2.8	2.6–2.8

γ_{sat} is the unit weight, γ, when S = 100%.

$$\boxed{\gamma_d = \frac{\gamma}{1 + w}} \tag{3.3}$$

$$\boxed{w = S\left(\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}\right)} \tag{3.4}$$

Convert any parameters expressed as a percentage into decimal form before using them in these formulas.

Geotechnical engineers often use the term "density" instead of "unit weight" for the variable γ. Although density is technically mass/volume (not weight/volume), consider these two terms to be synonymous when used in the context of geotechnical engineering. However, this book uses only the more correct term "unit weight." Table 3.2 presents typical unit weights for various soils.

Most weight-volume parameters must be determined directly or indirectly from laboratory tests on soil samples. However, it is usually not necessary to measure the specific gravity of solids, G_s. For most projects, we can assume $G_s = 2.65$ for clays or 2.70 for sands.

TABLE 3.2 TYPICAL UNIT WEIGHTS

Soil Type and Unified Soil Classification (See Figure 3.3)	Typical Unit Weight, γ			
	Above Groundwater Table		Below Groundwater Table	
	(lb/ft^3)	(kN/m^3)	(lb/ft^3)	(kN/m^3)
GP—Poorly-graded gravel	110–130	17.5–20.5	125–140	19.5–22.0
GW—Well-graded gravel	110–140	17.5–22.0	125–150	19.5–23.5
GM—Silty gravel	100–130	16.0–20.5	125–140	19.5–22.0
GC—Clayey gravel	100–130	16.0–20.5	125–140	19.5–22.0
SP—Poorly-graded sand	95–125	15.0–19.5	120–135	19.0–21.0
SW—Well-graded sand	95–135	15.0–21.0	120–145	19.0–23.0
SM—Silty sand	80–135	12.5–21.0	110–140	17.5–22.0
SC—Clayey sand	85–130	13.5–20.5	110–135	17.5–21.0
ML—Low plasticity silt	75–110	11.5–17.5	80–130	12.5–20.5
MH—High plasticity silt	75–110	11.5–17.5	75–130	11.5–20.5
CL—Low plasticity clay	80–110	12.5–17.5	75–130	11.5–20.5
CH—High plasticity clay	80–110	12.5–17.5	70–125	11.0–19.5

Example 3.1

A 0.320 ft^3 sample of a certain soil has a weight of 38.9 lb, a moisture content of 19.2%, and a specific gravity of solids of 2.67. Find its void ratio and degree of saturation.

Solution

$$\gamma = \frac{W}{V} = \frac{38.9 \text{ lb}}{0.320 \text{ ft}^3} = 121.6 \text{ lb/ft}^3$$

$$\gamma_d = \frac{\gamma}{1 + w} = \frac{121.6 \text{ lb/ft}^3}{1 + 0.192} = 102.0 \text{ lb/ft}^3$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.67)(62.4 \text{ lb/ft}^3)}{102.0 \text{ lb/ft}^3} - 1 = \mathbf{0.633} \qquad \Leftarrow \textit{Answer}$$

$$e = \frac{w\,G_s}{S} \rightarrow S = \frac{w\,G_s}{e} = \frac{(0.192)(2.67)}{0.633} = \mathbf{81.0\%} \qquad \Leftarrow \textit{Answer}$$

Relative Density

The *relative density*, D_r, is a convenient way to express the void ratio of sands and gravels. It is based on the void ratio of the soil, e, the *minimum index void ratio*, e_{min}, and the *maximum index void ratio*, e_{max}:

$$\boxed{D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100\%} \tag{3.5}$$

The values of e_{min} and e_{max} are determined by conducting standard laboratory tests [ASTM D4254]. The in-situ e could be computed from the unit weight of the soil using Equation 3.2, but accurate measurements of the unit weight of clean sands and gravels are difficult or impossible to obtain. Therefore, engineers often obtain D_r from correlations based on in-situ tests, as described in Chapter 4.

If a soil has a relative density of 0%, it is supposedly in its loosest possible condition, while at 100 % it is supposedly in its densest possible condition. Although it is possible for natural soils to have relative densities outside this range, such conditions are very unusual. A relationship between consistency of sands and gravels and relative density is shown in Table 3.3.

Do not confuse relative density with relative compaction. The latter is based on the Proctor compaction test [ASTM D1557] and is typically used to evaluate compacted fills. Although these two parameters measure similar soil properties, and both are expressed as a percentage, they are not numerically equal.

The relative density applies only to sands and gravels with less than 15 percent fines. It can be an excellent indicator of the engineering properties of such soils, and it is therefore an important part of many analysis methods. However, other considerations,

TABLE 3.3 CONSISTENCY OF COARSE-GRAINED SOILS VARIOUS RELATIVE DENSITIES (Adapted from Lambe and Whitman, 1969)

Relative Density, D_r (%)	Classification
0–15	Very loose
15–35	Loose
35–65	Medium dense[a]
65–85	Dense
85–100	Very dense

[a]Lambe and Whitman used the term "medium," but "medium dense" is better because "medium" usually refers to the grain size distribution.

such as stress history, mineralogical content, grain-size distribution, and fabric (the configuration of the particles), also affect the engineering properties.

Particle Size

Because soil is a particulate material, it is natural to consider the size of these particles and their effect on the behavior of the soil. Several different classification schemes are available, but the one published by ASTM (American Society for Testing and Materials) is the most common system used by geotechnical engineers. This system classifies soil particles, as shown in Table 3.4.

TABLE 3.4 ASTM PARTICLE SIZE CLASSIFICATION (Per ASTM D2487)

Sieve Size		Particle Diameter		Soil Classification	
Passes	Retained on	(in)	(mm)		
	12 in	> 12	> 350	Boulder	Rock Fragments
12 in	3 in	3–12	75.0–350	Cobble	
3 in	3/4 in	0.75–3	19.0–75.0	Coarse gravel	Soil
3/4 in	#4	0.19–0.75	4.75–19.0	Fine gravel	
#4	#10	0.079–0.19	2.00–4.75	Coarse sand	
#10	#40	0.016–0.079	0.425–2.00	Medium sand	
#40	#200	0.0029–0.016	0.075–0.425	Fine sand	
#200		< 0.0029	< 0.075	Fines (silt + clay)	

Most natural soils contain a wide variety of particle sizes and thus do not fall completely within any of the categories listed in Table 3.4. Thus, the distribution of particle sizes in a particular soil is most easily expressed in the form of a *grain-size distribution curve*, which is a plot of the percentage of the dry soil by weight that is smaller than a certain particle diameter vs. the particle diameter.

Clays

Soils that consist of silt, sand, or gravel are primarily the result of physical and mild chemical weathering processes, and retain much of the chemical structure of their parent rocks. However, this is not the case with clay soils because they have experienced extensive chemical weathering and have been changed into a new material quite different from the parent rocks.

One of the important consequences of this extensive weathering is that individual clay particles are extremely small (less than 2×10^{-3} mm in diameter). They cannot be seen with optical microscopes, and require an electron microscope for scientific study. This small size is significant because the weights of individual particles are small compared to the ionic forces between particles, so their engineering behavior (shear strength, compressibility, etc.) depends more on these interparticle forces. This is quite different from coarser soils, whose behavior depends primarily on gravitational forces. Another important difference between clays and other soils is that they offer much more resistance to the flow of water. This low *hydraulic conductivity* impacts many of its engineering properties.

Because of these many differences, we often have different analysis methods for clays. Many of the discussions in this book treat clays and clayey silts separately from sands, nonplastic silts, and gravels.

Plasticity and the Atterberg Limits

The moisture content, *w*, is a basic and useful indicator of soil properties, especially in cohesive soils. For example, clays with a low moisture content are stronger and less compressible than those with a high moisture content.

In 1911, the Swedish soil scientist Albert Atterberg (1846–1916) developed a series of tests to evaluate the relationship between moisture content and soil consistency. In the 1930s, Karl Terzaghi and Arthur Casagrande adapted these tests for civil engineering purposes and they soon became a routine part of geotechnical engineering. This series includes three separate tests: the *liquid limit test*, the *plastic limit test*, and the *shrinkage limit test* (ASTM D427 and D4318). Together they are known as the *Atterberg limits tests*. Figure 3.2 shows qualitative descriptions of the changes in consistency of a fine-grained soil that occur as its moisture content changes. Dry cohesive soils are hard and brittle, whereas wet cohesive soils are soft and pliable. Although these changes in consistency are gradual, the Atterberg limits tests define the boundaries between the various

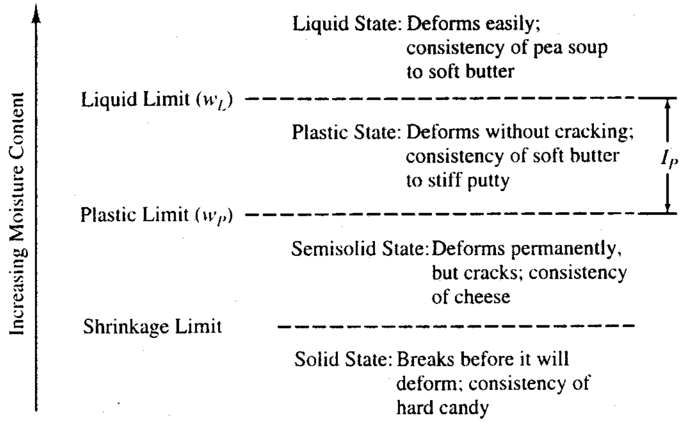


Figure 3.2 Consistency of fine-grained soils at different mositure contents (Sowers, 1979).

states in a somewhat arbitrary but standardized way. The test results are expressed in terms of the moisture content with the percent sign dropped:

w_s = shrinkage limit (or SL)
w_p = plastic limit (or PL)
w_L = liquid limit (or LL)

By comparing the moisture content of a soil with its Atterberg limits, an engineer could gain a qualitative sense of its consistency. For example, if a certain soil has a liquid limit of 55, a plastic limit of 20, and a moisture content of 25%, then it would have a consistency comparable to that of a stiff putty (i.e., it is slightly wetter than the plastic limit).

The liquid limit and plastic limit tests are part of many laboratory test programs. They are inexpensive and the results can be quite useful. In contrast, the shrinkage limit has little practical significance for engineers and is rarely measured.

When the soil is at a moisture content between the liquid limit and the plastic limit, it is said to be in a *plastic state*. It can be easily molded without cracking or breaking. The children's toy play-dough has a similar consistency. This property relates to the amount and type of clay in the soil. The *plasticity index*, PI or I_p, is a measure of the range of moisture contents that encompass the plastic state:

$$\boxed{I_p = w_L - w_p} \tag{3.6}$$

Soils with a large clay content retain this plastic state over a wide range of moisture contents, and thus have a high plasticity index. The opposite is true of silty soils. Clean sands and gravels are considered to be nonplastic (NP).

The Atterberg limits give the engineer a feel for the soil's behavior, and they form the basis for the Unified Soil Classification System (described in the next section). Engineers also have developed empirical correlations between the Atterberg limits and soil properties such as compressibility and shear strength.

3.2 SOIL CLASSIFICATION

Standardized systems of classifying soil are very important, and many such systems have been developed. A proper classification reveals much useful information to a foundation engineer.

Unified Soil Classification System

The most common soil classification system for foundation engineering problems is the *Unified Soil Classification System* (USCS) [ASTM D2487]. This system assigns a two or four-letter *group symbol* to the soil, along with standardized descriptions called the *group name*. Several potential group names are associated with each group symbol. For example, a certain soil might be classified as "SC–Clayey sand," where SC is the group symbol and clayey sand is the group name.

The first letter of the group symbol tells the general type of soil:

G = gravel
S = sand
M = silt
C = clay
O = organic

The second letter is a supplementary description:

W = well-graded
P = poorly-graded
M = silty
C = clayey
L = low plasticity
H = high plasticity

A special group symbol, Pt, is assigned to peat, which is a highly organic soil that is generally unsuitable for supporting structural foundations.

Figure 3.3 presents an abbreviated outline of the USCS group symbols. See *Geotechnical Engineering: Principles and Practices* or ASTM D2487 for a complete explanation of group symbols and group names.

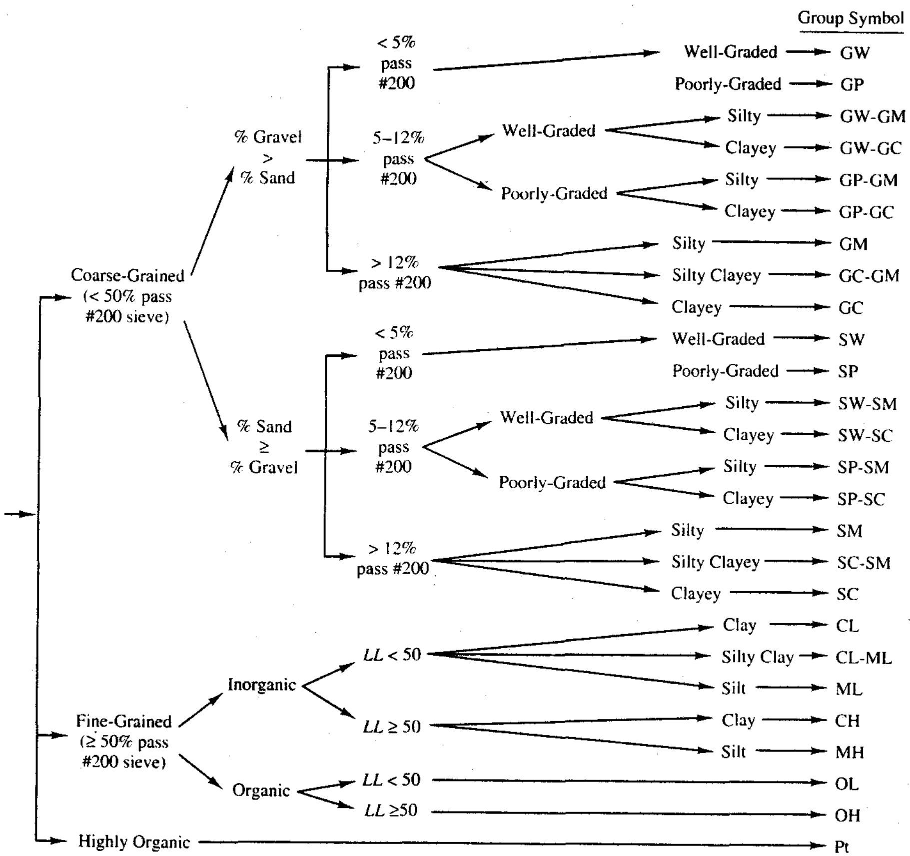


Figure 3.3 Abbreviated outline of the Unified Soil Classification System (Adapted from ASTM D2487. Used with permission of ASTM).

QUESTIONS AND PRACTICE PROBLEMS

3.1 Explain the difference between moisture content and degree of saturation.

3.2 A certain saturated sand ($S = 100\%$) has a moisture content of 25.1% and a specific gravity of solids of 2.68. It also has a maximum index void ratio of 0.84 and a minimum index void ratio of 0.33. Compute its relative density and classify its consistency.

3.3 Consider a soil that is being placed as a fill and compacted using a sheepsfoot roller (a piece of construction equipment). Will the action of the roller change the void ratio of the soil? Explain.

3.4 A sample of soil has a volume of 0.45 ft^3 and a weight of 53.3 lb. After being dried in an oven, it has a weight of 45.1 lb. It has a specific gravity of solids of 2.70. Compute its moisture content and degree of saturation before it was placed in the oven.

3.3 GROUNDWATER

The presence of water in soil has a dramatic impact on its behavior. Karl Terzaghi (1939) once said:

> . . . in engineering practice, difficulties with soils are almost exclusively due not to the soils themselves but to the water contained in their voids. On a planet without any water there would be no need for soil mechanics.

The adverse effects of water in foundation engineering problems include:

- Softening of clay bonds with resulting decrease in strength and increase in compressibility
- Reduction of effective stress, with corresponding decrease in shear strength
- Expansion (in the case of certain clays) or collapse (in the case of certain loose, dry soils) when dry soils become wetted
- Hydrostatic uplift pressures

Therefore, the assessment of groundwater conditions is an important part of foundation engineering.

Groundwater Table

Groundwater conditions are often very complex, and can be the subject of extensive investigation and monitoring. These complexities can include *artesian* conditions, *perched groundwater*, and complex interbedded *aquicludes* and *aquifers*. However, in this book we will consider only simple groundwater conditions defined solely by the position of a horizontal *groundwater table* (also called the *phreatic surface*). It represents the level to which groundwater would rise in an observation well, as shown in Figure 3.4. In this simple groundwater profile, all of the soil voids below the groundwater table are completely filled with water ($S = 100\%$).

Pore Water Pressure

The *pore water pressure*, u, is the pressure in the water within the soil "pores" (voids). There are two sources of pore water pressure: hydrostatic pore water pressure and excess pore water pressure.

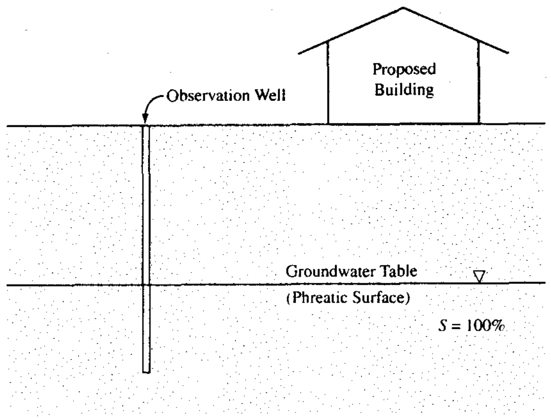


Figure 3.4 Simple groundwater conditions defined solely by the position of the groundwater table. An observation well is a perforated pipe placed in a boring for the purpose of locating and monitoring the groundwater table.

Hydrostatic Pore Water Pressure

The *hydrostatic pore water pressure*, u_h, is that solely caused by the force of gravity acting on the pore water. In the simple groundwater conditions just defined, the hydrostatic pore water pressure is:

$$u_h = \gamma_w z_w \tag{3.7}$$

Where:

u_h = hydrostatic pore water pressure at a point in the ground
γ_w = unit weight of water = 62.4 lb/ft^3 = 9.8 kN/m^3
z_w = vertical distance from groundwater table to the point

For example, if the groundwater table is at elevation 215 ft and the point is at elevation 190 ft, then $u_h = \gamma_w z_w = (62.4 \text{ lb/ft}^3)(215 - 190 \text{ ft}) = 1600 \text{ lb/ft}^2$.

Excess Pore Water Pressure

If the soil is in the process of settling, heaving, or shearing, the resulting squeezing or expansion of the soil voids can produce an additional pressure called the *excess pore water pressure*, u_e. The magnitude of u_e depends on many factors, including:

- The rate of application of the normal and/or shear loads that are causing the void size to change

- The change in the void size
- The rate of drainage into or out of the voids

The excess pore water pressure is positive when the voids are in the process of becoming smaller, as when the soil is settling, and negative when the voids are becoming larger, as when the soil is heaving. Shear strains can produce either positive or negative excess pore water pressures.

The pore water pressure, u, is the sum of the hydrostatic and excess pore water pressures:

$$u = u_h + u_e \tag{3.8}$$

Since excess pore water pressures are the consequence of settling, heaving, or shearing, they dissipate when these processes are completed. Therefore, excess pore water pressures are always temporary phenomena.

Although it is sometimes possible to explicitly consider excess pore water pressures in geotechnical analyses (such as when performing consolidation rate analyses), we will not be doing so in this book. Our discussions of excess pore water pressures will be limited to helping us understand the physical processes that control soil behavior. All of the pore water pressure computations in this book will consider only the hydrostatic pressure. Thus, for our purposes:

$$\boxed{u = \gamma_w z_w} \tag{3.9}$$

The impact of excess pore water pressures on soil properties will be considered implicitly, as described later in this chapter.

3.4 STRESS

Many foundation engineering analyses require a knowledge of the stresses in the soil. These stresses have two kinds of sources:

- *Geostatic stresses* are the result of the force of gravity acting directly on the soil mass.
- *Induced stresses* are caused by external loads, such as foundations.

Both normal and shear stresses may be present in a soil. These are represented by the variables σ and τ, respectively. We will identify the direction of normal stresses using subscripts σ_x and σ_y for the two perpendicular horizontal stresses (which, for foundation design purposes are usually assumed to be equal in magnitude) and σ_z is the vertical stress.

Normal stresses in the ground are nearly always compressive, so geotechnical engineers use a sign convention opposite that used by structural engineers: compressive stresses are positive, while tensile stresses are negative. This way we virtually always work with positive numbers.

When using English units, express stresses in units of lb/ft^2 in the soil and lb/in^2 in structural members. With SI units, use kPa in the soil and MPa in structural members. Engineers in some non-SI metric countries use units of kg_f/cm^2 or bars (1 bar = 100 kPa).

Geostatic Stresses

The most important geostatic stress is the vertical compressive stress because it is directly caused by gravity. Geotechnical engineers compute this stress more often than any other. The geostatic vertical total stress, σ_z, is:

$$\boxed{\sigma_z = \Sigma \gamma H} \tag{3.10}$$

Where:

σ_z = geostatic vertical total stress
γ = unit weight of soil stratum
H = thickness of soil stratum

We carry this summation from the ground surface down to the point at which the stress is to be computed. This computation is similar to the procedure for computing pressure in a body of water, except for the additional consideration that unit weight may vary with depth.

Part of the vertical total stress is carried by the solid particles, and the rest is carried by the pore water. We are especially interested in the part carried by the solids, and have given it a special name: the *effective stress*, σ'. It is computed as follows:

$$\boxed{\sigma' = \sigma - u} \tag{3.11}$$

In the case of vertical stress:

$$\boxed{\sigma'_z = \sigma_z - u} \tag{3.12}$$

Where:

σ_z' = vertical effective stress
σ_z = vertical total stress
u = pore water pressure

Combining equations 3.10 and 3.12 gives:

$$\boxed{\sigma'_z = \Sigma \gamma H - u} \tag{3.13}$$

Note how the distribution of force between the solids and water is *not* proportional to their respective cross-sectional areas.

Horizontal Stress

The geostatic horizontal stress also is important for many engineering analyses. For example, the design of retaining walls depends on the horizontal stresses in the soil being retained. It may be expressed either as total horizontal stress, σ_x, or effective horizontal stress, σ_x'. The ratio of the horizontal to vertical effective stresses is defined as the *coefficient of lateral earth pressure*, K:

$$\boxed{K = \frac{\sigma'_x}{\sigma'_z}} \tag{3.14}$$

The value of K in undisturbed ground is known as the *coefficient of lateral earth pressure at rest*, K_0, and can vary between about 0.2 and 6. Typical values are between 0.35 and 0.7 for normally consolidated soils and between 0.5 and 3 for overconsolidated soils.[1] The most accurate way to determine K_0 is by measuring σ_x in-situ using methods such as the pressuremeter, dilatometer, or stepped blade, and combining it with computed values of σ_z and u. However, these methods are not commonly used in North America, and are probably justified only for critical projects. A less satisfactory method is to measure K_0 in the laboratory using a triaxial compression machine or other equipment. Unfortunately, this method suffers from problems because of sample disturbance, stress history, and other factors.

The most common method of assessing K_0 is by using empirical correlations with other soil properties. Several such correlations have been developed, including the following by Mayne and Kulhawy (1982), which is based on laboratory tests on 170 soils that ranged from clay to gravel. This formula is applicable only when the ground surface is level:

$$\boxed{K_0 = (1 - \sin\phi')\mathrm{OCR}^{\sin\phi'}} \tag{3.15}$$

Where:

K_0 = coefficient of lateral earth pressure at rest
ϕ' = effective friction angle (see definition later in this chapter)
OCR = overconsolidation ratio (see definition later in this chapter)

Chapter 23 discusses lateral earth pressures in more detail.

---

[1]The terms "normally consolidated soils" and "overconsolidated soils" are defined later in this chapter.

Induced Stresses

Induced stresses are those caused by external loads, such as structural foundations. These stresses are a very important part of foundation engineering, and most of the related geotechnical analyses focus on the soil's response to these stresses. We will discuss them in the context of the various types of foundations.

Example 3.2

The soil profile beneath a certain site consists of 5.0 m of silty sand underlain by 13.0 m of clay. The groundwater table is at a depth of 2.8 m below the ground surface. The sand has a unit weight of 19.0 kN/m³ above the groundwater table and 20.0 kN/m³ below. The clay has a unit weight of 15.7 kN/m³, an effective friction angle of 35°, and an overconsolidation ratio of 2. Compute σ_x, σ'_x, σ_z, and σ'_z at a depth of 11.0 m.

Solution

$$\sigma_z = \Sigma \gamma H$$

$$= (19.0 \text{ kN/m}^3)(2.8 \text{ m}) + (20.0 \text{ kN/m}^3)(2.2 \text{ m}) + (15.7 \text{ kN/m}^3)(6.0 \text{ m})$$

$$= \mathbf{191 \text{ kPa}} \quad \Leftarrow \textit{Answer}$$

$$u = \gamma_w z_w = (9.8 \text{ kN/m}^3)(8.2 \text{ m}) = 80 \text{ kPa}$$

$$\sigma'_z = \sigma_z - u = 191 \text{ kPa} - 80 \text{ kPa} = \mathbf{111 \text{ kPa}} \quad \Leftarrow \textit{Answer}$$

$$K_0 = (1 - \sin\phi')\text{OCR}^{\sin\phi'} = (1 - \sin 35°)2^{\sin 35°} = 0.635$$

$$\sigma'_x = K\sigma'_z = (0.635)(111 \text{ kPa}) = \mathbf{70 \text{ kPa}} \quad \Leftarrow \textit{Answer}$$

$$\sigma_x = \sigma'_x + u = 70 \text{ kPa} + 80 \text{ kPa} = \mathbf{150 \text{ kPa}} \quad \Leftarrow \textit{Answer}$$

QUESTIONS AND PRACTICE PROBLEMS

3.5 A site is underlain by a soil that has a unit weight of 18.7 kN/m³ above the groundwater table and 19.9 kN/m³ below. The groundwater table is located at a depth of 3.5 m below the ground surface. Compute the total vertical stress, pore water pressure, and effective vertical stress at the following depths below the ground surface:

a. 2.2 m
b. 4.0 m
c. 6.0 m

3.6 The subsurface profile at a certain site is shown in Figure 3.5. Compute u, σ_x, σ_z, σ_x', and σ_z' at Point A.

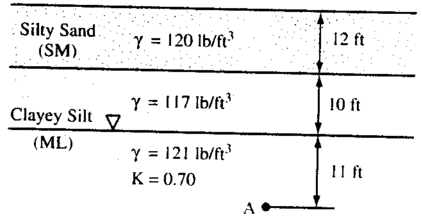


Figure 3.5 Soil profile for Problem 3.6.

3.5 COMPRESSIBILITY AND SETTLEMENT

Settlement requirements often control the design of foundations, as discussed in Chapter 2, so we need to have methods of computing the settlement in soils. In this section we will review the physical processes that control settlement and the methods of computing settlement due to the weight of extensive fills. Chapters 7 and 14 use these concepts to compute settlements due to the loads applied to foundations.

Physical Processes

Settlement is the result of various physical processes. These include the following:

- **Consolidation settlement**—When a fill is placed on the ground, the vertical stress in the underlying soil increases. This increase causes the solid particles to pack together more tightly, which in turn causes the soil to settle. We call this process *consolidation*, and the resulting settlement is *consolidation settlement.*
- **Secondary compression settlement**—The process called *secondary compression* is the result of creep, viscous behavior of the clay-water system, the compression and decomposition of organic matter, and other physical and chemical processes. *Secondary compression settlement* is negligible in sands and in overconsolidated clays. It can be important in some normally consolidated clays, and is usually very important in organic soils.
- **Distortion settlement**—When heavy loads are applied over a small area, the soil can deform laterally. Similar lateral deformations also can occur near the perimeter of larger loaded areas. These deformations produce *distortion settlement*, which can be important in foundations. Chapter 7 discusses this topic in more detail.
- **Settlement caused by compression or collapse of underground mines, sinkholes, or tunnels**—These can be important at some sites, but are beyond the scope of this book.
- **Settlement caused by hydrocollapse**—This occurs in certain soils when they become wetted. We will discuss this phenomena in Chapter 20.

- **Settlement or heave caused by wetting or drying of expansive soils**—This occurs in certain clayey soils, and is discussed in Chapter 19.

Consolidation

Consolidation is usually the most important source of settlement, and it is the only one we will discuss in this chapter. The consolidation process begins with the application of a load, which produces an increase in the vertical stress, $\Delta\sigma_z$, in the underlying ground. If the load is caused by the weight of an extensive fill, then:

$$\boxed{\Delta\sigma_z = \gamma_{fill} H_{fill}} \tag{3.16}$$

Where:

$\Delta\sigma_z$ = change in vertical stress in the ground beneath the fill
γ_{fill} = unit weight of the fill
H_{fill} = thickness of the fill

If the length and width of the fill are large (i.e., an *extensive* fill), then $\Delta\sigma_z$ is essentially constant with depth.

The introduction of $\Delta\sigma_z$ causes an immediate increase in the vertical total stress, σ_z, in the underlying soils. In addition, if these soils are saturated, $\Delta\sigma_z$ also causes an equal amount of excess pore water pressure, u_e. In other words, immediately after the fill is placed, its weight is carried entirely by the pore water. This means the vertical effective stress immediately after loading is unchanged from its pre-construction value.

The presence of these excess pore water pressures produces a hydraulic gradient, which forces some of the pore water to flow out of the soil. As the water flows out, the soil consolidates and $\Delta\sigma_z$ is slowly transferred from the pore water to the solids. Eventually, the excess pore water pressure becomes zero and the vertical effective stress increases to its final value, σ_{zf}':

$$\boxed{\sigma_{zf}' = \sigma_{z0}' + \Delta\sigma_z} \tag{3.17}$$

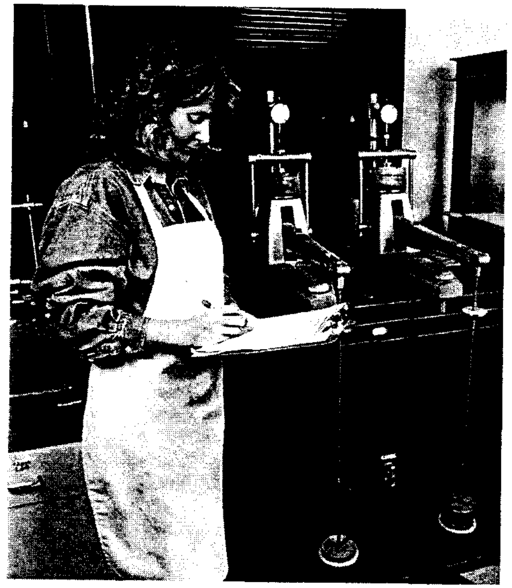
Where:

σ_{z0}' = initial vertical effective stress at a point in the soil beneath the fill
σ_{zf}' = final vertical effective stress at a point in the soil beneath the fill
$\Delta\sigma_z$ = change in vertical stress

The consolidation settlement, δ_c, is the result of this increase in vertical effective stress.

Consolidation (Oedometer) Tests

To predict the magnitude of δ_c in a soil, we need to know its stress–strain properties. This normally required obtaining soil samples in the field, bringing them to the laboratory, subjecting them to a series of loads, and measuring the corresponding settlements. We do



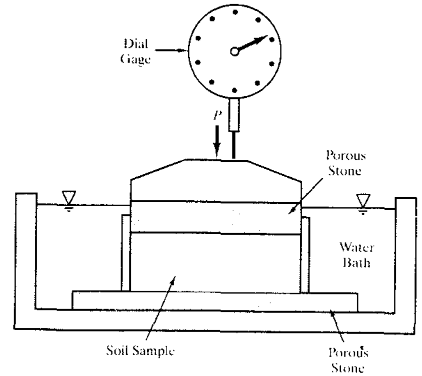


Figure 3.6 (a) Performing consolidation tests in the laboratory. The two consolidometers use the weights in the foreground to load the samples; (b) cross section of a consolidometer.

this by conducting a *consolidation test* (also known as an *oedometer test*), which is performed in a *consolidometer* (or *oedometer*) as shown in Figure 3.6.

Because we are mostly interested in the engineering properties of natural soils as they exist in the field, consolidation tests are usually performed on high quality "undisturbed" samples. It is fairly simple to obtain these samples in soft to medium clays, but quite difficult in clean sands. It also is important for samples that were saturated in the field to remain so during storage and testing, because irreversible changes can occur if they are allowed to dry.

Sometimes engineers need to evaluate the consolidation characteristics of proposed compacted fills, and do so by performing consolidation tests on samples that have been remolded and compacted in the laboratory. These tests are usually less critical because well-compacted fills generally have a low compressibility.

Test Procedure and Results

The test procedure consists of applying a series of normal loads to the sample, allowing it to consolidate under each load, and measuring the corresponding settlements. The results are presented in a semilogarithmic plot as shown in Figure 3.7. This plot represents consolidation both in terms of vertical strain, ϵ_z, and change in void ratio, e. The first part of the test results, indicated by Curve AB, is called the *recompression curve*. The middle part, indicated by Curve BC, is called the *virgin curve*. Upon reaching Point C, the sample is progressively unloaded, thus producing the *rebound curve*, CD.

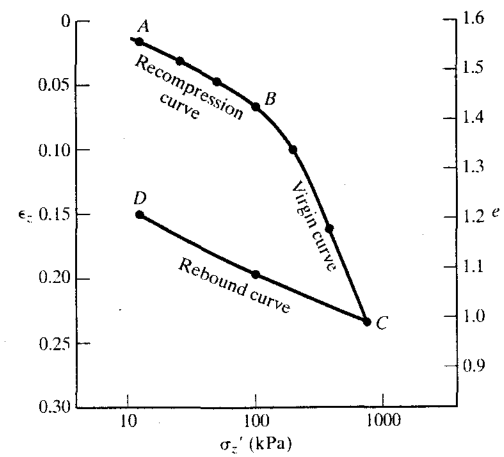


Figure 3.7 Results of a laboratory consolidation test. The initial void ratio, e_0, is 1.60.

The slope of the virgin curve is called the *compression index*, C_c:

$$C_c = -\frac{d\,e}{d \log \sigma_z'} \tag{3.18}$$

Since the virgin curve is a straight line (on a semilogarithmic plot), we can obtain a numerical value for C_c by selecting any two points, a and b, on this line, as shown in Figure 3.8, and rewriting Equation 3.18 as:

$$C_c = -\frac{e_a - e_b}{(\log \sigma_z')_b - (\log \sigma_z')_a} \tag{3.19}$$

Equation 3.19 is used when the test results have been plotted in terms of void ratio ($e - \sigma_z'$). Alternatively, if the data is plotted only in vertical strain form ($\epsilon_z - \sigma_z'$) form then:

$$\frac{C_c}{1 + e_0} = \frac{(\epsilon_z)_b - (\epsilon_z)_a}{(\log \sigma_z')_b - (\log \sigma_z')_a} \tag{3.20}$$

Where:

e_0 = initial void ratio (i.e., at beginning of test)

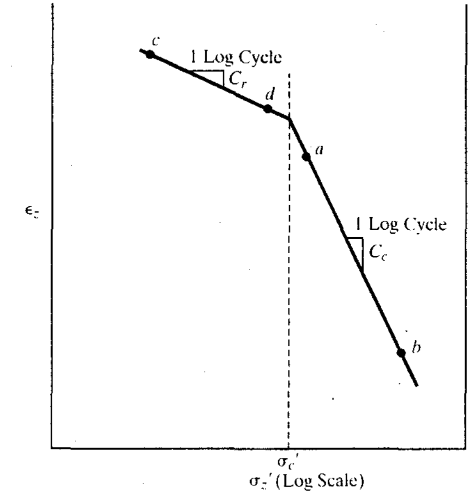


Figure 3.8 The slopes of consolidation curves on a semilogarithmic e vs. σ_z' plot are C_c and C_r. The break in slope occurs at the preconsolidation stress, σ_c'.

When using Equations 3.19 and 3.20, it is convenient to select points a and b such that $\log (\sigma_z')_b = 10 \log (\sigma_z')_a$. This makes the denominator of both equations equal to 1, which simplifies the computation. This choice also demonstrates that C_c could be defined as the reduction in void ratio per tenfold increase (one log-cycle) in effective stress.

In theory, the recompression and rebound curves have nearly equal slopes, but the rebound curve is more reliable because it is less sensitive to sample disturbance effects. This slope, which we call the *recompression index*, C_r, is defined in the same way as C_c and can be found using Equations 3.21 or 3.22 with points c and d on the decompression curve.

Using void ratio data:

$$C_r = \frac{e_c - e_d}{(\log \sigma_z')_d - (\log \sigma_z')_c} \tag{3.21}$$

Using strain data:

$$\frac{C_r}{1 + e_0} = \frac{(\epsilon_z)_d - (\epsilon_z)_c}{(\log \sigma_z')_d - (\log \sigma_z')_c} \tag{3.22}$$

Another important parameter from the consolidation test results is the stress that corresponds to the break-in-slope in Figure 3.9. This is called the *preconsolidation stress*, σ_c', and represents the greatest vertical effective stress the soil has ever experienced. The value of σ_c' obtained from the consolidation test represents only the conditions at the point where the sample was obtained. If the sample had been taken at a different elevation, the preconsolidation stress would change accordingly.

Consolidation Status in the Field

Normally Consolidated, Overconsolidated, and Underconsolidated Soils

When performing consolidation analyses, we need to compare the preconsolidation stress, σ_c', with the initial vertical effective stress, σ_{z0}'. The former is determined from laboratory test data, as described earlier, while the later is determined using Equation 3.13 with the original field conditions (i.e., without the new load) and the original hydrostatic pore water pressures (i.e., Equation 3.9). Both values must be determined at the same depth, which normally is the depth of the sample on which the consolidation test was performed. Once these values have been determined, we need to assess which of the following three conditions exist in the field:

1. If $\sigma_{z0}' \approx \sigma_c'$, then the vertical effective stress in the field has never been higher than its current magnitude. This condition is known as being *normally consolidated* (NC). For example, this might be the case at the bottom of a lake, where sediments brought in by a river have slowly accumulated. In theory these two values should be exactly equal, but both are subject to error because of sample disturbance and other factors. Therefore, we will consider the soil to be normally consolidated if they are equal within about ± 20%.
2. If $\sigma_{z0}' < \sigma_c'$, then the vertical effective stress in the field was once higher than its current magnitude. This condition is known as being *overconsolidated* (OC) or *preconsolidated*. There are many processes that can cause a soil to become overconsolidated, including (Brumund, et al., 1976):
   - Extensive erosion or excavation such that the ground surface was once much higher than its current elevation.
   - Surcharge loading from a glacier, which has since melted.
   - Surcharge loading from a structure, such as a storage tank, which has since been removed.
   - Increases in the pore water pressure, such as from a rising groundwater table.
   - Dessiccation (drying) and the resulting surface tension forces in the remaining pore water.
   - Chemical changes in the soil.

   The term *overconsolidated* can be misleading since it implies there has been excessive consolidation. Although there are a few situations, such as cut slopes, where heavily overconsolidated soils are less desirable, overconsolidation is almost always a good thing.
3. If $\sigma_{z0}' > \sigma_c'$, the soil is said to be *underconsolidated*, which means the soil is still in the process of consolidating under a previously applied load. We will not be dealing with this case.

TABLE 3.5 CLASSIFICATION OF SOIL COMPRESSIBILITY

$\frac{C_c}{1+e_0}$ or $\frac{C_r}{1+e_0}$	Classification
0–0.05	Very slightly compressible
0.05–0.10	Slightly compressible
0.10–0.20	Moderately compressible
0.20–0.35	Highly compressible
> 0.35	Very highly compressible

Table 3.5 gives a classification of soil compressibility based on $C_c / (1+e_0)$ for normally consolidated soils or $C_r / (1+e_0)$ for overconsolidated soils.

Overconsolidation Margin and Overconsolidation Ratio

The σ_c' values from the laboratory only represent the preconsolidation stress at the sample depth. To estimate σ_c' at other depths in the same strata (i.e., in a soil strata with the same geologic origin), compute the *overconsolidation margin*, σ_m', at the sample depth using:

$$\boxed{\sigma_m' = \sigma_c' - \sigma_{z0}'} \qquad (3.23)$$

The overconsolidation margin should be approximately constant in a strata with common geologic origins, so we can estimate the preconsolidation stress at other depths in that strata by using Equation 3.23 with σ_{z0}' at the desired depth. In normally consolidated soils, $\sigma_m' = 0$. Table 3.6 presents typical ranges of σ_m'.

Another useful parameter is the *overconsolidation ratio* or OCR:

$$\boxed{\text{OCR} = \frac{\sigma_c'}{\sigma_{z0}'}} \qquad (3.24)$$

TABLE 3.6 TYPICAL RANGES OF OVERCONSOLIDATION MARGINS

Overconsolidation Margin, σ_m'		
(kPa)	(lb/ft²)	Classification
0	0	Normally consolidated
0–100	0–2000	Slightly overconsolidated
100–400	2000–8000	Moderately overconsolidated
> 400	> 8000	Heavily overconsolidated

Unlike the overconsolidation margin, the OCR is not constant with depth in a given strata. For normally consolidated soils, OCR = 1.

Example 3.3

A proposed fill is to be placed on the soil profile shown in Figure 3.9. A consolidation test has been performed on a sample obtained from Point A, and this test indicates the preconsolidation pressure at that point is 200 kPa. Determine whether the silty clay stratum is normally consolidated or overconsolidated, and compute the overconsolidation margin and overconsolidation ratio. The proposed fill has not yet been placed.

Solution

At Point A:

$$\sigma'_{z0} = \Sigma \gamma H - u$$
$$= (19.1\ \text{kN/m}^3)(1.2\ \text{m}) + (20.2\ \text{kN/m}^3)(1.0\ \text{m}) + (17.8\ \text{kN/m}^3)(5.0\ \text{m}) - (9.8\ \text{kN/m}^3)(6.0\ \text{m})$$
$$= 73\ \text{kPa}$$

$\sigma'_{z0} < \sigma'_c$ by more than 20%, so **soil is overconsolidated** ⇐ ***Answer***

$\sigma'_m = \sigma'_c - \sigma'_{z0} = 200\ \text{kPa} - 73\ \text{kPa} = \mathbf{127\ kPa}$ ⇐ ***Answer***

$$\text{OCR} = \frac{\sigma'_c}{\sigma'_{z0}} = \frac{200\ \text{kPa}}{73\ \text{kPa}} = \mathbf{2.7} \quad \Leftarrow \textit{Answer}$$

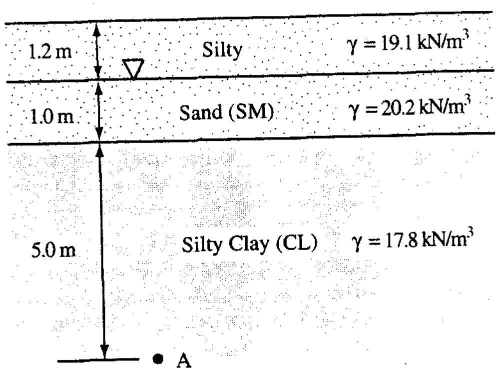


Figure 3.9 Soil profile for Example 3.3.

Compressibility of Sands and Gravels

The principles of consolidation apply to all soils, but the consolidation test is primarily applicable to clays and silts. It is very difficult to perform reliable consolidation tests on most sands because they are more prone to sample disturbance, and this disturbance has a significant effect on the test results. Clean sands are especially troublesome. Gravels have similar sample disturbance problems, plus their large grain size requires very large samples and a very large consolidometer, neither of which is practical.

Fortunately, sands and gravels subjected to static loads are much less compressible than silts and clays, so it often is sufficient to use estimated values of C_c or C_r in lieu of laboratory tests. For sands, these estimates can be based on the data gathered by Burmister (1962) as interpreted in Table 3.7. He performed a series of consolidation tests on samples reconstituted to various relative densities. Engineers can estimate the in-situ relative density using the methods described in Chapter 4, then select an appropriate $C_c/(1+e_0)$ from this table. Note that all of these values are "very slightly compressible" as defined in Table 3.5.

For saturated overconsolidated sands, $C_r/(1+e_0)$ is typically about one-third of the values listed in Table 3.7, which makes such soils nearly incompressible. Compacted

TABLE 3.7 TYPICAL CONSOLIDATION PROPERTIES OF SATURATED NORMALLY CONSOLIDATED SANDY SOILS AT VARIOUS RELATIVE DENSITIES (Adapted from Burmister, 1962)

	$C_c/(1+e_0)$					
Soil Type	$D_r = 0\%$	$D_r = 20\%$	$D_r = 40\%$	$D_r = 60\%$	$D_r = 80\%$	$D_r = 100\%$
Medium to coarse sand, some fine gravel (SW)	—	—	0.005	—	—	—
Medium to coarse sand (SW/SP)	0.010	0.008	0.006	0.005	0.003	0.002
Fine to coarse sand (SW)	0.011	0.009	0.007	0.005	0.003	0.002
Fine to medium sand (SW/SP)	0.013	0.010	0.008	0.006	0.004	0.003
Fine sand (SP)	0.015	0.013	0.010	0.008	0.005	0.003
Fine sand with trace fine to coarse silt (SP-SM)	—	—	0.011	—	—	—
Find sand with little fine to coarse silt (SM)	0.017	0.014	0.012	0.009	0.006	0.003
Fine sand with some fine to coarse silt (SM)	—	—	0.014	—	—	—

fills can be considered to be overconsolidated, as can soils that have clear geologic evidence of preloading, such as glacial tills. Therefore, many settlement analyses simply consider the compressibility of such soils to be zero. If it is unclear whether a soil is normally consolidated or overconsolidated, it is conservative to assume it is normally consolidated.

Very few consolidation tests have been performed on gravelly soils, but the compressibility of these soils is probably equal to or less than those for sand, as listed in Table 3.7.

Another characteristic of sands and gravels is their high hydraulic conductivity, which means any excess pore water drains very quickly. Thus, the rate of consolidation is very fast, and typically occurs nearly as fast as the load is applied. Therefore, if the load is caused by a fill, the consolidation of these soils may have little practical significance.

Another way to assess the compressibility of sands is to use in-situ tests. We will discuss these test methods in Chapter 4, and will apply them to sand compressibility in Chapter 7. This method is especially useful for settlements due to loads on foundations.

Consolidation Settlement Predictions

The purpose of performing consolidation tests is to define the stress–strain properties of the soil and thus allow us to predict consolidation settlements in the field. We perform this computation by projecting the laboratory test results (as contained in the parameters C_c, C_r, e_0, and σ_c') back to the field conditions.

For simplicity, the discussions of consolidation settlement predictions in this chapter consider only the case of one-dimensional consolidation, and we will be computing only the ultimate consolidation settlement. *One-dimensional consolidation* means only vertical strains occur in the soil (i.e., $\epsilon_x = \epsilon_y = 0$). This is a reasonable assumption when computing settlements due to the weight of fills, but is not quite true for settlements due to loads on foundations. We will return to this issue in Chapter 7. The *ultimate consolidation settlement* is the value of δ_c at the end of the consolidation process.

Normally Consolidated Soils ($\sigma_{z0}' \approx \sigma_c'$)

If $\sigma_{z0}' \approx \sigma_c'$, the soil is, by definition, normally consolidated. Thus, the initial and final conditions are as shown in Figure 3.10, and the compressibility is defined by C_c, the slope of the virgin curve.

To compute the ultimate consolidation settlement, we divide the soil into layers, compute the settlement of each layer, and sum as follows:

$$\delta_c = \Sigma \frac{C_c}{1 + e_0} H \log\left(\frac{\sigma'_{zf}}{\sigma'_{z0}}\right) \tag{3.25}$$

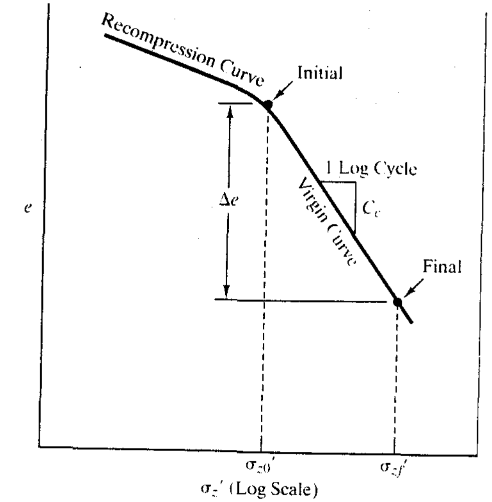


Figure 3.10 Consolidations of normally consolidated soils.

Where:

δ_c = ultimate consolidation settlement at the ground surface
C_c = compression index
e_0 = initial void ratio
H = thickness of soil layer
σ_{z0}' = initial vertical effective stress
σ_{zf}' = final vertical effective stress

When using Equation 3.25, compute σ_{z0}' and σ_{zf}' at the midpoints of each layer.

Overconsolidated Soils — Case I ($\sigma_{z0}' < \sigma_{zf}' \leq \sigma_c'$)

If both σ_{z0}' and σ_{zf}' do not exceed σ_c', the entire consolidation process occurs on the recompression curve as shown in Figure 3.11. The analysis is thus identical to that for normally consolidated soils except we use the recompression index, C_r instead of the compression index, C_c:

$$\delta_c = \Sigma \frac{C_r}{1 + e_0} H \log\left(\frac{\sigma'_{zf}}{\sigma'_{z0}}\right) \tag{3.26}$$

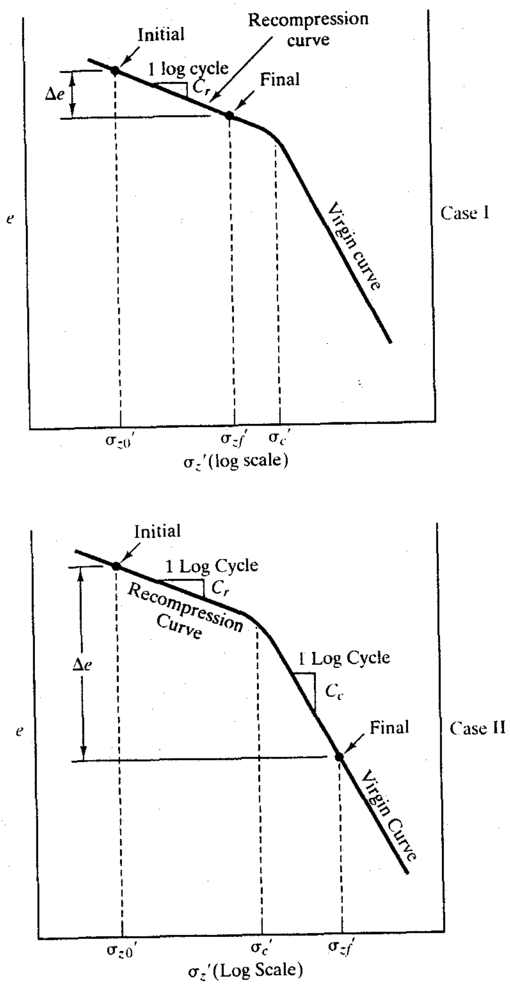


Figure 3.11 Consolidation of overconsolidated soils.

Overconsolidated Soils — Case II ($\sigma_{z0}' < \sigma_c' < \sigma_{zf}'$)

If the consolidation process begins on the recompression curve and ends on the virgin curve, as shown in Figure 3.11, then the analysis must consider both C_c and C_r:

$$\delta_c = \Sigma \left[\frac{C_r}{1 + e_0} H \log \left(\frac{\sigma_c'}{\sigma_{z0}'} \right) + \frac{C_c}{1 + e_0} H \log \left(\frac{\sigma_{zf}'}{\sigma_c'} \right) \right] \tag{3.27}$$

This condition is quite common, since many soils that might appear to be normally consolidated from a geologic analysis actually have a small amount of overconsolidation (Mesri, Lo, and Feng, 1994).

Ultimate Consolidation Settlement Analysis Procedure

Use the following procedure to compute δ_c caused by the weight of extensive fills:

1. Beginning at the original ground surface, divide the soil profile into strata, where each stratum consists of a single soil type with a common geologic origin. For example, one stratum may consist of a dense sand, while another might be a soft-to-medium clay. Continue downward with this process until you have passed through all of the compressible strata (i.e., until you reach bedrock or some very hard soil). For each stratum, identify the unit weight, γ. Note: Boring logs usually report the dry unit weight, γ_d, and moisture content, w, but we can compute γ from this data using Equation 3.3. Also define the location of the groundwater table.
2. Each clay or silt stratum must have results from at least one consolidation test (or at least estimates of these results). Using the techniques described earlier, determine if each stratum is normally consolidated or overconsolidated, then assign values for $C_c/(1+e_0)$ and/or $C_r/(1+e_0)$. For each overconsolidated stratum, compute σ_m' using Equation 3.23 and assume it is constant throughout that stratum. For normally consolidated soils, set $\sigma_m' = 0$.
3. For each sand or gravel stratum, assign a value for C_c / $(1+e_0)$ or C_r / $(1+e_0)$ based on the information in Table 3.7.
4. For any very hard stratum, such as bedrock or glacial till, that is virtually incompressible compared to the other strata, assign $C_c = C_r = 0$.
5. Working downward from the original ground surface (i.e., do not consider any proposed fills), divide the soil profile into horizontal layers. Begin a new layer whenever a new stratum is encountered, and divide any thick strata into multiple layers. When performing computations by hand, each strata should have layers no more than 2 to 5 m (5 to 15 ft) thick. Thinner layers are especially appropriate near the ground surface, because the strain is generally larger there. Computer-based computations can use much thinner layers throughout the entire depth, and achieve slightly more precise results.

6. Tabulate the following parameters at the midpoint of each layer:
   For normally consolidated strata:

$$\sigma'_{z0}$$
$$\sigma'_{zf}$$
$$C_c/(1 + e_0)$$
$$H$$

   For overconsolidated strata:

$$\sigma'_{z0}$$
$$\sigma'_{zf}$$
$$\sigma'_c = \sigma'_{z0} + \sigma'_m$$
$$C_c/(1 + e_0)$$
$$C_r/(1 + e_0)$$
$$H$$

   It is not necessary to record these parameters in incompressible strata.

   Normally we compute σ_{z0}' and σ_{zf}' using the hydrostatic pore water pressures (Equation 3.9) with no significant seepage force, and this is the only case we will consider in this book. However, if preexisting excess pore water pressures or significant seepage forces are present, they should be evaluated. Sometimes this may require the installation of piezometers to obtain accurate information on the in-situ pore water pressures.
7. Using Equation 3.25, 3.26, or 3.27, compute the consolidation settlement for each layer, then sum to find δ_c. Note that each layer will not necessarily use the same equation. If σ_c' is only slightly greater than σ_{z0}' (perhaps less than 20 percent greater), then it may not be clear if the soil is truly overconsolidated, or if the difference is only an apparent overconsolidation caused by uncertainties in assessing these two values. In such cases, it is acceptable to use either Equation 3.25 (normally consolidated) or 3.27 (overconsolidated case II).

Example 3.4

A 3.0 m deep compacted fill is to be placed over the soil profile shown in Figure 3.12. A consolidation test on a sample from point A produced the following results:

$$C_c = 0.40$$
$$C_r = 0.08$$
$$e_0 = 1.10$$
$$\sigma'_c = 70.0 \text{ kPa}$$

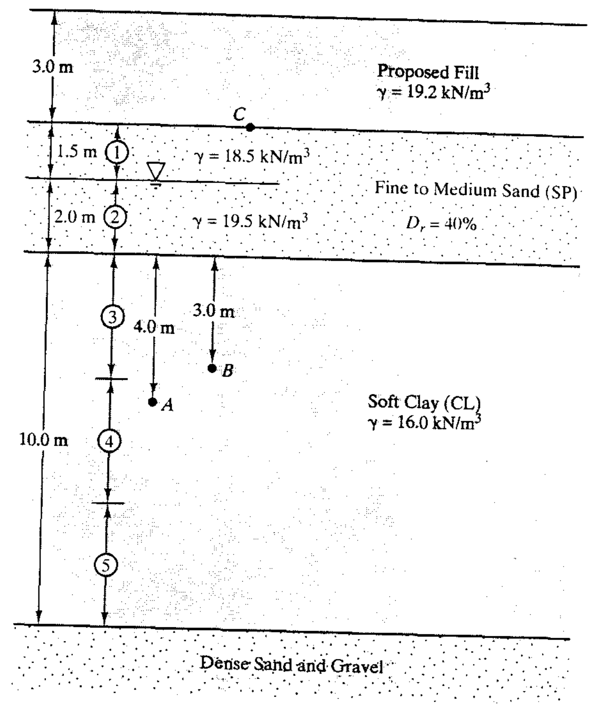


Figure 3.12 Soil profile for Example 3.4.

Compute the ultimate consolidation settlement due to the weight of this fill.

Solution

$$\begin{aligned}\sigma'_{zf} &= \sigma'_{z0} + \Delta\sigma_z \\ &= \sigma'_{z0} + \gamma_{fill}H_{fill} \\ &= \sigma'_{z0} + (19.2 \text{ kN/m}^3)(3.0 \text{ m}) \\ &= \sigma'_{z0} + 57.6 \text{ kPa}\end{aligned}$$

Compute the initial vertical stress at the sample location:

$$\begin{aligned}\sigma'_{z0} &= \Sigma\,\gamma H - u \\ &= (18.5 \text{ kN/m}^3)(1.5 \text{ m}) + (19.5 \text{ kN/m}^3)(2.0 \text{ m}) \\ &\quad + (16.0 \text{ kN/m}^3)(4.0 \text{ m}) - (9.8 \text{ kN/m}^3)(6.0 \text{ m}) \\ &= 72.0 \text{ kPa}\end{aligned}$$

$$\frac{C_c}{1+e_0} = \frac{0.40}{1+1.10} = 0.190$$

At the sample $\sigma_c' \approx \sigma_{z0}' \rightarrow \therefore$ normally consolidated

For the sand strata, use $C_c/(1+e_0) = 0.008$, per Table 3.7

		At midpoint of layer				
Layer	H (m)	σ_{z0}' (kPa) Eqn 3.13	σ_{zf}' (kPa)	$\frac{C_c}{1+e_0}$	Eqn.	$(\delta_c)_{ult}$ (mm)
1	1.5	13.9	71.5	0.008	3.25	8
2	2.0	37.4	95.0	0.008	3.25	6
3	3.0	56.4	114.0	0.19	3.25	174
4	3.0	75.0	132.6	0.19	3.25	141
5	4.0	96.7	154.3	0.19	3.25	154
					$\delta_c =$	483

Round off to:

$\delta_c =$ **480 mm** ⇐ ***Answer***

Notice how we have used the same analysis for soils above and below the groundwater table, and both are based on saturated $C_c / (1 + e_0)$ values. This is conservative (although in this case, very slightly so) because the soils above the groundwater table are probably less compressible.

Example 3.5

An 8.5-m deep compacted fill is to be placed over the soil profile shown in Figure 3.13. Consolidation tests on samples from points A and B produced the following results:

	Sample A	Sample B
C_c	0.25	0.20
C_r	0.08	0.06
e_0	0.66	0.45
σ_c'	101 kPa	510 kPa

Compute the ultimate consolidation settlement caused by the weight of this fill.

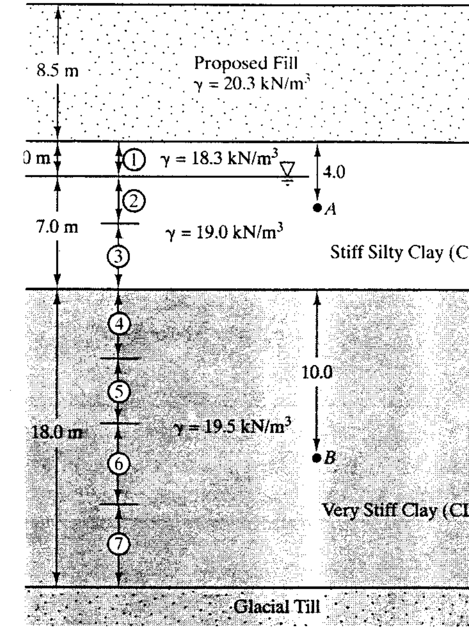


Figure 3.13 Soil profile for Example 3.5.

Solution

$$\sigma_{zf}' = \sigma_{z0}' + \Delta\sigma_z$$
$$= \sigma_{z0}' + \gamma_{fill}H_{fill}$$
$$= \sigma_{z0}' + (20.3\ \text{kN/m}^3)(8.5\ \text{m})$$
$$= \sigma_{z0}' + 172.6\ \text{kPa}$$

At sample A:

$$\sigma_{z0}' = \Sigma\,\gamma H - u$$
$$= (18.3\ \text{kN/m}^3)(2.0\ \text{m}) + (19.0\ \text{kN/m}^3)(2.0\ \text{m}) - (9.8\ \text{kN/m}^3)(2.0\ \text{m})$$
$$= 55.0\ \text{kPa}$$

$$\sigma_{zf}' = \sigma_{z0}' + \gamma_f H_f$$
$$= 55.0\ \text{kPa} + 172.6\ \text{kPa}$$
$$= 227.6\ \text{kPa}$$

$\sigma_{z0}' < \sigma_c'$ and $\sigma_{zf}' > \sigma_c'$ $\quad\therefore$ overconsolidated case II

$$\frac{C_c}{1 + e_0} = \frac{0.40}{1 + 1.10} = 0.190$$

At the sample $\sigma'_c \approx \sigma'_{z0} \rightarrow \therefore$ normally consolidated

For the sand strata, use $C_c/(1+e_0) = 0.008$, per Table 3.7

		At midpoint of layer				
Layer	H (m)	σ'_{z0} (kPa) Eqn 3.13	σ'_{zf} (kPa)	$\frac{C_c}{1+e_0}$	Eqn.	$(\delta_c)_{ult}$ (mm)
1	1.5	13.9	71.5	0.008	3.25	8
2	2.0	37.4	95.0	0.008	3.25	6
3	3.0	56.4	114.0	0.19	3.25	174
4	3.0	75.0	132.6	0.19	3.25	141
5	4.0	96.7	154.3	0.19	3.25	154
					$\delta_c =$	483

Round off to:

δ_c = **480 mm** ⇐***Answer***

Notice how we have used the same analysis for soils above and below the groundwater table, and both are based on saturated $C_c / (1 + e_0)$ values. This is conservative (although in this case, very slightly so) because the soils above the groundwater table are probably less compressible.

Example 3.5

An 8.5-m deep compacted fill is to be placed over the soil profile shown in Figure 3.13. Consolidation tests on samples from points A and B produced the following results:

	Sample A	Sample B
C_c	0.25	0.20
C_r	0.08	0.06
e_0	0.66	0.45
σ_c'	101 kPa	510 kPa

Compute the ultimate consolidation settlement caused by the weight of this fill.

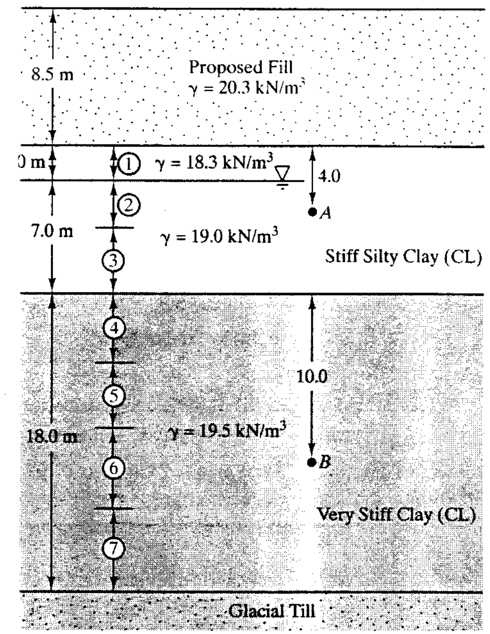


Figure 3.13 Soil profile for Example 3.5.

Solution

$$\begin{aligned}\sigma'_{zf} &= \sigma'_{z0} + \Delta\sigma_z \\ &= \sigma'_{z0} + \gamma_{fill}H_{fill} \\ &= \sigma'_{z0} + (20.3\ \text{kN/m}^3)(8.5\ \text{m}) \\ &= \sigma'_{z0} + 172.6\ \text{kPa}\end{aligned}$$

At sample A:

$$\begin{aligned}\sigma'_{z0} &= \Sigma\,\gamma H - u \\ &= (18.3\ \text{kN/m}^3)(2.0\ \text{m}) + (19.0\ \text{kN/m}^3)(2.0\ \text{m}) - (9.8\ \text{kN/m}^3)(2.0\ \text{m}) \\ &= 55.0\ \text{kPa}\end{aligned}$$

$$\begin{aligned}\sigma'_{zf} &= \sigma'_{z0} + \gamma_f H_f \\ &= 55.0\ \text{kPa} + 172.6\ \text{kPa} \\ &= 227.6\ \text{kPa}\end{aligned}$$

$\sigma'_{z0} < \sigma'_c$ and $\sigma'_{zf} > \sigma'_c \quad \therefore$ overconsolidated case II

At sample B:

$$\sigma'_{z0} = \Sigma \gamma H - u$$
$$= (18.3 \text{ kN/m}^3)(2.0 \text{ m}) + (19.0 \text{ kN/m}^3)(7.0 \text{ m})$$
$$+ (19.5 \text{ kN/m}^3)(10.0 \text{ m}) - (9.8 \text{ kN/m}^3)(17.0 \text{ m})$$
$$= 198.0 \text{ kPa}$$

$$\sigma'_{zf} = \sigma'_{z0} + \gamma_{fill} H_{fill}$$
$$= 198.0 \text{ kPa} + 172.6 \text{ kPa}$$
$$= 370.6 \text{ kPa}$$

$\sigma'_{z0} < \sigma'_c$ and $\sigma'_{zf} \leq \sigma'_c$ $\quad\therefore$ overconsolidated case I

		At midpoint of layer						
Layer	H (m)	σ_{z0}' (kPa) Eqn. 3.13	σ_c' (kPa) Eqn. 3.21	σ_{zf}' (kPa)	$\frac{C_r}{1+e_0}$	$\frac{C_c}{1+e_0}$	Eqn.	δ_c (mm)
1	2.0	18.3	64.3	190.9	0.05	0.15	3.27	196
2	3.0	50.4	96.4	223.0	0.05	0.15	3.27	206
3	4.0	82.6	128.6	255.2	0.05	0.15	3.27	217
4	4.0	120.4	—	293.0	0.04	0.14	3.26	62
5	4.0	159.2	—	331.8	0.04	0.14	3.26	51
6	5.0	202.8	—	375.4	0.04	0.14	3.26	53
7	5.0	251.4	—	424.0	0.04	0.14	3.26	45
							$\delta_c =$	830

δ_c = **830 mm** $\Leftarrow$ ***Answer***

Notice how most of the compression occurs in the upper strata, which is overconsolidated case II (i.e., some of the compression occurs along the virgin curve). The lower strata has much less compression even though it is twice as thick because it is overconsolidated case I and all of the compression occurs on the recompression curve, and because the ratio of σ_{zf}/σ_{z0} is smaller.

QUESTIONS AND PRACTICE PROBLEMS

3.7 A 2-m thick fill is to be placed on the soil shown in Figure 3.14. Once it is compacted, this fill will have a unit weight of 19.5 kN/m^3. Compute the ultimate consolidation settlement.

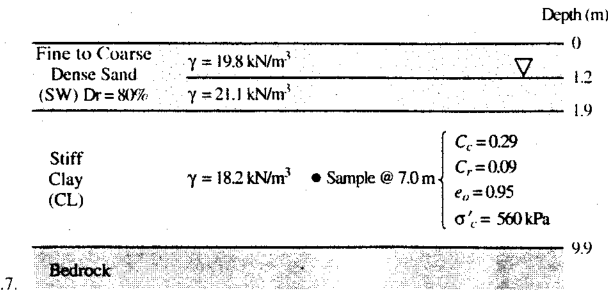


Figure 3.14 Soil profile for Problem 3.7.

3.6 STRENGTH

Structural foundations also induce significant shear stresses into the ground. If these shear stresses exceed the shear strength of the soil or rock, failure occurs. Therefore, we must be able to assess these shear stresses and strengths and design foundations such that the shear stresses are sufficiently smaller than the shear strength.

Sources of Shear Strength in Soil

The shear strength of common engineering materials, such as steel, is controlled by their molecular structure. Failure generally requires breaking the molecular bonds that hold the material together, and thus depends on the strength of these bonds. For example, steel has very strong molecular bonds, and thus has a high shear strength, while plastic has much weaker bonds and a correspondingly lower shear strength.

However, the physical mechanisms that control shear strength in soil are much different. Soil is a particulate material, so shear failure occurs when the stresses between the particles are such that they slide or roll past each other as shown in Figure 3.15. It is not

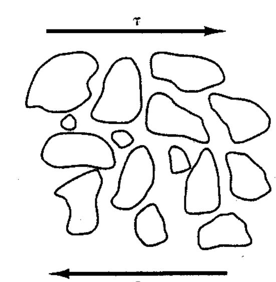


Figure 3.15 Shear failure in soil occurs when the shear stresses are large enough to make the particles roll and slide past each other.

necessary to fracture individual particles. Therefore, the shear strength depends on interactions between the particles, not on their internal strength.

When viewed on a microscopic scale, these interactions are very complex. For simplicity, we will divide them into two broad categories: frictional strength and cohesive strength.

Frictional Strength

Frictional strength is similar to classic sliding friction problems from basic physics. The force that resists sliding is equal to the normal force multiplied by the coefficient of friction, μ, as shown in Figure 3.16.

However, instead of using the coefficient of friction, μ, geotechnical engineers prefer to describe frictional strength using the *effective friction angle* (or *effective angle of internal friction*), ϕ', where:

$$\phi' = \tan^{-1}\mu \tag{3.28}$$

The value of ϕ' depends on both the frictional properties of the individual particles and the interlocking between particles. These are affected by many factors, including:

- **Mineralogy**—The effective friction angle in sands made of pure quartz is typically 30 to 36°. However, the presence of other minerals can change ϕ'. For example, sands contain significant quantities of mica, which is much smoother than quartz, have a smaller ϕ'. These are called *micaceous sands*. Clay minerals are typically even weaker (ϕ' values as low as 4° have been measured in pure montmorillonite).
- **Shape**—The friction angle of angular particles is much higher than that of rounded ones.
- **Gradation**—Well graded soils typically have more interlocking between the particles, and thus a higher friction angle than those that are poorly graded. For example, GW soils typically have ϕ' values about 2° higher than comparable GP soils.

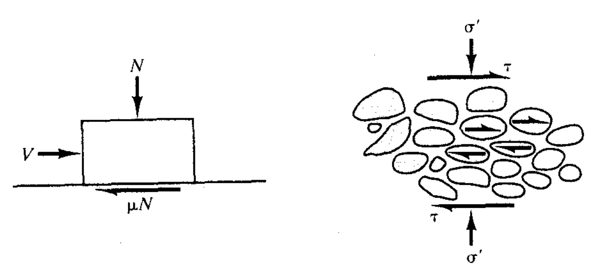


Figure 3.16 Comparison between friction on a sliding block and frictional strength in a soil.

- **Void ratio**—Decreasing the void ratio, such as by compacting a soil with a sheepsfoot roller, also increases interlocking, which results in a higher ϕ'.
- **Organic material**—Organics introduce many problems, including a decrease in the friction angle.

The impact of water on frictional strength is especially important, and many shear failures are induced by changes in the groundwater conditions. However, many people mistakenly believe water-induced changes in shear strength are primarily caused by lubrication effects. Although the process of wetting some dry soils can induce lubrication, the resulting decrease in ϕ' is very small. Sometimes the introduction of water has an antilubricating effect (Mitchell, 1993) and causes a small increase in ϕ'. Nevertheless, focusing on these small effects tends to obscure another far more important process: the impact of groundwater on the effective stress, σ'.

As the groundwater table rises, the pore water pressure, u, increases and the effective stress, σ', decreases. This causes a reduction in frictional strength, just as a reduction in the normal force, N, reduces the frictional resistance μN acting on the bottom of the sliding block in Figure 3.16. This is the primary way groundwater influences frictional strength in soils.

Cohesive Strength

Some soils have shear strength even when the effective stress, σ', is zero, or at least *appears* to be zero. This strength is called the *cohesive strength*, and we describe it using the variable c', the *effective cohesion*. There are two types of cohesive strength: true cohesion and apparent cohesion (Mitchell, 1993).

True cohesion is shear strength that is truly the result of bonding between the soil particles. These bonds include the following:

- *Cementation* is chemical bonding caused by the presence of cementing agents, such as calcium carbonate ($CaCO_3$) or iron oxide (Fe_2O_3) (Clough, et al., 1981). Even small quantities of these agents can provide significant cohesive strengths. *Caliche* is an example of a heavily cemented soil that has a large cohesive strength. Cementation also can be introduced artificially using Portland cement or special chemicals.
- *Electrostatic and electromagnetic attractions* hold particles together. However, these forces are very small and probably do not produce significant shear strength in soils.
- *Primary valence bonding (adhesion)* is a type of cold welding that occurs in clays when they become overconsolidated.

Apparent cohesion is shear strength that appears to be caused by bonding between the soil particles, but it is really frictional strength in disguise. Sources of apparent cohesion include the following:

- *Negative pore water pressures* that have not been considered in the stress analysis. These negative pore water pressures are present in soils above the groundwater table.
- *Negative excess pore water pressures caused by dilation*. Some soils tend to *dilate* or expand when they are sheared. In saturated soils, this dilation draws water into the voids. However, sometimes the rate of shearing is more rapid than the rate at which water can flow (i.e., the voids are trying to expand more rapidly than they can draw in the extra water). This is especially likely in saturated clays, because their hydraulic conductivity is so low. When this occurs, large negative excess pore water pressures can develop in the soil.
- *Apparent mechanical forces* are those caused by particle interlocking, and can develop in soils where this interlocking is very difficult to overcome.

Geotechnical engineers often use the term "cohesive soil" to describe clays. Although this term is convenient, it also is very misleading (Santamarina, 1997). Most of the so-called cohesive strength in clays is really apparent cohesion caused by pore water pressures that are negative, or at least less than the hydrostatic pore water pressure. In such soils it is better to think of "cohesive strength" as a mathematical idealization rather than a physical reality.

In cemented soils, cohesive strength really does reflect bonding between the soil particles. In some cases, we may rely on this strength in our designs. However, in other cases, it is wise to ignore this source of strength. For example, if the cementing agent is water-soluble, it may disappear if the soil becomes wetted during the life of the project.

Methods of Shear Strength Analysis

There are two principal methods of conducting shear strength analyses: An effective stress analysis and a total stress analysis. Both methods are used in foundation design.

Effective Stress Analyses

The shear strength in a soil is developed only by the solid particles, because the water and air phases have no shear strength. Therefore, it seems reasonable to evaluate strength problems using the effective stress, σ', because it is the portion of the total stress, σ, carried by the solid particles. When using an effective stress analysis, we describe shear strength using the *Mohr-Coulomb failure criterion*:

$$\boxed{s = c' + \sigma' \tan \phi'} \tag{3.29}$$

Where:

s = shear strength
c' = effective cohesion
σ' = effective stress acting on the shear surface
ϕ' = effective friction angle

The values of c' and ϕ' for a particular soil are usually obtained from special laboratory tests, as discussed later in this chapter.

Total Stress Analyses

Analyses based on effective stresses are possible only if we can predict the effective stresses in the field. This is a simple matter when only hydrostatic pore water pressures are present, but this can become very complex when there are excess pore water pressures. For example, when a foundation is placed over a saturated clay, the applied load produces excess pore water pressures as discussed earlier. In addition, some soils also develop additional excess pore water pressures as they are sheared. Often these excess pore water pressures are difficult or impossible to predict. Unfortunately, if we cannot predict the pore water pressure, then we do not know the magnitude of the effective stress and cannot solve Equation 3.29.

Because of these difficulties, geotechnical engineers are sometimes forced to evaluate problems based on total stresses instead of effective stresses. This approach involves reducing the lab data in terms of total stress and expressing it using the parameters c_T and ϕ_T. Equation 3.29 then needs to be rewritten as:

$$\boxed{s = c_T + \sigma \tan \phi_T} \tag{3.30}$$

Where:

s = shear strength
c_T = total cohesion
σ = total stress
ϕ_T = total friction angle

This method assumes the pore water pressures developed in the lab are the same as those in the field, and thus are implicitly incorporated into c_T and ϕ_T. This assumption introduces some error in the analysis, but it becomes an unfortunate necessity when we cannot predict the magnitudes of excess pore water pressures in the field.

Volume Changes and Excess Pore Water Pressures

When loads such as those from structural foundations are placed on the ground, the resulting normal and shear stresses cause the soil voids to expand or contract. If the soil is saturated, this expansion or contraction produces excess pore water pressures, as discussed earlier. However, this build-up of excess pore water pressures induces a hydraulic gradient that forces some of the water out of the voids and causes these pressures to dissipate. Thus, there are two processes acting simultaneously: the build-up of excess pore water pressure, which depends primarily on the rate of loading, and the dissipation of these pressures, which depends on the rate of drainage.

Geotechnical engineers often evaluate these two competing processes by considering two drainage conditions: the drained condition and the undrained condition.

If the rate of drainage is faster than the rate of loading, then any excess pore water pressures will be small and short-lived. We call this the *drained condition*. It exists when the rate of loading is very slow, the rate of drainage is very high, or both. Figure 3.17a shows the progression of various parameters with time when drained conditions exist.

If drained conditions exist, we assume the pore water pressure always equals the hydrostatic pore water pressure, and thus may be computed using Equation 3.29. This is very convenient and simplifies the computations.

Conversely, if the rate of drainage is slower than the rate of loading, significant excess pore water pressures may develop in the soil. We call this the *undrained condition*, and it is described in Figure 3.17b. It occurs when the load is applied rapidly, or the soil drains slowly.

The undrained condition is more difficult to analyze because we need to account for the excess pore water pressures. This may be done either explicitly or implicitly, as discussed later.

Shear Strength of Saturated Sands and Gravels

The rate of drainage in sands and gravels is very rapid because of their high hydraulic conductivity. In other words, any excess pore water pressures that may develop in these soils dissipate very rapidly because water flows through them very quickly and easily. In addition, most of the load acting on foundations usually consists of dead and live loads, which are applied over a period of days to weeks. This is much slower than the rate of drainage in sands and gravels. Therefore, when designing foundations on sands and gravels, we can nearly always assume drained conditions are present. Therefore, the pore water pressure equals the hydrostatic pore water pressure, as defined in Equation 3.7, and can compute the vertical effective stress using Equation 3.13.

The effective cohesion, c', and effective friction angle, ϕ', are obtained from laboratory tests (as discussed later in this chapter) or from in-situ tests (as discussed in Chapter 4). For clean or silty sands and gravels (USCS group symbols SM, SP, SW, GM, GP, and GW) it is normally best to use $c' = 0$. Some cohesion may be present in clayey sands and gravels (group symbols SC and GC), but it should be used only with great caution, because it may not be present in the field. Figure 3.18 presents typical values of ϕ', and may be used to check the lab or field test results.

Finally, knowing the values of σ', c', and ϕ', we can compute the shear strength using Equation 3.29.

Shear Strength of Saturated Clays and Silts

The hydraulic conductivity of clays is about one million times smaller than that of sands, so the rate of drainage in these soils is very slow—much slower than the rate of loading. Therefore, undrained conditions are typically present in such soils. This means significant excess pore water pressures may be present during and immediately after loading. Eventually these excess pore water pressures dissipate, as shown in Figure 3.17b.

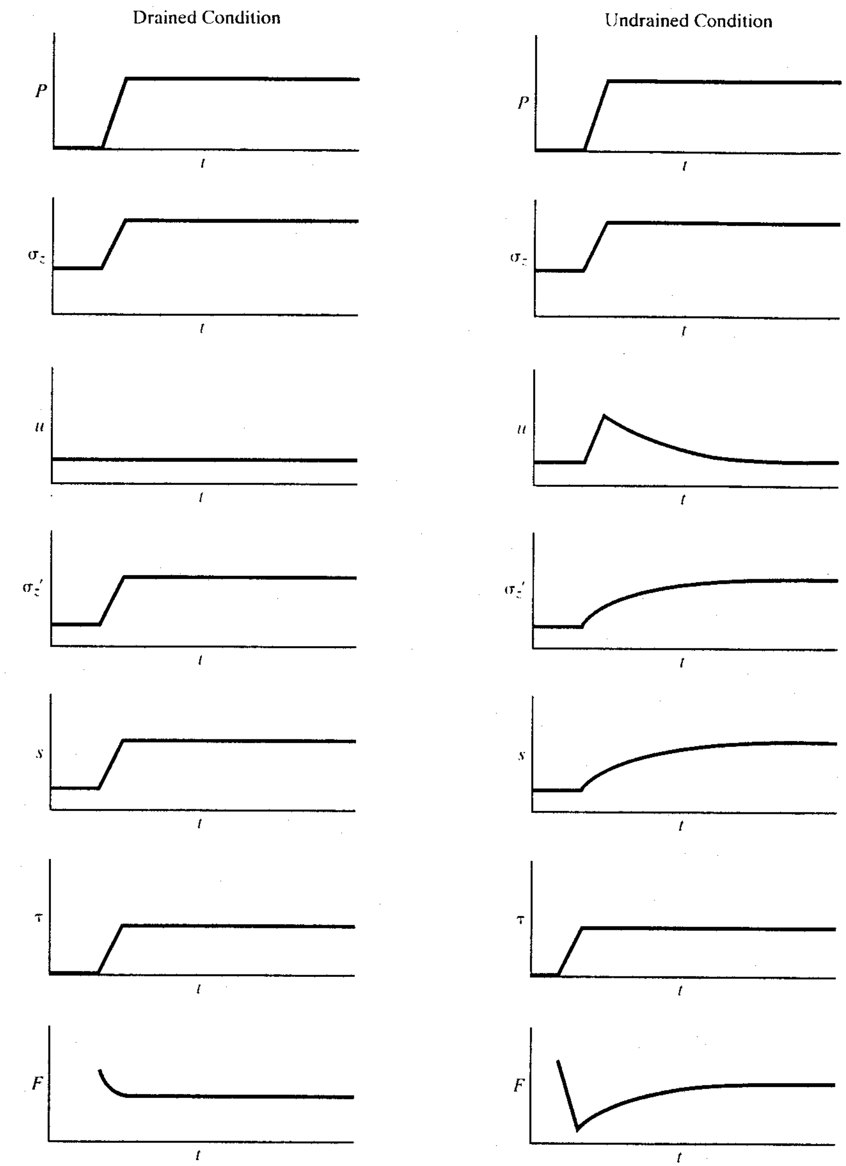


Figure 3.17 Changes in normal and shear stresses, ϕ and τ; shear strength, s; and factor of safety, F, with time at a point in a saturated soil subjected to a load P under drained vs. undrained conditions.

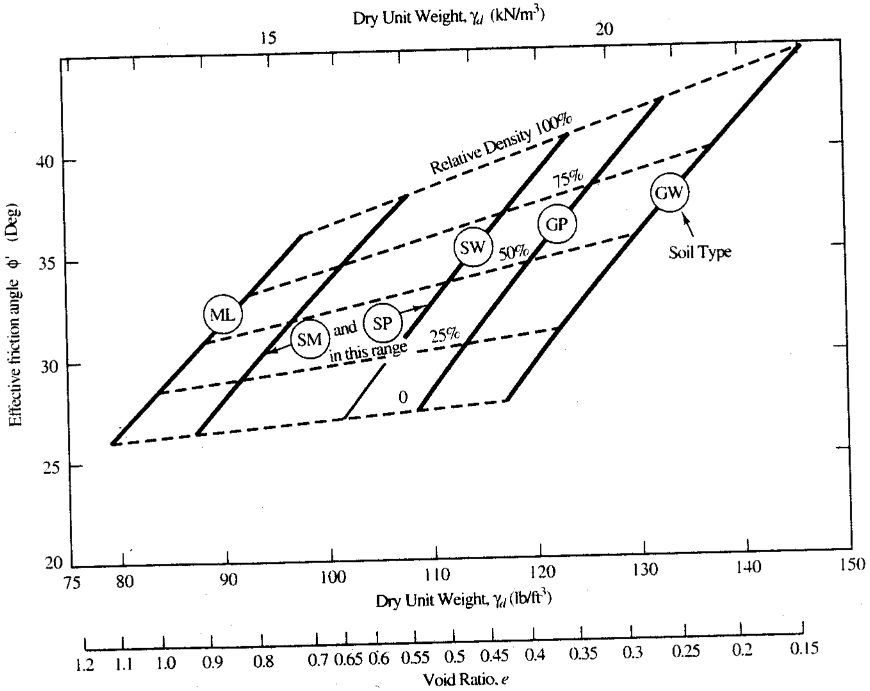


Figure 3.18 Typical ϕ' values for sands, gravels, and silts without plastic fines (Adapted from U.S. Navy, 1982a).

The hydraulic conductivity of silts is greater than that of clays, but it still is much smaller than that in sands. Once again, undrained conditions will prevail, although less time is required for the excess pore water pressures to dissipate.

To understand the impact of these excess pore water pressures, compare the two sets of plots in Figure 3.17, along with Equations 3.13 and 3.29. In both cases, the vertical total stress, σ_z, and the shear stress, τ, increase as the load P is applied. Under drained conditions, the vertical effective stress, σ_z', and the shear strength, s, both rise concurrent with σ_z, so both τ and s reach their peak value at the end of loading.

However, under undrained conditions, the temporary presence of excess pore water pressure (the spike in the u plot) causes a lag in the increase in shear strength. Although τ reaches its peak at the end of loading, s is still low, thus producing a temporary drop in the factor of safety, F. Then, as the excess pore water pressures dissipate, both s and F slowly climb.

The most likely time for failure is immediately after construction (i.e., when F is at a minimum). Therefore, foundations are normally designed to have a certain minimum

factor of safety at this critical moment. To accomplish this goal, we need to consider the excess pore water pressure, either explicitly or implicitly.

In principle, we should be able to determine the magnitude of the excess pore water pressure, u_e, and use Equations 3.7, 3.8, 3.13, and 3.29 to compute the shear strength immediately after construction. This method is called an effective stress analysis. We then could use this shear strength as the basis for a foundation design that provides the minimum acceptable factor of safety. Unfortunately, the magnitude of u_e is difficult to determine, especially in overconsolidated soils, so this method is not economically viable for most foundation projects.

Because of this problem, engineers normally resort to performing a total stress analysis and compute the shear strength using Equation 3.30. This equation is based on c_T and ϕ_T, which are determined by evaluating the laboratory test data in terms of total stress instead of effective stress. Presumably, the consequences of the excess pore water pressures are implicit within these values. We then assume the excess pore water pressures in the field are the same as those in the lab, and apply Equation 3.30 to the field conditions without needing to know the magnitude of the excess pore water pressures.

If the soil is truly saturated and truly undrained, then $\phi_T = 0$ (even though $\phi' > 0$) because newly applied loads are carried entirely by the pore water and do not change σ' or s. This is very convenient because the second term in Equation 3.30 drops out and we no longer need to compute σ. We call this a "$\phi = 0$ analysis," and the shear strength is called the *undrained shear strength*, s_u, where $s_u = c_T$. Table 3.8 gives typical values of s_u, which may be used for preliminary analyses or to check laboratory test results.

Usually we assign an appropriate s_u value for each saturated undrained strata based on laboratory or field test results. Many geotechnical analysis methods use this s_u

TABLE 3.8 RELATIONSHIP BETWEEN CONSISTENCY OF COHESIVE SOILS AND UNDRAINED SHEAR STRENGTH (Adapted from Terzaghi & Peck, 1967 and ASTM D2488-90; used with permission).

Consistency	Undrained Shear Strength, s_u (lb/ft²)	Undrained Shear Strength, s_u (kPa)	Visual Identification
Very soft	<250	<12	Thumb can penetrate more than 1 in (25 mm)
Soft	250–500	12–25	Thumb can penetrate about 1 in (25 mm)
Medium	500–1000	25–50	Penetrated with thumb with moderate effort
Stiff	1000–2000	50–100	Thumb will indent soil about 1/4 in (8 mm)
Very stiff	2000–4000	100–200	Thumb will not indent, but readily indented with thumbnail
Hard	>4000	>200	Indented by thumbnail with difficulty or cannot indent with thumbnail

value directly. Other analysis methods require the shear strength to be defined using c and ϕ. When using the later type with saturated undrained analyses, we set $c = s_u$ and $\phi = 0$.

In reality, s_u is probably not constant throughout a particular soil strata, even if it appears to be homogeneous. In general, s_u increases with depth because the lower portions of the strata have been consolidated to correspondingly greater loads, and thus have a higher shear strength. The shallow portions also have higher strengths if they had once dried out (desiccated) and formed a crust. Finally, the natural non-uniformities in a soil strata produce variations in s_u. We can accommodate these variations by simply taking an average value, or by dividing the strata into smaller layers.

Shear Strength of Saturated Intermediate Soils

Thus far we have divided soils into two distinct categories. Sands and gravels do not develop excess pore water pressures during static loading, and thus may be evaluated using effective stress analyses and hydrostatic pore water pressures. Conversely, silts and clays do develop excess pore water pressures, and thus require more careful analysis. They also may have problems with sensitivity and creep. Although many "real-world" soils neatly fit into one of these two categories, others behave in ways that are intermediate between these two extremes. Their field behavior is typically somewhere between being drained and undrained (i.e., they develop some excess pore water pressures, but not as much as would occur in a clay).

Although there are no clear-cut boundaries, these intermediate soils typically include those with unified classification SC, GC, SC-SM, or GC-GM, as well as some SM, GM, and ML soils. Proper shear strength evaluations for engineering analyses require much more engineering judgment, which is guided by a thorough understanding of soil strength principles. When in doubt, it is usually conservative to evaluate these soils using the techniques described for silts and clays.

Shear Strength of Unsaturated Soils

Thus far we have only considered soils that are saturated ($S = 100\%$). The strength of unsaturated soils ($S < 100\%$) is generally greater, but more difficult to evaluate. Nevertheless, many engineering projects encounter these soils, so geotechnical engineers need to have methods of evaluating them. This has been a topic of ongoing research (Fredlund and Rahardjo, 1993), and standards of practice are not yet as well established as those for saturated soils.

Some of the additional strength in unsaturated soils is caused by negative pore water pressures. These negative pore water pressures increase the effective stress, and thus increase the shear strength. However, this additional strength is very tenuous and is easily lost if the soil becomes wetted.

Geotechnical engineers usually base designs on the assumption that unsaturated soils could become wetted in the future. This wetting could come from a rising groundwater table, irrigation, poor surface drainage, broken pipelines, or other causes. Therefore, we usually saturate (or at least "soak") soil samples in the laboratory before performing strength tests. This is intended to remove the apparent cohesion and thus simulate the "worst case" field conditions. We then determine the highest likely elevation for the groundwater table, which may be significantly higher than its present location, and compute positive pore water pressures accordingly. Finally, we assume $u = 0$ in soils above the groundwater table.

Laboratory Shear Strength Tests

The shear strength parameters, c' and ϕ' (or c_T and ϕ_T) may be determined by performing laboratory or in-situ tests. This section discusses some of the more common laboratory tests, and Chapter 4 discusses in-situ tests.

Several different laboratory tests are commonly used to measure the shear strength of soils. Each has its advantages and disadvantages, and no one test is suitable for all circumstances. When selecting a test method, we must consider many factors, including the following:

- Soil type
- Initial moisture content and need, if any, to saturate the sample
- Required drainage conditions (drained or undrained)

Direct Shear Test

The French engineer Alexandre Collin may have been the first to measure the shear strength of a soil (Head, 1982). His tests, conducted in 1846, were similar to the modern direct shear test. The test as we now know it [ASTM D3080] was perfected by several individuals during the first half of the twentieth century.

The apparatus, shown in Figure 3.19, typically accepts a 60 to 75 mm (2.5–3.0 in) diameter cylindrical sample and subjects it to a certain effective stress. The shear stress is then slowly increased until the soil fails along the surface shown in the figure. The test is normally repeated on new samples of the same soil until three sets of effective stress and shear strength measurements are obtained. A plot of this data produces values of the cohesion, c, and friction angle, ϕ.

The direct shear test has the advantage of being simple and inexpensive and it is an appropriate method when we need the drained strength of sands. It also can be used to obtain the drained strength of clays, but produces less reliable results because we have no way of controlling the drainage conditions other than varying the speed of the test. The direct shear test also has the disadvantages of forcing the shear to occur along a specific plane instead of allowing the soil to fail along the weakest zone, and it produces nonuniform strains in the sample, which can produce erroneous results in strain-softening soils.

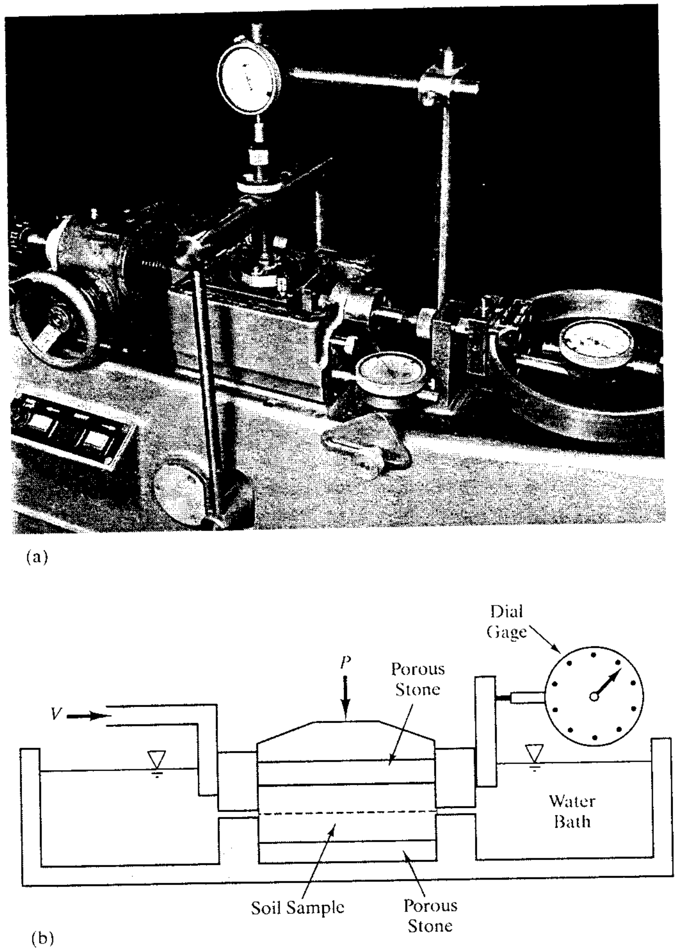


Figure 3.19 (a) A direct shear machine. The sample is inside the sample holder, directly below the upper dial gauge. (b) Cross section through the sample holder showing the soil sample and shearing action.

Example 3.6

A series of three direct shear tests has been conducted on a certain saturated soil. Each test was performed on a 2.375-inch diameter, 1.00-inch tall sample. The test has been performed slowly enough to produce drained conditions. The results of these tests are as follows:

Test Number	Normal Load (lb)	Shear Load at Failure (lb)
1	75	51
2	150	110
3	225	141

Determine c' and ϕ'.

Solution

$$A = \frac{\pi D^2}{4} = \frac{\pi(2.375\text{ in})^2}{4}\left(\frac{1\text{ ft}^2}{144\text{ in}^2}\right) = 0.0308\text{ ft}^2$$

Based on this area and the measured forces:

Test Number	σ' (lb/ft^2)	τ (lb/ft^2)
1	2435	1656
2	4870	3571
3	7305	4578

This data is plotted in Figure 3.20. It does not form a perfect line. This is because of experimental error, slight differences in the three samples, true nonlinearity, and other factors. We have drawn a best-fit line through these three points to obtain $c' = 400$ lb/ft^2 and $\phi' = 31°$. Note: The shear area changes as the direct shear test progresses, and a more thorough analysis would account for this change. However, most engineers neglect this change because it has very little effect on the final test results and because it is slightly conservative to do so.

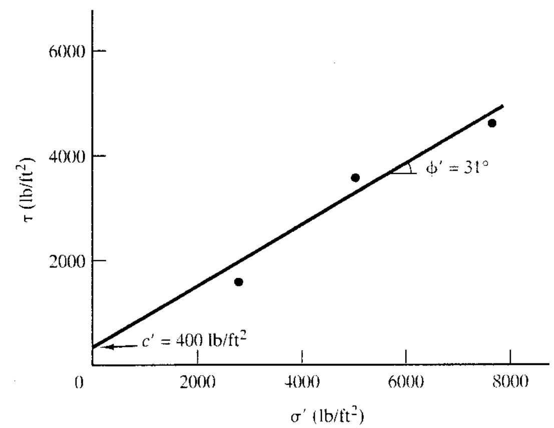
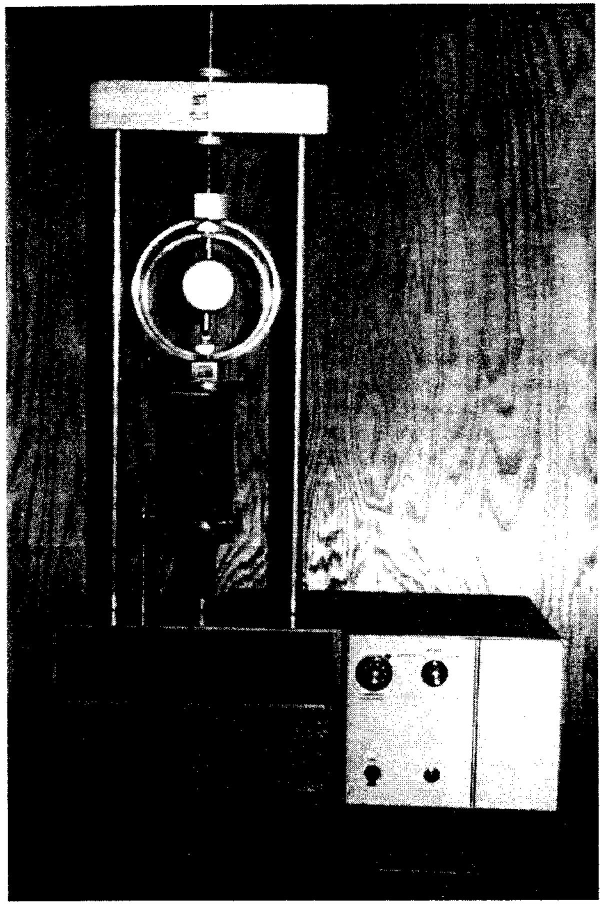


Figure 3.20 Direct shear test results.

Unconfined Compression Test

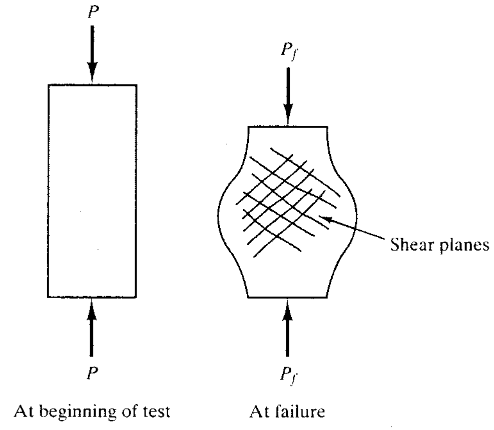
The unconfined compression test [ASTM D2166], shown in Figure 3.21a, uses a tall, cylindrical sample of cohesive soil subjected to an axial load. This load is applied quickly (i.e., only a couple of minutes to failure) to maintain undrained conditions.

At the beginning of the test, this load and the stresses in the soil are both equal to zero. As the load increases, the stresses in the soil increase, as shown by the Mohr's cir-



(a)

Figure 3.21 (a) Unconfined compression test machine; (b) Force analysis of an unconfined compression test.



(b)

Figure 3.21 *Continued*

cles in Figure 3.22, until the soil fails. The soil appears to fail in compression, and the test results are often expressed in terms of the compressive strength[2]; it actually fails in shear on diagonal planes, as shown in Figure 3.21b. The cross-sectional area of the sample increases as the test progresses, and the area at failure, A_f, is:

$$A_f = \frac{A_0}{1 - \epsilon_f} \tag{3.31}$$

The undrained shear strength, s_u, is:

$$s_u = \frac{P_f}{2 A_f} \tag{3.32}$$

Where:

A_f = cross-sectional area at failure
A_0 = initial cross-sectional area
ϵ_f = axial strain at failure
s_u = undrained shear strength
P_f = axial load at failure

[2]When reviewing unconfined compression test results, be sure to note whether they are expressed in terms of "compressive" strength or shear strength. In this test, the shear strength is equal to half of the compressive strength.

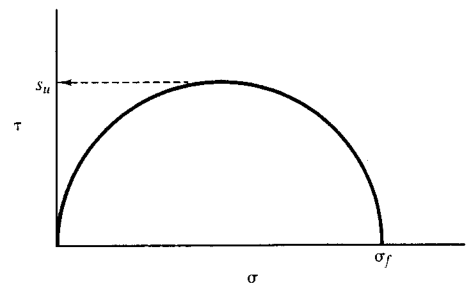
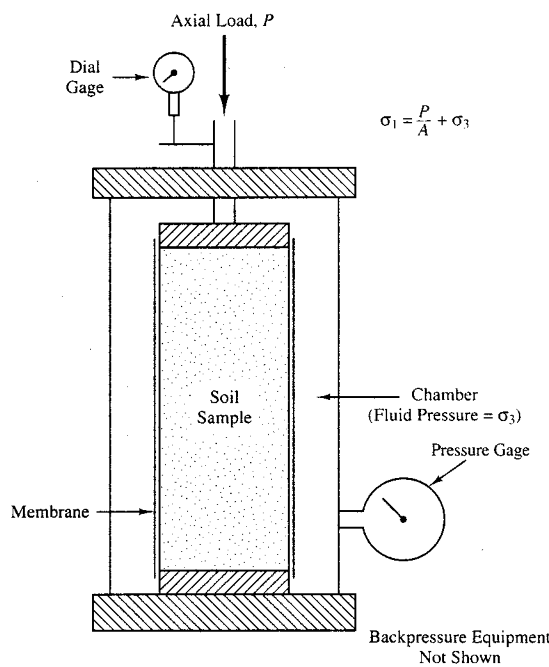


Figure 3.22 Mohr's circle at failure in an unconfined compression test.



(a)

Figure 3.23 Triaxial compression test. The soil sample is enclosed in a flexible membrane and subjected to a horizontal chamber pressure, σ_3. This pressure remains constant during the test. The axial load, P, is then increased until the soil fails. The dial gauge is used to measure the axial strain during the test. The chamber in the photograph is covered with a protective wire mesh.

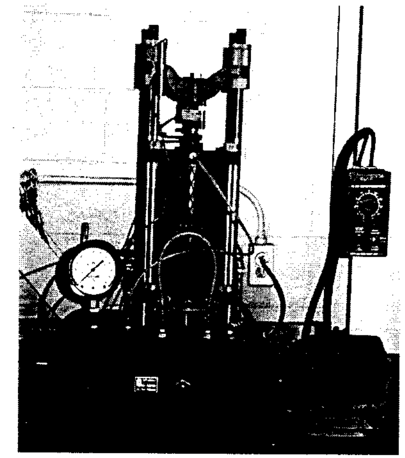


Figure 3.23 *Continued*

This test is inexpensive and in common use. It does not force failure to occur along a predetermined surface, and thus it reflects the presence of weak zones or planes. It usually provides conservative (low) results because the horizontal stress is zero rather than what is was in the field and because of sample disturbance. Tests of fissured clays are often misleading unless a large sample is used because the fissures in small samples are rarely representative of those in the field.

Triaxial Compression Test

The triaxial compression test [ASTM D2850] may be thought of as an extension of the unconfined compression test. The cylindrical soil sample is now located inside a pressurized chamber that supplies the desired lateral stress, as shown in Figure 3.23. Although the apparatus required to perform this test is much more complex, it also allows more flexibility and greater control over the test. It can measure either the drained or undrained strength of nearly any type of soil. In addition, unsaturated soils can be effectively saturated before testing.

The three most common types of triaxial compression tests are as follows:

- The ***unconsolidated-undrained (UU) test*** (also known as a *quick* or *Q* test): Horizontal and vertical stresses, usually equal to the vertical stress that was present in the field, are applied to the sample. No consolidation is permitted, and the soil is sheared under undrained conditions. The result is expressed as an s_u value.

- The ***consolidated-drained (CD) test*** (also known as a *slow* or *S* test): Horizontal and vertical stresses, usually equal to or greater than the vertical stress that was present in the field, are applied and the soil is allowed to consolidate. Then, it is sheared under drained conditions. Typically, three of these tests are performed at different confining stresses to find the drained values of c and ϕ.
- The ***consolidated-undrained test*** (also known as a *rapid* or *R* test): The initial stresses are applied as with the CD test and the soil is allowed to consolidate. However, the shearing occurs under undrained conditions. The results could be expressed as a value of s_u, but it is also possible to obtain the drained c and ϕ by measuring pore water pressures during the test and computing the effective stresses.

QUESTIONS AND PRACTICE PROBLEMS

3.8 Estimate the effective friction angle of the following soils:

a. Silty sand with a dry unit weight of 110 lb/ft^3.

b. Poorly-graded gravel with a relative density of 70%.

c. Very dense well-graded sand.

3.9 Explain the difference between the drained condition and the undrained condition.

3.10 A soil has $c' = 5$ kPa and $\phi' = 32°$. The effective stress at a point in the soil is 125 kPa. Compute the shear strength at this point.

SUMMARY

Major Points

1. Soil is a particulate material, which means it is an assemblage of individual particles. Its engineering properties are largely dependant on the interactions between these particles.
2. Soil can potentially include all three phases of matter (solid, liquid, and gas) simultaneously. It is helpful to measure the relative proportions of these three phases, and we express these proportions using standard weight-volume parameters.
3. The relative density is a special weight-volume parameter often used to describe the void ratio of sands and gravels.
4. The particle sizes in soil vary over several orders of magnitude, from sub-microscopic clay particles to gravel, cobbles, and boulders.
5. Clays are a special kind of soil because of their extremely small particle sizes and because of the special interactions between these particles and between the solids and the pore water.
6. Plasticity describes the relationship between moisture content and consistency in clays and silts. These relationships are quantified using the Atterberg limits.

7. Various soil classification systems are used in civil engineering. The most common system is the Unified Soil Classification System, which uses a standard system of group symbols and group names.
8. Groundwater has a profound impact on soil properties. Groundwater conditions can be very complex, but in this book, we will consider only the simple case of a horizontal groundwater table.
9. The pore water pressure is the pressure in the water within the soil voids. There are two kinds: hydrostatic pore water pressure is due solely to gravity acting on the pore water, while excess pore water pressure is due to the squeezing or expanding of the soil voids. Excess pore water pressures are always temporary.
10. Soils have both normal and shear stresses. They have two sources: Geostatic stresses are those due to the force of gravity acting on the soil mass, while induced stresses are due to applied external loads, such as foundations.
11. Settlement in soils can be the result of various processes. The most important of these is consolidation, which is the rearranging of particles into a tighter packing. Consolidation settlement analyses are based on data from a consolidation test, which is performed in the laboratory.
12. Soils in the field can be either normally consolidated or overconsolidated, depending on the difference between the current vertical effective stress and the preconsolidation stress, which is the greatest past value of this stress.
13. The degree of overconsolidation may be expressed using the overconsolidation margin or the overconsolidation ratio.
14. Shear strength in soil has two sources: frictional strength and cohesive strength.
15. Shear strength analyses may be based on effective stresses or on total stresses. Effective stress analyses are more accurate models of soil behavior, but are difficult to perform when excess pore water pressures are present.
16. The drained conditions are present when the rate of loading is slow compared to the rate of drainage. This is the case in sands and gravels. The undrained condition occurs when the reverse is true, which occurs in silts and clays.
17. Various tests are available to measure shear strength in the laboratory.

Vocabulary

Apparent cohesion	Cohesion	Drained condition
Atterberg limits	Cohesive strength	Dry unit weight
Boulder	Compression index	Effective stress
Clay	Consolidation	Effective stress analysis
Cobble	Consolidation test	Excess pore water pressure
Coefficient of lateral earth pressure	Degree of saturation	Fines
	Direct shear test	Friction angle

Frictional strength
Geostatic stress
Gravel
Groundwater
Groundwater table
Group symbol
Horizontal stress
Hydraulic conductivity
Hydrostatic pore water pressure
Induced stress
Liquid limit
Moisture content
Normally consolidated
Overconsolidated
Overconsolidation margin
Overconsolidation ratio
Particulate material
Phase diagram
Plastic limit
Plasticity
Pore water pressure
Porosity
Rebound curve
Recompression index
Recompression curve
Relative density
Sand
Settlement
Silt
Specific gravity
Stress
Total stress
Total stress analysis
Triaxial compression test
True cohesion
Unconfined compression test
Undrained condition
Undrained shear strength
Unified soil classification system
Unit weight
Virgin curve
Void ratio
Weight-volume relationship

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

3.11 Which laboratory tests would be appropriate for finding s_u of a clay?

3.12 Which laboratory tests would be appropriate for finding ϕ' of a sand?

3.13 The following data were obtained from direct shear tests on a series of 60-mm diameter samples of a certain soil:

Test No.	Effective Stress at Failure (kPa)	Shear Strength (kPa)
1	75.0	51.2
2	150.0	82.7
3	225.0	110.1

Find the values of c' and ϕ'.

3.14 The soil profile at a certain site is as follows:

Depth (ft)	γ (lb/ft^3)	c' (lb/ft^2)	ϕ' (deg)	s_u (lb/ft^2)
0–12	119			1000
12–20	126	200	20	
20–32	129	0	32	

The groundwater table is at a depth of 15 ft.

Develop plots of pore water pressure, total stress, effective stress, and shear strength vs. depth. All four of these plots should be superimposed on the same diagram with the parameters on the horizontal axis (increasing to the right) and depth on the vertical axis (increasing downward).

Hint: Because the cohesion and friction angle suddenly change at the strata boundaries, the shear strength also may change suddenly at these depths.

3.15 Repeat Problem 3.14 using the following data:

Depth (m)	γ (kN/m^3)	c' (kPa)	ϕ' (deg)	s_u (kPa)
0–5	18.5			50.0
5–12	20.0	8.4	21	
12–20	20.5	0.0	35	

The groundwater table is at a depth of 7 m.

3.16 A 9 ft thick fill is to be placed on the soil shown in Figure 3.24. Once it is compacted, this fill will have a unit weight of 122 lb/ft^3. Compute the ultimate settlement caused by consolidation of the underlying clay.

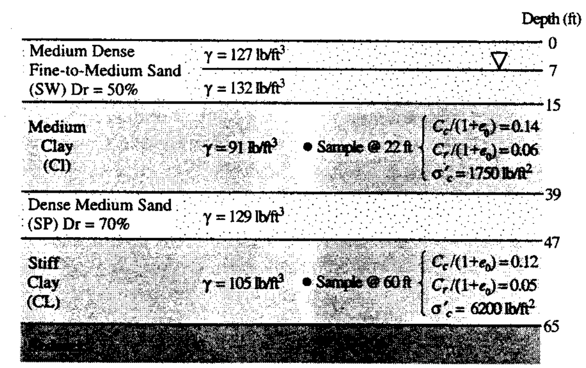


Figure 3.24 Soil profile for Problem 3.16.

4

Site Exploration and Characterization

Make a large number of trial holes to find the different strata in order to be sure that an apparently good soil does not overlay a clay, a sandy soil, or some other soil which can be compressed under a load. If the trial holes cannot be made, then the earth may be beaten with a wooden rafter six or eight feet long; if the sound is dry and light, and the soil offers resistance, then the earth is firm, but a heavy sound and poor resistance mean a worthless foundation.

Paraphrased from *L'Architecture Pratique,* a 1691 book of practical design and construction guidelines by the French engineer Bullet (after Heyman, 1972; Reprinted with permission of Cambridge University Press)

One of the fundamental differences between the practices of structural engineering and geotechnical engineering is the way each determines the engineering properties of the materials with which they work. For practical design problems, structural engineers normally find the necessary material properties by referring to handbooks. For example, if one wishes to use A36 steel, its engineering properties (strength, modulus of elasticity, etc.) are well known and can be found from a variety of sources. It is not necessary to measure the strength of A36 steel each time we use it in a design (although routine strength tests may be performed later as a quality control function). Conversely, the geotechnical engineer works with soil and rock, both of which are natural materials with unknown engineering properties. Therefore, we must identify and test the materials at each new site before conducting any analyses.

Modern soil investigation and testing techniques have progressed far beyond Bullet's method of beating the earth with wooden rafters. A variety of techniques are available, as discussed i [illegible] this continues to be the single largest source of

uncertainties in foundation engineering. Our ability to perform analyses far exceeds our ability to determine the appropriate soil properties to input into these analyses. Therefore, it is very important for the foundation engineer to be familiar with the available techniques, know when to use them, and understand the degree of precision (or lack of precision!) associated with them.

For purposes of this discussion, we will divide these techniques into three categories:

- **Site investigation** includes methods of defining the soil profile and other relevant data and recovering soil samples.
- **Laboratory testing** includes testing the soil samples in order to determine relevant engineering properties.
- **In-situ testing** includes methods of testing the soils in-place, thus avoiding the difficulties associated with recovering samples.

4.1 SITE EXPLORATION

The objectives of the site investigation phase include:

- Determining the locations and thicknesses of the soil strata.
- Determining the location of the groundwater table as well as any other groundwater-related characteristics.
- Recovering soil samples.
- Defining special problems and concerns.

Typically, we accomplish these goals using a combination of literature searches and on-site exploration techniques.

Background Literature Search

Before conducting any new exploration at a project site, gather whatever information is already available, both for the proposed structure and the subsurface conditions at the site. Important information about the structure would include:

- Its location and dimensions.
- The type of construction, column loads, column spacing, and allowable settlements.
- Its intended use.
- The finish floor elevation.
- The number and depth of any basements.
- The depth and extent of any proposed grading.
- Local building code requirements.

The literature search also should include an effort to obtain at least a preliminary idea of the subsurface conditions. It would be very difficult to plan an exploration program with no such knowledge. Fortunately, many methods and resources are often available to gain a preliminary understanding of the local soil conditions. These may include one or more of the following:

- Determining the geologic history of the site, including assessments of anticipated rock and soil types, the proximity of faults, and other geologic features.
- Gathering copies of boring logs and laboratory test results from previous investigations on this or other nearby sites.
- Reviewing soil maps developed for agricultural purposes.
- Reviewing old and new aerial photographs and topographic maps (may reveal previous development or grading at the site).
- Reviewing water well logs (helps establish historic groundwater levels).
- Locating underground improvements, such as utility lines, both onsite and immediately offsite.
- Locating foundations of adjacent structures, especially those that might be impacted by the proposed construction.

At some sites, this type of information may be plentiful, whereas at others it may be scarce or nonexistent.

Field Reconnaissance

Along with the background literature search, the foundation engineer should visit the site and perform a field reconnaissance. Often such visits will reveal obvious concerns that may not be evident from the literature search or the logs of the exploratory borings.

The field reconnaissance would include obtaining answers to such questions as the following:

- Is there any evidence of previous development on the site?
- Is there any evidence of previous grading on the site?
- Is there any evidence of landslides or other stability problems?
- Are nearby structures performing satisfactorily?
- What are the surface drainage conditions?
- What types of soil and/or rock are exposed at the ground surface?
- Will access problems limit the types of subsurface exploration techniques that can be used?
- Might the proposed construction affect existing improvements? (For example, a fragile old building adjacent to the site might be damaged by vibrations from pile driving.)
- Do any offsite conditions affect the proposed development? (For example, potential flooding, mudflows, rockfalls, etc.)

Subsurface Exploration and Sampling

The heart of the site investigation phase consists of exploring the subsurface conditions and sampling the soils. These efforts provide most of the basis for developing a design soil profile. A variety of techniques are available to accomplish these goals.

Exploratory Borings

The most common method of exploring the subsurface conditions is to drill a series of vertical holes in the ground. These are known as *borings* or *exploratory borings* and are typically 75 to 600 mm (3–24 in) in diameter and 3 to 30 m (10–100 ft) deep. They can be drilled with hand augers or with portable power equipment, but they are most commonly drilled using a truck-mounted rig, as shown in Figure 4.1.

A wide variety of drilling equipment and techniques are available to accommodate the various subsurface conditions that might be encountered. Sometimes it is possible to drill an open hole using a *flight auger* or a *bucket auger*, as shown in Figure 4.2. However, if the soil is subject to *caving* (i.e., the sides of the boring fall in) or *squeezing* (the soil moves inward, reducing the diameter of the boring), then it will be necessary to provide some type of lateral support during drilling. Caving is likely to be encountered in clean sands, especially below the groundwater table, while squeezing is likely in soft saturated clays.

One method of dealing with caving or squeezing soils is to use *casing*, as shown in Figure 4.3a. This method involves temporarily lining some or all of the boring with a steel

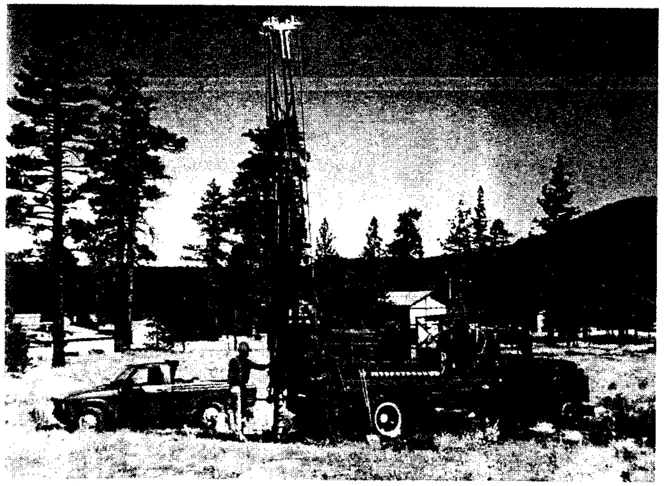


Figure 4.1 A truck-mounted drill rig.

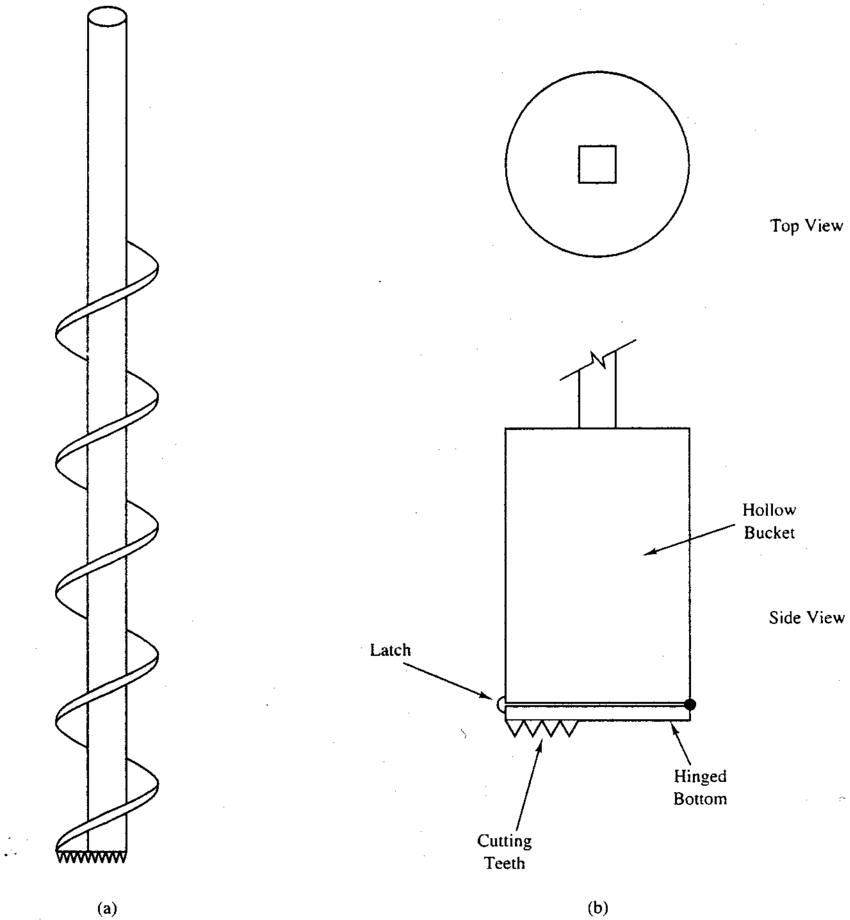


Figure 4.2 (a) Flight auger; and (b) Bucket auger.

pipe. Alternatively, we could use a *hollow-stem auger*, as shown in Figure 4.3b. The driller screws each of these augers into the ground and obtains soil samples by lowering sampling tools through a hollow core. When the boring is completed, the augers are removed. Finally, we could use a *rotary wash boring*, as shown in Figure 4.3c. These borings are filled with a bentonite slurry (a combination of bentonite clay and water) to provide hydrostatic pressure on the sides of the boring and thus prevent caving.

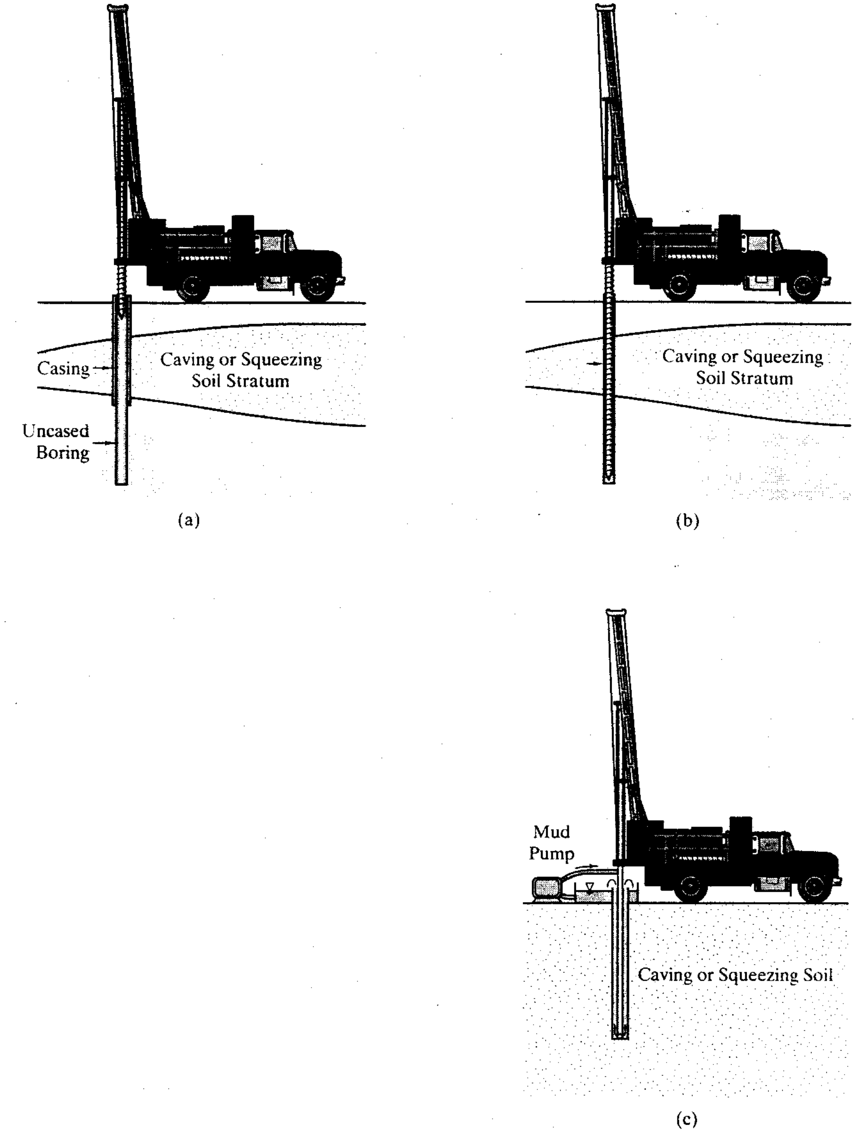


Figure 4.3 Methods of dealing with caving or squeezing soils: (a) casing; (b) hollow-stem auger; and (c) rotary wash boring.

Drilling through rock, especially hard rock, requires different methods and equipment. Engineers often use *coring*, which recovers intact cylindrical specimens of the rock.

There are no absolute rules to determine the required number, spacing, and depth of exploratory borings. Such decisions are based on the findings from the field reconnaissance, along with engineering judgement and a knowledge of customary standards of practice. This is a subjective process that involves many factors, including:

- How large is the site?
- What kinds of soil and rock conditions are expected?
- Is the soil profile erratic, or is it consistent across the site?
- What is to be built on the site (small building, large building, bridge, etc.)?
- How critical is the proposed project (i.e., what would be the consequences of a failure?)?
- How large and heavy are the proposed structures?
- Are all areas of the site accessible to drill rigs?

Although we will not know the final answers to some of these questions until the site characterization program is completed, we should have at least a preliminary idea based on the literature search and field reconnaissance.

For buildings taller than three stories or 12.2 m (40 ft), the *BOCA National Building Code* (BOCA, 1996) requires at least one boring for every 230 m^2 (2500 ft^2) of built-over area. Table 4.1 presents rough guidelines for determining the normal spacing of exploratory borings at sites not governed by the BOCA code. However, it is important to recognize that there is no single "correct" solution for the required number and depth of borings, and these guidelines must be tempered with appropriate engineering judgement.

Borings generally should extend at least to a depth such that the change in vertical effective stress due to the new construction is less than 10 percent of the initial vertical effective stress. For buildings on spread footing or mat foundations, this criteria is met by following the guidelines in Table 4.2. If fill is present, the borings must extend through it and into the natural ground below, and if soft soils are present, the borings should extend through them and into firmer soils below. For heavy structures, at least some of the borings should be carried down to bedrock, if possible, but certainly well below the depth of any proposed pile foundations.

TABLE 4.1 ROUGH GUIDELINES FOR SPACING EXPLORATORY BORINGS FOR PROPOSED MEDIUM TO HEAVY WEIGHT BUILDINGS, TANKS, AND OTHER SIMILAR STRUCTURES

	Structure Footprint Area for Each Exploratory Boring	
Subsurface Conditions	(m^2)	(ft^2)
Poor quality and/or erratic	100–300	1,000–3,000
Average	200–400	2,000–4,000
High quality and uniform	300–1,000	3,000–10,000

TABLE 4.2 GUIDELINES FOR DEPTHS OF EXPLORATORY BORINGS FOR BUILDINGS ON SHALLOW FOUNDATIONS (Adapted from Sowers, 1979)

	Minimum Depth of Borings (S = number of stories; D = anticipated depth of foundation)	
Subsurface Conditions	(m)	(ft)
Poor	$6\,S^{0.7}+D$	$20\,S^{0.7}+D$
Average	$5\,S^{0.7}+D$	$15\,S^{0.7}+D$
Good	$3\,S^{0.7}+D$	$10\,S^{0.7}+D$

On large projects, the drilling program might be divided into two phases: a preliminary phase to determine the general soil profile, and a final phase based on the results of the preliminary borings.

The conditions encountered in an exploratory boring are normally presented in the form of a *boring log*, as shown in Figure 4.4. These logs also indicate the sample locations and might include some of the laboratory test results.

Soil Sampling

One of the primary purposes of drilling the exploratory borings is to obtain representative soil samples. We use these samples to determine the soil profile and to perform laboratory tests.

There are two categories of samples: disturbed and undisturbed. A *disturbed sample* (sometimes called a *bulk* sample) is one in which there is no attempt to retain the in-place structure of the soil. The driller might obtain such a sample by removing the cuttings off the bottom of a flight auger and placing them in a bag. Disturbed samples are suitable for many purposes, such as classification and Proctor compaction tests.

A truly *undisturbed sample* is one in which the soil is recovered completely intact and its in-place structure and stresses are not modified in any way. Such samples are desirable for laboratory tests that depend on the structure of the soil, such as consolidation tests and shear strength tests. Unfortunately, the following problems make it almost impossible to obtain a truly undisturbed soil sample:

- Shearing and compressing the soil during the process of inserting the sampling tool.
- Relieving the sample of its in-situ stresses.
- Possible drying and desiccation.
- Vibrating the sample during recovery and transport.

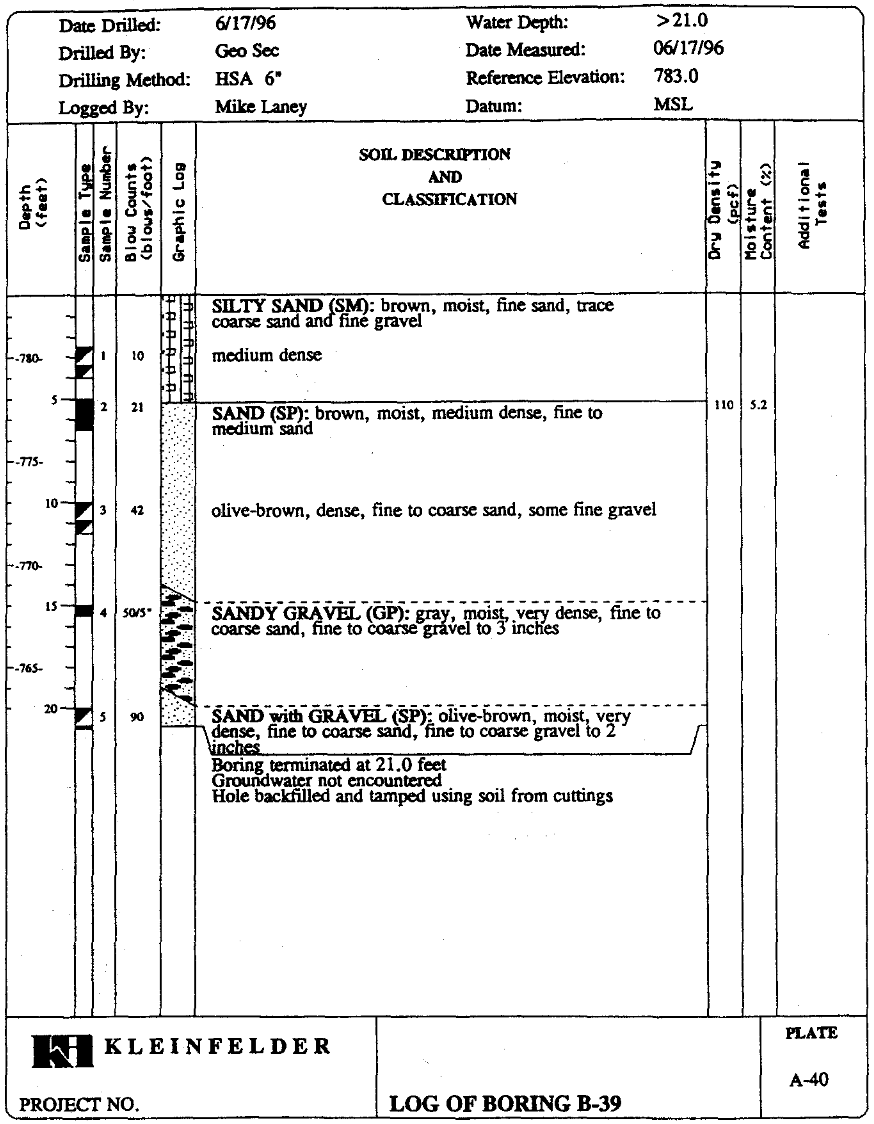


Figure 4.4 A boring log. Samples 2 and 4 were obtained using a heavy-wall sampler, and the corresponding blow counts are the number of hammer blows required to drive the sampler. Samples 1, 3, and 5 are standard penetration tests, and the corresponding blow counts are the N_{60} values, as discussed later in this chapter. If the groundwater table had been encountered, it would have been shown on the log (Kleinfelder, Inc.).

Some soils are more subject to disturbance than others. For example, techniques are available to obtain good quality samples of medium clays, while clean sands are almost impossible to sample without extensive disturbance. However, even the best techniques produce samples that are best described as "relatively undisturbed."

A variety of sampling tools is available. Some of these tools are shown in Figure 4.5. Those with thin walls produce the least disturbance, but they may not have the integrity needed to penetrate hard soils.

Groundwater Monitoring

The position and movements of the groundwater table are very important factors in foundation design. Therefore, subsurface investigations must include an assessment of groundwater conditions. This is often done by installing an *observation well* in the com-

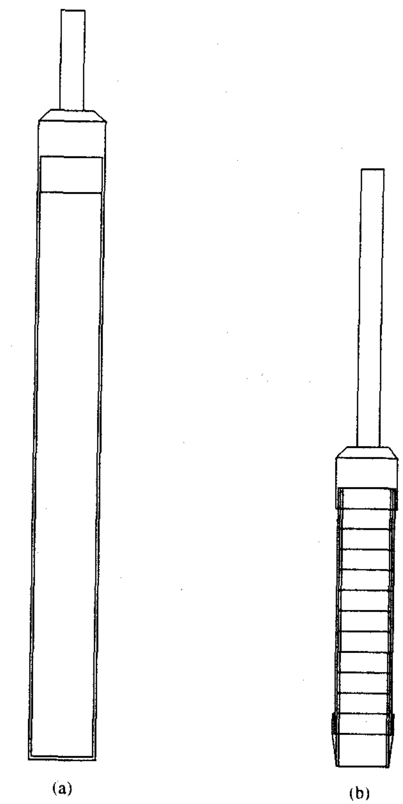


Figure 4.5 Common soil sampling tools: (a) Shelby tube samples have thin walls to reduce sample disturbance; and (b) Ring-lined barrel samplers have thicker walls to withstand harder driving. Both are typically 60–100 mm (2.5–4 in) in diameter.

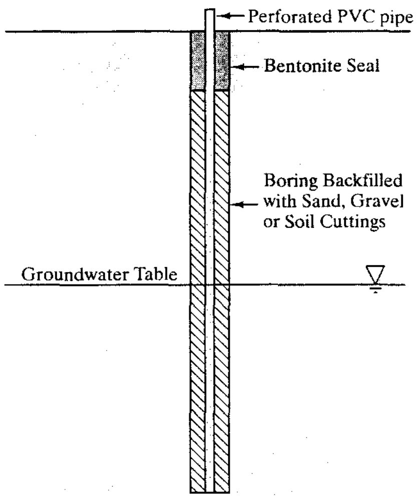


Figure 4.6 A typical observation well.

pleted boring to monitor groundwater conditions. Such wells typically consist of slotted or perforated PVC pipe, as shown in Figure 4.6. Once the groundwater level has stabilized, we can locate it by lowering a probe into the observation well.

We usually compare observation well data with historic groundwater records, or at least consider the season of the year and the recent precipitation patterns to determine the design groundwater level. This design level should represent the worst (i.e., shallowest) groundwater level that is likely to occur during the design life of the project. This level is often shallower than that observed in the observation wells.

Exploratory Trenches

Sometimes it is only necessary to explore the upper 3 m (10 ft) of soil. This might be the case for lightweight structures on sites where the soil conditions are good, or on sites with old shallow fills of questionable quality. Additional shallow investigations might also be necessary to supplement a program of exploratory borings. In such cases, it can be very helpful to dig *exploratory trenches* (also known as *test pits*) using a backhoe as shown in Figure 4.7. They provide more information than a boring of comparable depth (because more of the soil is exposed) and are often less expensive. Disturbed samples can easily be recovered with a shovel, and undisturbed samples can be obtained using hand-held sampling equipment.

Two special precautions are in order when using exploratory trenches: First, these trenches must be adequately shored before anyone enters them (see OSHA Excavation Regulations). Many individuals, including one of the author's former colleagues, have

Figure 4.7 This exploratory trench was dug by the backhoe in the background, and has been stabilized using aluminum-hydraulic shoring. An engineering geologist is logging the soil conditions in one wall of the trench.

been killed by neglecting to enforce this basic safety measure. Second, these trenches must be properly backfilled to avoid creating an artificial soft zone that might affect future construction.

4.2 LABORATORY TESTING

Soil samples obtained from the field are normally brought to a soil mechanics laboratory for further classification and testing. This method is sometimes called *ex-situ testing*. The purpose of the testing program is to determine the appropriate engineering properties of the soil, as follows:

- **Classification, weight-volume, and index tests**—Several routine tests are usually performed on many of the samples to ascertain the general characteristics of the soil profile. These include:
  - Moisture content
  - Unit weight (density)
  - Atterberg limits (plastic limit, liquid limit)
  - Grain-size distribution (sieve and hydrometer analyses)

These tests are inexpensive and can provide a large quantity of valuable information.

- **Shear strength tests**—Virtually all foundation designs require an assessment of shear strength. Common shear strength tests include the direct shear test, the unconfined compression test, and the triaxial compression test, as discussed in Chapter 3. Shear strength also may be determined from in-situ tests, as discussed later in this chapter.
- **Consolidation tests**—Foundation designs also require an assessment of soil compressibility, which provides the necessary data for settlement analyses. In clays and silts, this data is most often obtained by conducting laboratory consolidation tests, as discussed in Chapter 3.
- **Compaction tests**—Sometimes it is necessary to place compacted fills at a site, and place the foundations on these fills. In such cases, we perform Proctor compaction tests to assess the compaction characteristics of the soil.
- **Corrosivity tests**—When corrosion or sulfate attack is a concern, as discussed in Chapter 2, we need to perform special tests such as resistivity tests and sulfate content tests, and then use the results to design appropriate protective measures.

QUESTIONS AND PRACTICE PROBLEMS

4.1 Describe a scenario that would require a very extensive site investigation and laboratory testing program (i.e., one in which a large number of borings and many laboratory and/or in-situ tests would be necessary).

4.2 How would you go about determining the location of the groundwater table in the design soil profile. Recall that this is not necessarily the same as the groundwater table that was present when the borings were made.

4.3 A five-story office building is to be built on a site underlain by moderately uniform soils. Bedrock is at a depth of over 200 m. This building will be 50 m wide and 85 m long, and the foundations will be founded at a depth of 1 m below the ground surface. Determine the required number and depth of the exploratory borings.

4.4 A two-story reinforced concrete building is to be built on a vacant parcel of land. This building will be 100 ft wide and 200 ft long. Based on information from other borings on adjacent properties, you are reasonably certain that the soils below a depth of 5 to 8 feet (1.5 to 2.5 m) are strong and relatively incompressible. However, the upper soils are questionable because several uncompacted fills have been found in the neighborhood. Not only are these uncompacted fills loose, they have often contained various debris such as wood, rocks, and miscellaneous trash. However, none of these deleterious materials is present at the ground surface at this site.

Plan a site investigation program for this project and present your plan in the form of written instructions to your field crew. This plan should include specific instructions regarding what to do, where to do it, and any special instructions. You should presume that the field crew is experienced in soil investigation work, but is completely unfamiliar with this site.

4.3 IN-SITU TESTING

Laboratory tests on "undisturbed" soil samples are no better than the quality of the sample. Depending on the type of test, the effects of sample disturbance can be significant, especially in sands. Fortunately, we can often circumvent these problems by using *in-situ* (in-place) testing methods. These entail bringing the test equipment to the field and testing the soils in-place.

In addition to bypassing sample disturbance problems, in-situ tests have the following advantages:

- They are usually less expensive, so a greater number of tests can be performed, thus characterizing the soil in more detail.
- The test results are available immediately.

However, they also have disadvantages, including:

- Often no sample is obtained, thus making soil classification more difficult.
- The engineer has less control over confining stresses and drainage.

In most cases, we must use empirical correlations and calibrations to convert in-situ test results to appropriate engineering properties for design. Many such methods have been published, and some of them are included in this book. Most of these correlations were developed for clays of low to moderate plasticity or for quartz sands, and thus may not be appropriate for special soils such as very soft clays, organic soils, sensitive clays, fissured clays, cemented soils, calcareous sands, micaceous sands, collapsible soils, and frozen soils.

Some in-situ test methods have been in common use for several decades, while others are relative newcomers. Many of these tests will probably continue to become more common in engineering practice.

Standard Penetration Test (SPT)

One of the most common in-situ tests is the standard penetration test, or SPT. This test was originally developed in the late 1920s and has been used most extensively in North and South America, the United Kingdom, and Japan. Because of this long record of experience, the test is well established in engineering practice. Unfortunately, it is also plagued by many problems that affect its accuracy and reproducibility and is slowly being replaced by other test methods, especially on larger and more critical projects.

Test Procedure

The test procedure was not standardized until 1958 when ASTM standard D1586 first appeared. It is essentially as follows[1]:

1. Drill a 60 to 200 mm (2.5–8 in) diameter exploratory boring to the depth of the first test.
2. Insert the SPT sampler (also known as a *split-spoon sampler*) into the boring. The shape and dimensions of this sampler are shown in Figure 4.8. It is connected via steel rods to a 63.5 kg (140 lb) hammer, as shown in Figure 4.9.
3. Using either a rope and cathead arrangement or an automatic tripping mechanism, raise the hammer a distance of 760 mm (30 in) and allow it to fall. This energy drives the sampler into the bottom of the boring. Repeat this process until the sampler has penetrated a distance of 450 mm (18 in), recording the number of hammer blows required for each 150 mm (6 in) interval. Stop the test if more than fifty blows are required for any of the intervals, or if more than one hundred total blows are required. Either of these events is known as *refusal* and is so noted on the boring log.
4. Compute the N value by summing the blow counts for the last 300 mm (12 in) of penetration. The blow count for the first 150 mm (6 in) is retained for reference purposes, but not used to compute N because the bottom of the boring is likely to be disturbed by the drilling process and may be covered with loose soil that fell from the sides of the boring. Note that the N value is the same regardless of whether the engineer is using English or SI units.
5. Remove the SPT sampler; remove and save the soil sample.
6. Drill the boring to the depth of the next test and repeat steps 2 through 6 as required.

Thus, N values may be obtained at intervals no closer than 450 mm (18 in).

Unfortunately, the procedure used in the field varies, partially due to changes in the standard, but primarily as a result of variations in the test procedure and poor workmanship. The test results are sensitive to these variations, so the N value is not as repeatable as we would like. The principal variants are as follows:

- Method of drilling
- How well the bottom of the hole is cleaned before the test
- Presence or lack of drilling mud
- Diameter of the drill hole
- Location of the hammer (surface type or down-hole type)
- Type of hammer, especially whether it has a manual or automatic tripping mechanism

[1]See the ASTM D1586 standard for the complete procedure.

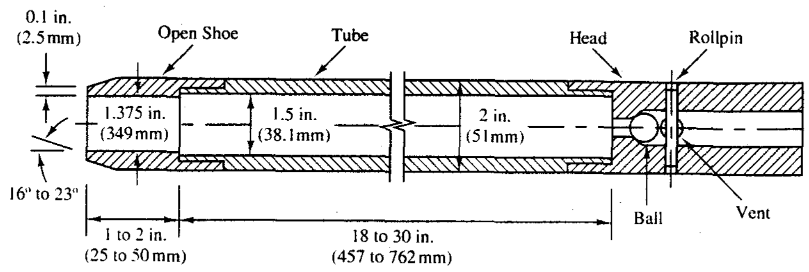


Figure 4.8 The SPT sampler (Adapted from ASTM D1586: Copyright ASTM, used with permission).

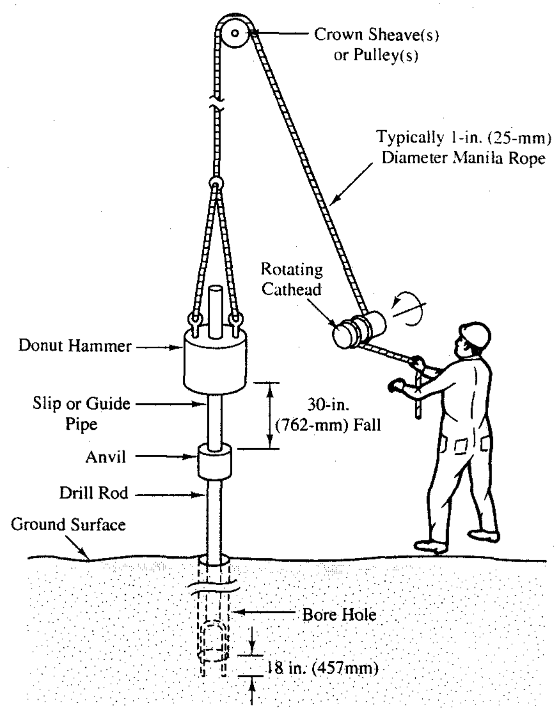


Figure 4.9 The SPT sampler in place in the boring with hammer, rope, and cathead (Adapted from Kovacs, et al., 1981).

- Number of turns of the rope around the cathead
- Actual hammer drop height (manual types are often as much as 25 percent in error)
- Mass of the anvil that the hammer strikes
- Friction in rope guides and pulleys
- Wear in the sampler drive shoe
- Straightness of the drill rods
- Presence or absence of liners inside the sampler (this seemingly small detail can alter the test results by 10 to 30 percent)
- Rate at which the blows are applied

As a result of these variables, the test results will vary depending on the crew and equipment. These variations, as well as other aspects of the test, were the subject of increased scrutiny during the 1970s and 1980s along with efforts to further standardize the "standard" penetration test (DeMello, 1971; Nixon, 1982). Based on these studies, Seed et al. (1985) recommended the following additional criteria be met when conducting standard penetration tests:

- Use the rotary wash method to create a boring that has a diameter between 200 and 250 mm (4–5 in). The drill bit should provide an upward deflection of the drilling mud (tricone or baffled drag bit).
- If the sampler is made to accommodate liners, then these liners should be used so the inside diameter is 35 mm (1.38 in).
- Use A or AW size drill rods for depths less than 15 m (50 ft) and N or NW size for greater depths.
- Use a hammer that has an efficiency of 60 percent.
- Apply the hammer blows at a rate of 30 to 40 per minute.

Fortunately, automatic hammers are becoming more popular. They are much more consistent than hand-operated hammers, and thus improve the reliability of the test.

Although much has been said about the disadvantages of the SPT, it does have at least three important advantages over other in-situ test methods: First, it obtains a sample of the soil being tested. This permits direct soil classification. Most of the other methods do not include sample recovery, so soil classification must be based on conventional sampling from nearby borings and on correlations between the test results and soil type. Second, it is very fast and inexpensive because it is performed in borings that would have been drilled anyway. Finally, nearly all drill rigs used for soil exploration are equipped to perform this test, whereas other in-situ tests require specialized equipment that may not be readily available.

Corrections to the Test Data

We can improve the raw SPT data by applying certain correction factors. The variations in testing procedures may be at least partially compensated by converting the measured N to N_{60} as follows (Skempton, 1986):

TABLE 4.3 SPT HAMMER EFFICIENCIES (Adapted from Clayton, 1990).

Country	Hammer Type (per Figure 4.10)	Hammer Release Mechanism	Hammer Efficiency E_m
Argentina	Donut	Cathead	0.45
Brazil	Pin weight	Hand dropped	0.72
China	Automatic	Trip	0.60
	Donut	Hand dropped	0.55
	Donut	Cathead	0.50
Colombia	Donut	Cathead	0.50
Japan	Donut	Tombi trigger	0.78–0.85
	Donut	Cathead 2 turns + special release	0.65–0.67
UK	Automatic	Trip	0.73
US	Safety	2 turns on cathead	0.55–0.60
	Donut	2 turns on cathead	0.45
Venezuela	Donut	Cathead	0.43

$$N_{60} = \frac{E_m C_B C_S C_R N}{0.60} \tag{4.1}$$

Where:

N_{60} = SPT N value corrected for field procedures
E_m = hammer efficiency (from Table 4.3)
C_B = borehole diameter correction (from Table 4.4)
C_S = sampler correction (from Table 4.4)
C_R = rod length correction (from Table 4.4)
N = measured SPT N value

Many different hammer designs are in common use, none of which is 100 percent efficient. Some common hammer designs are shown in Figure 4.10, and typical hammer efficiencies are listed in Table 4.3. Many of the SPT-based design correlations were developed using hammers that had an efficiency of about 60 percent, so Equation 4.2 corrects the results from other hammers to that which would have been obtained if a 60 percent efficient hammer was used.

The SPT data also may be adjusted using an *overburden correction* that compensates for the effects of effective stress. Deep tests in a uniform soil deposit will have higher N values than shallow tests in the same soil, so the overburden correction adjusts the measured N values to what they would have been if the vertical effective stress, σ_z', was 100 kPa (2000 lb/ft^2). The corrected value, $(N_1)_{60}$, is (Liao and Whitman, 1985):

TABLE 4.4 BOREHOLE, SAMPLER, AND ROD CORRECTION FACTORS (Adapted from Skempton, 1986).

Factor	Equipment Variables	Value
Borehole diameter factor, C_B	65–115 mm (2.5–4.5 in)	1.00
	150 mm (6 in)	1.05
	200 mm (8 in)	1.15
Sampling method factor, C_S	Standard sampler	1.00
	Sampler without liner (not recommended)	1.20
Rod length factor, C_R	3–4 m (10–13 ft)	0.75
	4–6 m (13–20 ft)	0.85
	6–10 m (20–30 ft)	0.95
	>10 m (>30 ft)	1.00

$$(N_1)_{60} = N_{60}\sqrt{\frac{2000\ \text{lb/ft}^2}{\sigma_z'}} \quad (4.2\ \text{English})$$

$$(N_1)_{60} = N_{60}\sqrt{\frac{100\ \text{kPa}}{\sigma_z'}} \quad (4.2\ \text{SI})$$

Where:

$(N_1)_{60}$ = SPT N value corrected for field procedures and overburden stress

σ_z' = vertical effective stress at the test location

N_{60} = SPT N value corrected for field procedures

Although Liao and Whitman did not place any limits on this correction, it is probably best to keep $(N_1)_{60} \leq 2\,N_{60}$. This limit avoids excessively high $(N_1)_{60}$ values at shallow depths.

The use of correction factors is often a confusing issue. Corrections for field procedures are always appropriate, but the overburden correction may or may not be appropriate depending on the procedures used by those who developed the analysis method under consideration. In this book, the overburden correction should be applied only when the analysis procedure calls for an $(N_1)_{60}$ value.

Uses of SPT Data

The SPT N value, as well as many other test results, is only an *index* of soil behavior. It does not directly measure any of the conventional engineering properties of soil and is

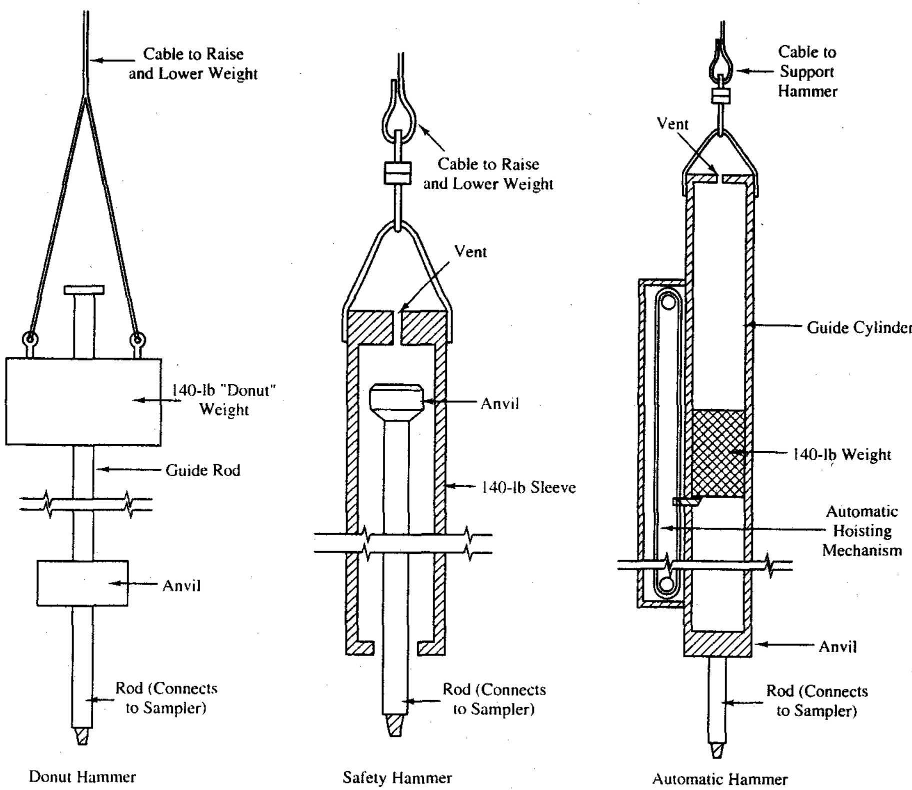


Figure 4.10 Types of SPT hammers.

useful only when appropriate correlations are available. Many such correlations exist, all of which were obtained empirically.

Unfortunately, most of these correlations are very approximate, especially those based on fairly old data that were obtained when test procedures and equipment were different from those now used. In addition, because of the many uncertainties in the SPT results, all of these correlations have a wide margin of error.

Be especially cautious when using correlations between SPT results and engineering properties of clays because these functions are especially crude. In general, the SPT should be used only in sandy soils.

Correlation with Relative Density

The relative density is a useful way to describe the consistency of sands, as discussed in Chapter 3. It may be determined from SPT data using the following empirical correlation (Kulhawy and Mayne, 1990):

$$D_r = \sqrt{\frac{(N_1)_{60}}{C_P C_A C_{OCR}}} \times 100\% \tag{4.3}$$

$$C_P = 60 + 25 \log D_{50} \tag{4.4}$$

$$C_A = 1.2 + 0.05 \log\left(\frac{t}{100}\right) \tag{4.5}$$

$$C_{OCR} = OCR^{0.18} \tag{4.6}$$

Where:

D_r = relative density (in decimal form)
$(N_1)_{60}$ = SPT N value corrected for field procedures and overburden stress
C_P = grain-size correction factor
C_A = aging correction factor
C_{OCR} = overconsolidation correction factor
D_{50} = grain size at which 50 percent of the soil is finer (mm)
t = age of soil (time since deposition) (years)
OCR = overconsolidation ratio

Rarely will we have test data to support all of the parameters in Equations 4.4 to 4.6, so it often is necessary to estimate some of them. If a grain size curve is not available, D_{50} may be estimated from a visual examination of the soil with reference to Table 3.4. Geologists can sometimes estimate the age of sand deposits, but Equation 4.5 is not very sensitive to the chosen value, so a "wild guess" is probably sufficient. A value of t = 1000 years is probably sufficient for most analyses. The overconsolidation ratio is rarely known in sands, but values of about 1 in loose sands (N_{60} < 10) to about 4 in dense sands (N_{60} > 50) should be sufficient for Equation 4.6.

Correlation with Shear Strength

DeMello (1971) suggested a correlation between SPT results and the effective friction angle of uncemented sands, ϕ', as shown in Figure 4.11. This correlation should be used only at depths greater than about 2 m (7 ft).

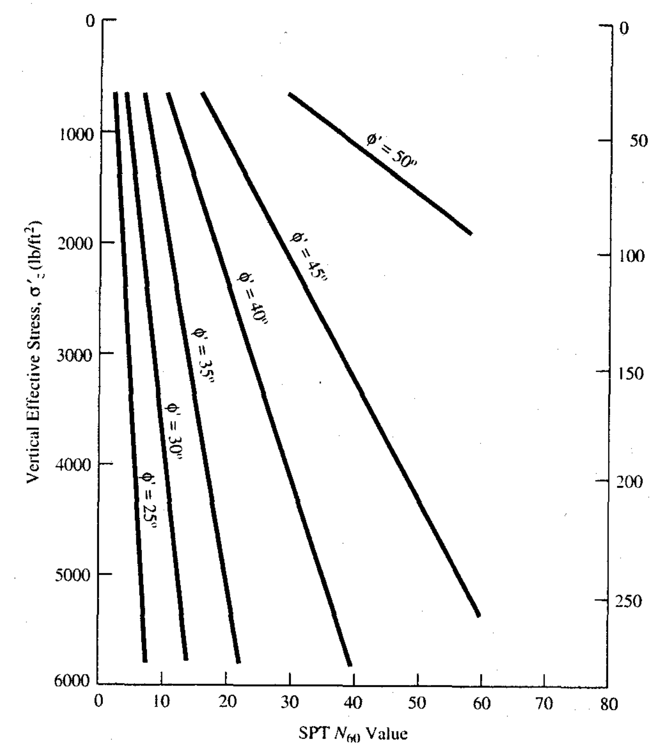


Figure 4.11 Empirical correlation between N_{60} and ϕ' for uncemented sands (Adapted from DeMello, 1971).

Example 4.1

A 6-inch diameter exploratory boring has been drilled through a fine sand to a depth of 19 ft. An SPT N-value of 23 was obtained at this depth using a USA safety hammer with a standard sampler. The boring then continued to greater depths, eventually encountering the groundwater table at a depth of 35 ft. The unit weight of the sand is unknown. Compute $(N_1)_{60}$, ϕ', and D_r at the test location, and use this data to classify the consistency of the sand.

Solution

Per Table 3.2, SP soils above the groundwater table typically have a unit weight of 95 to 125 lb/ft³. The measured N-value of 23 suggests a moderately dense sand, so use γ = 115 lb/ft³. Therefore, at the sample depth:

$$\sigma'_z = \Sigma \gamma H - u = (115\ \text{lb/ft}^3)(19\ \text{ft}) - 0 = 2185\ \text{lb/ft}^2$$

$$N_{60} = \frac{E_m C_B C_S C_R N}{0.60} = \frac{(0.57)(1.05)(1.00)(0.85)(23)}{0.60} = 19.5$$

$$(N_1)_{60} = N_{60}\sqrt{\frac{2000\ \text{lb/ft}^2}{\sigma'_z}} = 19.5\sqrt{\frac{2000\ \text{lb/ft}^2}{2185\ \text{lb/ft}^2}} = \mathbf{18.7} \qquad \Leftarrow \textit{Answer}$$

Per Figure 4.11: $\phi' = 40°$ ⇐ ***Answer***

D_{50} was not given. However, the soil is classified as a fine sand, and Table 3.4 indicates such soils have grain sizes of 0.075 to 0.425 mm. Therefore, use $D_{50} = 0.2$ mm.

$$C_P = 60 + 25 \log D_{50} = 60 + 25 \log(0.2) = 42.5$$

t was not given, so use $t = 1000$ years.

$$C_A = 1.2 + 0.05 \log\left(\frac{t}{100}\right) = 1.2 + 0.05 \log\left(\frac{1000}{100}\right) = 1.25$$

OCR was not given, so use OCR = 2.5.

$$C_{OCR} = \text{OCR}^{0.18} = 2.5^{0.18} = 1.18$$

$$D_r = \sqrt{\frac{(N_1)_{60}}{C_P C_A C_{OCR}}} \times 100\% = \sqrt{\frac{18.7}{(42.5)(1.25)(1.18)}} \times 100\% = 55\% \qquad \Leftarrow \textit{Answer}$$

Per Table 3.3, the soil is **Medium Dense** ⇐ ***Answer***

Cone Penetration Test (CPT)

The *cone penetration test* [ASTM D3441], or CPT, is another common in-situ test (Schmertmann, 1978; De Ruiter, 1981; Meigh, 1987; Robertson and Campanella, 1989; Briaud and Miran, 1991). Most of the early development of this test occurred in western Europe in the 1930s and again in the 1950s. Although many different styles and configurations have been used, the current standard grew out of work performed in the Netherlands, so it is sometimes called the *Dutch Cone*. The CPT has been used extensively in Europe for many years and is becoming increasingly popular in North America and elsewhere.

Two types of cones are commonly used: the *mechanical cone* and the *electric cone*, as shown in Figure 4.12. Both have two parts, a 35.7-mm diameter cone-shaped tip with a 60° apex angle and a 35.7 mm diameter × 133.7 mm long cylindrical sleeve. A hydraulic ram pushes this assembly into the ground and instruments measure the resistance to penetration. The *cone resistance*, q_c, is the total force acting on the cone divided by the projected area of the cone (10 cm^2); and the *cone side friction*, f_{sc}, is the total frictional force acting on the friction sleeve divided by its surface area (150 cm^2). It is common to express the side friction in terms of the *friction ratio*, R_f, which is equal to $f_{sc}/q_c \times 100\%$.

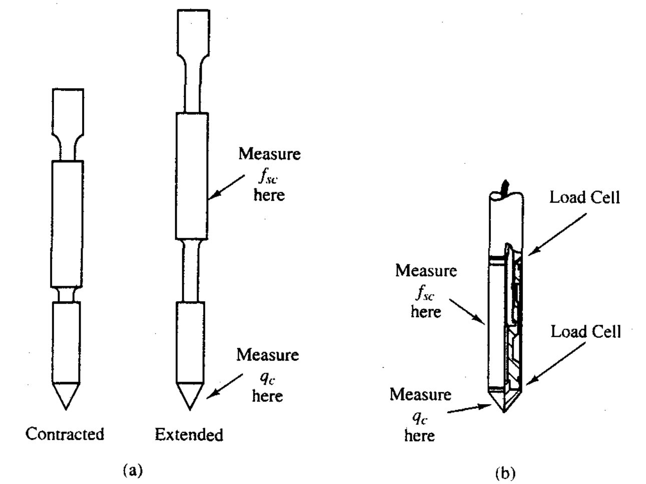


Figure 4.12 Types of cones: (a) A mechanical cone (also known as a Begemann Cone); and (b) An electric cone (also known as a Fugro Cone).

The operation of the two types of cones differs in that the mechanical cone is advanced in stages and measures q_c and f_{sc} at intervals of about 20 cm, whereas the electric cone includes built-in strain gauges and is able to measure q_c and f_{sc} continuously with depth. In either case, the CPT defines the soil profile with much greater resolution than does the SPT.

CPT rigs are often mounted in large three-axle trucks such as the one in Figure 4.13. These are typically capable of producing maximum thrusts of 10 to 20 tons (100–200 kN). Smaller, trailer-mounted or truck-mounted rigs also are available.

The CPT has been the object of extensive research and development (Robertson and Campanella, 1983) and thus is becoming increasingly useful to the practicing engineer. Some of this research effort is now being conducted using cones equipped with pore pressure transducers in order to measure the excess pore water pressures that develop while conducting the test. These are known as *piezocones*, and the enhanced procedure is known as a CPTU test. These devices promise to be especially useful in saturated clays.

A typical plot of CPT results is shown in Figure 4.14.

Although the CPT has many advantages over the SPT, there are at least three important disadvantages:

- No soil sample is recovered, so there is no opportunity to inspect the soils.
- The test is unreliable or unusable in soils with significant gravel content.
- Although the cost per foot of penetration is less than that for borings, it is necessary to mobilize a special rig to perform the CPT.

Figure 4.13 A truck-mounted CPT rig. A hydraulic ram, located inside the truck, pushes the cone into the ground using the weight of the truck as a reaction.

Uses of CPT Data

The CPT is an especially useful and inexpensive way to evaluate soil profiles. Since it retrieves data continuously with depth (with electric cones) or at very close intervals (with mechanical cones), the CPT is able to detect fine changes in the stratigraphy. Therefore, engineers often use the CPT in the first phase of subsurface investigation, saving boring and sampling for the second phase.

The CPT also can be used to assess the engineering properties of the soil through the use of empirical correlations. Those correlations intended for use in cohesionless soils are generally more accurate and most commonly used. They are often less accurate in cohesive soils because of the presence of excess pore water pressures and other factors. However, the piezocone may overcome this problem.

Correlations are also available to directly relate CPT results with foundation behavior. These are especially useful in the design of deep foundations, as discussed in Chapter 14.

Correlation with Soil Classification

Because the CPT does not recover any soil samples, it is not a substitute for conventional exploratory borings. However, it is possible to obtain an approximate soil classification using the correlation shown in Figure 4.15.

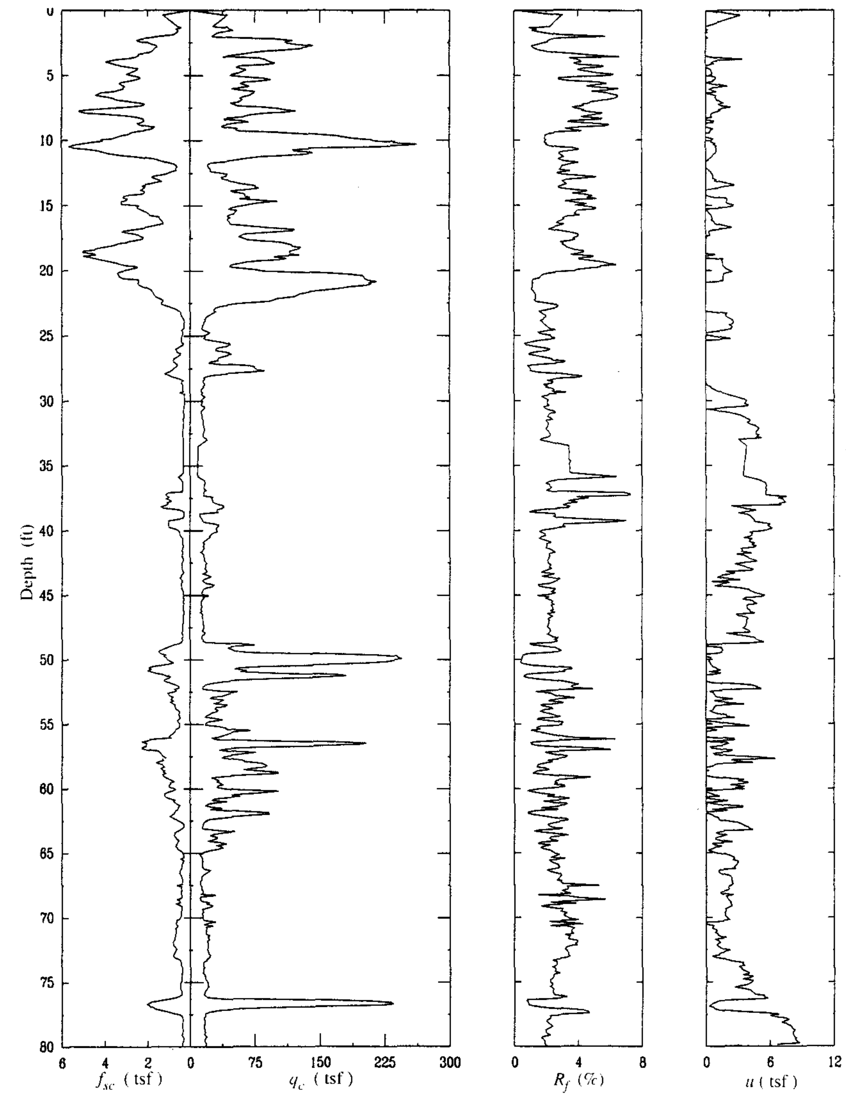


Figure 4.14 Sample CPT test results. These results were obtained from a piezocone, and thus also include a plot of pore water pressure, u, vs. depth. All stresses and pressures are expressed in tons per square foot (tsf). For practical purposes, 1 tsf = 1 kg/cm^2 (Alta Geo Cone Penetrometer Testing Services, Sandy, Utah).

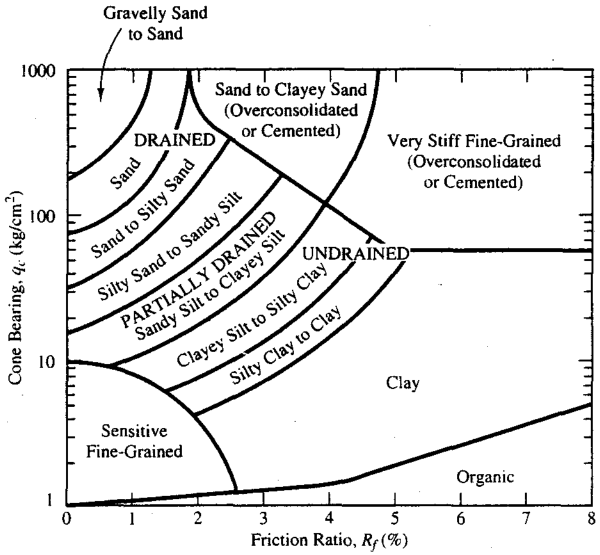


Figure 4.15 Classification of soil based on CPT test results (Adapted from Robertson and Campanella, 1983).

Correlation with Relative Density

The following approximate relationship between CPT results and the relative density of sands (Adapted from Kulhawy and Mayne, 1990):

$$D_r = \sqrt{\left(\frac{q_c}{315\,Q_c\,\mathrm{OCR}^{0.18}}\right)}\sqrt{\frac{2000\ \mathrm{lb/ft^2}}{\sigma'_z}} \times 100\% \qquad (4.7\ \text{English})$$

$$D_r = \sqrt{\left(\frac{q_c}{315\,Q_c\,\mathrm{OCR}^{0.18}}\right)}\sqrt{\frac{100\ \mathrm{kPa}}{\sigma'_z}} \times 100\% \qquad (4.7\ \text{SI})$$

Where:

q_c = cone resistance (ton/ft² or kg/cm²)

Q_c = compressibility factor

= 0.91 for highly compressible sands

= 1.00 for moderately compressible sands

= 1.09 for slightly compressible sands

For purposes of solving this formula, a sand with a high fines content or a high mica content is "highly compressible," whereas a pure quartz sand is "slightly compressible."

OCR = overconsolidation ratio

σ'_z = vertical effective stress (lb/ft² or kPa)

Correlation with Shear Strength

The CPT results also have been correlated with shear strength parameters, especially in sands. Figure 4.16 presents Robertson and Campanella's 1983 correlation for uncemented, normally consolidated quartz sands. For overconsolidated sands, subtract 1° to 2° from the effective friction angle obtained from this figure.

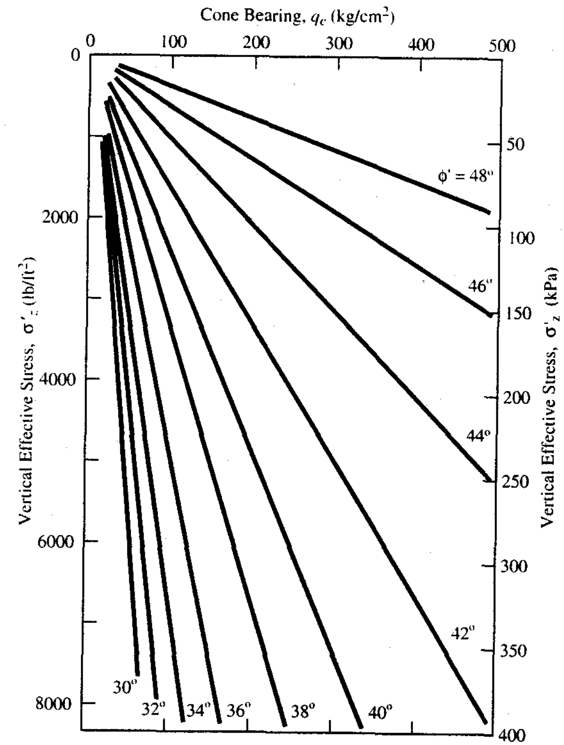


Figure 4.16 Relationship between CPT results, overburden stress and effective friction angle for uncemented, normally consolidated quartz sands (Adapted from Robertson and Campanella, 1983).

Correlation with SPT N-Value

Since the SPT and CPT are the two most common in-situ tests, it often is useful to convert results from one to the other. The ratio q_c/N_{60} as a function of the mean grain size, D_{50}, is shown in Figure 4.17. Note that N_{60} does not include an overburden correction.

Be cautious about converting CPT data to equivalent N values, and then using SPT-based analysis methods. This technique compounds the uncertainties because it uses two correlations—one to convert to N, and then another to compute the desired quantity.

Example 4.2

Using the CPT log in Figure 4.14, classify the soil stratum located between depths of 28 and 48 ft, and compute the equivalent SPT N-value.

Solution

This stratum has q_c = 15 tsf and R_f = 2.0%. According to Figure 4.15, this soil is probably a **clayey silt.** ⇐ ***Answer***

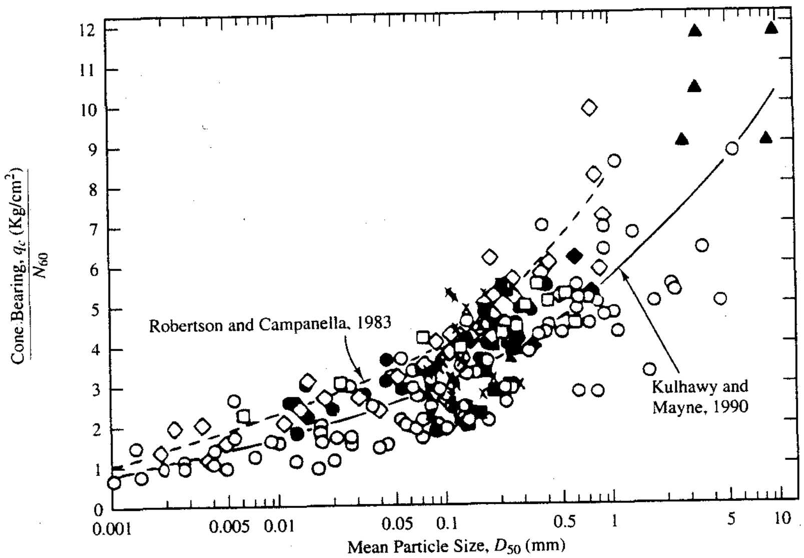


Figure 4.17 Correlation between q_c/N_{60} and the mean grain size, D_{50}. (Adapted from Kulhawy and Mayne, 1990.) Copyright © 1990 Electric Power Research Institute, reprinted with permission.

No grain-size data is available. However, based on the soil classification and Table 3.4, the mean grain size, D_{50}, is probably about 0.02 mm. According to Figure 4.17, q_c/N_{60} = 1. Therefore N_{60} **= 15** ⇐ ***Answer***

Commentary

Equation 4.8 and Figure 4.16 do not apply to this soil because it is not a sand.

Vane Shear Test (VST)

The Swedish engineer John Olsson developed the vane shear test (VST) in the 1920s to test the sensitive Scandinavian marine clays in-situ. The VST has grown in popularity, especially since the Second World War, and is now used around the world.

This test [ASTM D2573] consists of inserting a metal vane into the soil, as shown in Figure 4.18, and rotating it until the soil fails in shear. The undrained shear strength may be determined from the torque at failure, the vane dimensions, and other factors. The vane can be advanced to greater depths by simply pushing it deeper (especially in softer soils) or the test can be performed below the bottom of a boring and repeated as the boring is advanced. However, because the vane must be thin to minimize soil disturbance, it is only strong enough to be used in soft to medium cohesive soils. The test is performed rapidly (about 1 minute to failure) and therefore measures only the undrained strength.

The shear surface has a cylindrical shape, and the data analysis neglects any shear resistance along the top and bottom of this cylinder. Usually the vane height-to-diameter ratio is 2, which, when combined with the applied torque, produces the following theoretical formula:

$$s_u = \frac{6T_f}{7\pi d^3} \tag{4.8}$$

Where:

s_u = undrained shear strength

T_f = torque at failure

d = diameter of vane

However, several researchers have analyzed failures of embankments, footings, and excavations using vane shear tests (knowing that the factor of safety was 1.0) and found that Equation 4.9 often overestimates s_u. Therefore, an empirical correction factor, λ, as shown in Figure 4.19, is applied to the test results:

$$\boxed{s_u = \frac{6\lambda T_f}{7\pi d^3}} \tag{4.9}$$

An additional correction factor of 0.85 should be applied to test results from organic soils other than peat (Terzaghi, Peck, and Mesri, 1996).

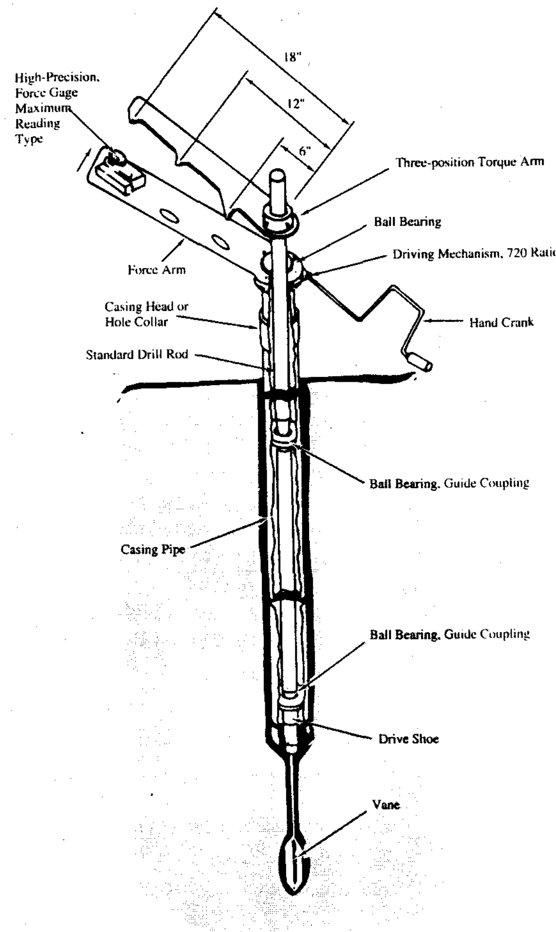


Figure 4.18 The vane shear test (U.S. Navy, 1982a).

Pressuremeter Test (PMT)

In 1954, a young French engineering student named Louis Ménard began to develop a new type of in-situ test: the pressuremeter test. Although Kögler had done some limited work on a similar test some twenty years earlier, it was Ménard who made it a practical reality.

The pressuremeter is a cylindrical balloon that is inserted into the ground and inflated, as shown in Figures 4.20 and 4.21. Measurements of volume and pressure can be

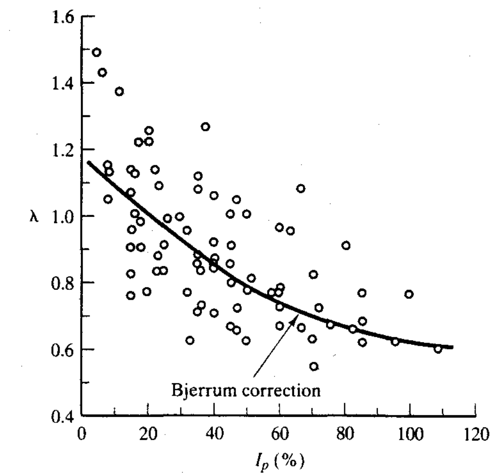


Figure 4.19 Vane shear correction factor, λ (from *Soil Mechanics in Engineering Practice*, 3rd ed. By Terzaghi, Peck, and Mesri; copyright © 1996; used by permission of John Wiley and Sons).

used to evaluate the in-situ stress, compressibility, and strength of the adjacent soil and thus the behavior of a foundation (Baguelin et al., 1978; Briaud, 1992).

The PMT may be performed in a carefully drilled boring or the test equipment can be combined with a small auger to create a self-boring pressuremeter. The latter design provides less soil disturbance and more intimate contact between the pressuremeter and the soil.

The PMT produces much more direct measurements of soil compressibility and lateral stresses than do the SPT and CPT. Thus, in theory, it should form a better basis for settlement analyses, and possibly for pile capacity analyses. However, the PMT is a difficult test to perform and is limited by the availability of the equipment and personnel trained to use it.

Although the PMT is widely used in France and Germany, it is used only occasionally in other parts of the world. However, it may become more popular in the future.

Dilatometer Test (DMT)

The dilatometer (Marchetti, 1980; Schmertmann, 1986b, 1988a, and 1988b), which is one of the newest in-situ test devices, was developed during the late 1970s in Italy by Silvano Marchetti. It is also known as a *flat dilatometer* or a *Marchetti dilatometer* and consists of a 95-mm wide, 15-mm thick metal blade with a thin, flat, circular, expandable steel membrane on one side, as shown in Figure 4.22.

The dilatometer test (DMT) is conducted as follows (Schmertmann, 1986a):

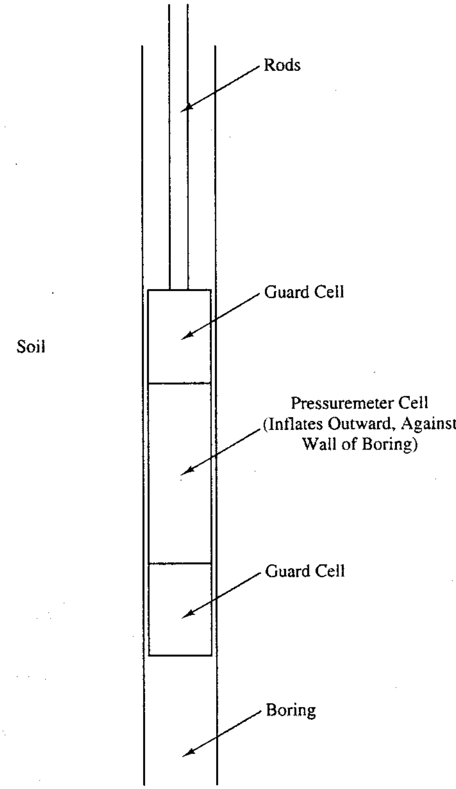


Figure 4.20 Schematic of the pressuremeter test.

1. Press the dilatometer into the soil to the desired depth using a CPT rig or some other suitable device.
2. Apply nitrogen gas pressure to the membrane to press it outward. Record the pressure required to move the center of the membrane 0.05 mm into the soil (the *A* pressure) and that required to move its center 1.10 mm into the soil (the *B* pressure).
3. Depressurize the membrane and record the pressure acting on the membrane when it returns to its original position. This is the *C* pressure and is a measure of the pore water pressure in the soil.
4. Advance the dilatometer 150 to 300 mm deeper into the ground and repeat the test. Continue until reaching the desired depth.

Each of these test sequences typically requires 1 to 2 minutes to complete, so a typical *sounding* (a complete series of DMT tests between the ground surface and the desired

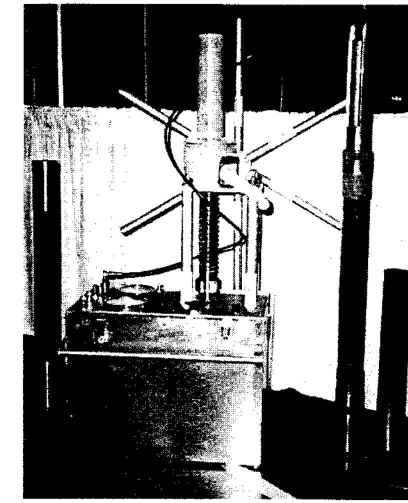


Figure 4.21 A complete pressuremeter set, including three different cell assemblies and the control unit.

depth) may require about 2 hours. In contrast, a comparable CPT sounding might be completed in about 30 minutes.

The primary benefit of the DMT is that it measures the lateral stress condition and compressibility of the soil. These are determined from the *A*, *B*, and *C* pressures and certain equipment calibration factors and expressed as the *DMT indices:*

I_D = material index (a normalized modulus)
K_D = horizontal stress index (a normalized lateral stress)
E_D = dilatometer modulus (theoretical elastic modulus)

Researchers have developed correlations between these indices and certain engineering properties of the soil (Schmertmann, 1988b; Kulhawy and Mayne, 1990), including:

- Classification
- Coefficient of lateral earth pressure, K_0
- Overconsolidation ratio, OCR
- Modulus of elasticity, E, or constrained modulus, M

The CPT and DMT are complementary tests (Schmertmann, 1988b). The cone is a good way to evaluate soil strength, whereas the dilatometer assesses compressibility and in-situ stresses. These three kinds of information form the basis for most foundation engineering analyses. In addition, the dilatometer blade is most easily pressed into the ground

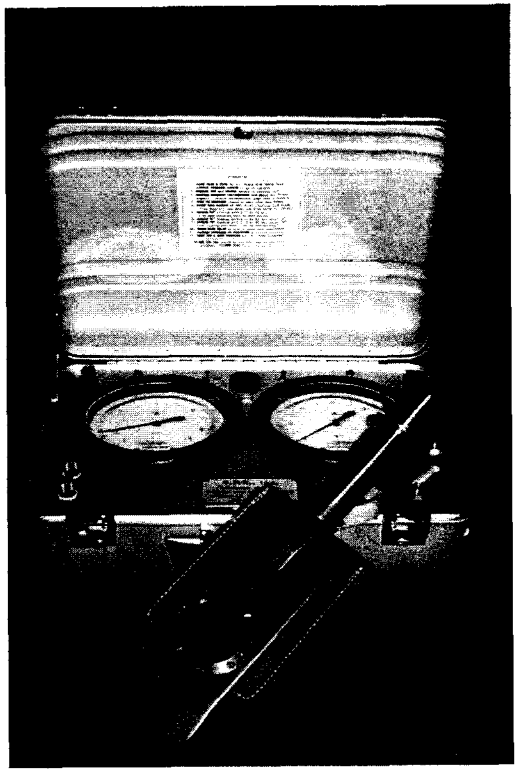


Figure 4.22 The Marchetti dilatometer along with its control unit and nitrogen gas bottle (GPE, Inc., Gainesville, FL).

using a conventional CPT rig, so it is a simple matter to conduct both CPT and DMT tests while mobilizing only a minimum of equipment.

The dilatometer test is a relative newcomer, and thus has not yet become a common engineering tool. Engineers have had only limited experience with it and the analysis and design methods based on DMT results are not yet well developed. However, its relatively low cost, versatility, and compatibility with the CPT suggest that it may enjoy widespread use in the future. It has very good repeatability, and can be used in soft or moderately stiff soils (i.e., those with $N \leq 40$), and provides more direct measurements of stress-strain properties.

Becker Penetration Test

Soils that contain a large percentage of gravel and those that contain cobbles or boulders create problems for most in-situ test methods. Often, the in-situ test device is not able to

penetrate through such soils (it meets *refusal*) or the results are not representative because the particles are about the same size as the test device. Frequently, even conventional drilling equipment cannot penetrate through these soils.

One method of penetrating through these very large-grained soils is to use a *Becker hammer drill*. This device, developed in Canada, uses a small diesel pile-driving hammer and percussion action to drive a 135 to 230 mm (5.5–9.0 inch) diameter double-wall steel casing into the ground. The cuttings are sent to the top by blowing air through the casing. This technique has been used successfully on very dense and coarse soils.

The Becker hammer drill also can be used to assess the penetration resistance of these soils using the *Becker penetration test*, which is monitoring the hammer blowcount. The number of blows required to advance the casing 300 mm (1 ft) is the Becker blowcount, N_B. Several correlations are available to convert it to an equivalent SPT N value (Harder and Seed, 1986). One of these correlation methods also considers the bounce chamber pressure in the diesel hammer.

Comparison of In-Situ Test Methods

Each of the in-situ test methods has its strengths and weaknesses. Table 4.5 compares some of the important attributes of the tests described in this chapter.

TABLE 4.5 ASSESSMENT OF IN-SITU TEST METHODS (Adapted from Mitchell, 1978a; used with permission of ASCE)

	Standard Penetration Test	Cone Penetration Test	Pressure-meter Test	Dilatometer Test	Becker Penetration Test
Simplicity and Durability of Apparatus	Simple; rugged	Complex; rugged	Complex; delicate	Complex; moderately rugged	Simple rugged
Ease of Testing	Easy	Easy	Complex	Easy	Easy
Continuous Profile or Point Values	Point	Continuous	Point	Point	Continuous
Basis for Interpretation	Empirical	Empirical; theory	Empirical; theory	Empirical; theory	Empirical
Suitable Soils	All except gravels	All except gravels	All	All except gravels	Sands through boulders
Equipment Availability and Use in Practice	Universally available: used routinely	Generally available; used routinely	Difficult to locate; used on special projects	Difficult to locate; used on special projects	Difficult to locate; used on special projects
Potential for Future Development	Limited	Great	Great	Great	Uncertain

QUESTIONS AND PRACTICE PROBLEMS

4.5 Discuss the advantages of the cone penetration test over the standard penetration test.

4.6 A standard penetration test was performed in a 150-mm diameter boring at a depth of 9.5 m below the ground surface. The driller used a UK-style automatic trip hammer and a standard SPT sampler. The actual blow count, N, was 19. The soil is a normally consolidated fine sand with a unit weight of 18.0 kN/m^3 and D_{50} = 0.4 mm. The groundwater table is at a depth of 15 m. Compute the following:

a. N_{60}

b. $(N_1)_{60}$

c. D_r

d. Consistency (based on Table 3.3)

e. ϕ'

4.7 Using the cone penetration test data in Figure 4.14, a unit weight of 115 lb/ft^3, and an overconsolidation ratio of 3, compute the following for the soil between depths of 21 and 23 ft. Use a groundwater depth of 15 ft below the ground surface.

a. Soil classification

b. D_r (assume the soil has some fines, but no mica)

c. Consistency (based on Table 3.3)

d. ϕ'

e. N_{60} (use an estimated D_{50} of 0.60 mm)

4.4 SYNTHESIS OF FIELD AND LABORATORY DATA

Investigation and testing programs often generate large amounts of information that can be difficult to sort through and synthesize. Real soil profiles are nearly always very complex, so the borings will not correlate and the test results will often vary significantly. Therefore, we must develop a simplified soil profile before proceeding with the analysis. In many cases, this simplified profile is best defined in terms of a one-dimensional function of soil type and engineering properties vs. depth; an idealized boring log. However, when the soil profile varies significantly across the site, one or more vertical cross-sections may be in order.

The development of these simplified profiles requires a great deal of engineering judgment along with interpolation and extrapolation of the data. It is important to have a feel for the approximate magnitude of the many uncertainties in this process and reflect them in an appropriate degree of conservatism. This judgment comes primarily with experience combined with a thorough understanding of the field and laboratory methodologies.

4.5 ECONOMICS

The site investigation and soil testing phase of foundation engineering is the single largest source of uncertainties. No matter how extensive it is, there is always some doubt whether the borings accurately portray the subsurface conditions, whether the samples are representative, and whether the tests are correctly measuring the soil properties. Engineers attempt to compensate for these uncertainties by applying factors of safety in our analyses. Unfortunately, this solution also increases construction costs.

In an effort to reduce the necessary level of conservatism in the foundation design, the engineer may choose a more extensive investigation and testing program to better define the soils. The additional costs of such efforts will, to a point, result in decreased construction costs, as shown in Figure 4.23. However, at some point, this becomes a matter of diminishing returns, and eventually the incremental cost of additional investigation and testing does not produce an equal or larger reduction in construction costs. The minimum on this curve represents the optimal level of effort.

An example of this type of economic consideration is a decision regarding the value of conducting full-scale pile load tests (see Chapter 13). On large projects, the potential savings in construction costs will often justify one or more tests, whereas on small projects, a conservative design developed without the benefit of load tests may result in a lower total cost.

We also must decide whether to conduct a large number of moderately precise tests (such as the SPT) or a smaller number of more precise but expensive tests (such as the PMT). Handy (1980) suggested the most cost-effective test is the one with a variability

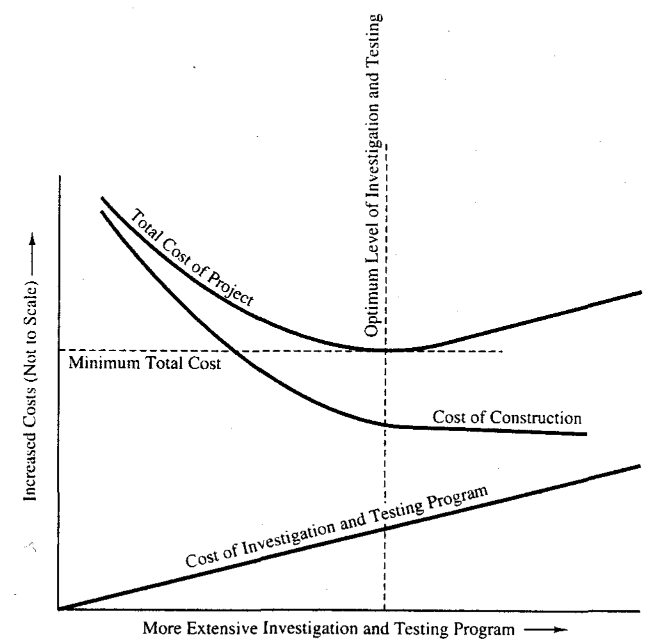


Figure 4.23 Cost effectiveness of more extensive investigation and testing programs.

consistent with the variability of the soil profile. Thus, a few precise tests might be appropriate in a uniform soil deposit, but more data points, even if they are less precise, are more valuable in an erratic deposit.

SUMMARY

Major Points

1. Soil and rock are natural materials. Therefore, their engineering properties vary from site to site and must be determined individually for each project. This process is known as site exploration and characterization.
2. The first step in a site exploration and characterization program typically consists of conducting a background literature search and a field reconnaissance.
3. Site exploration efforts usually include drilling exploratory borings and/or digging exploratory trenches, as well as obtaining soil and rock samples from these borings or trenches.
4. We bring these soil and rock samples to a laboratory to conduct standard laboratory tests. These tests help classify the soil, determine its strength, and assess its compressibility. Other tests might also be conducted.
5. In-situ tests are those conducted in the ground. These techniques are especially useful in soils that are difficult to sample, such as clean sands. In-situ methods include the standard penetration test (SPT), the cone penetration test (CPT), the vane shear test (VST), the pressuremeter test (PMT), the dilatometer test (DMT), and the Becker penetration test.
6. An investigation and testing program will typically generate large amounts of data, even though only a small portion of the soil is actually tested. The engineer must synthesize this data into a simplified form to be used in analyses and design.
7. Investigation and testing always involves uncertainties and risks. These can be reduced, but not eliminated, by drilling more borings, retrieving more samples, and conducting more tests. However, there are economic limits to such endeavors, so we must determine what amount of work is most cost effective.

Vocabulary

Becker penetration test	Cone penetration test	Field reconnaissance
Blow count	Coring	Flight auger
Boring log	Dilatometer test	Hammer
Bucket auger	Disturbed sample	Hollow-stem auger
Casing	Exploratory trench	In-situ test
Caving	Exploratory boring	Observation well

Overburden correction	Shelby tube sampler	Standard penetration test
Pressuremeter test	Site investigation	Undistrubed sample
Rotary wash boring	Squeezing	Vane shear test

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

4.8 Classify the soil stratum between depths of 66 and 80 ft in Figure 4.14. What is the significance of the spike in the plots at a depth of 77 ft?

4.9 The following standard penetration tests results were obtained in a uniform silty sand:

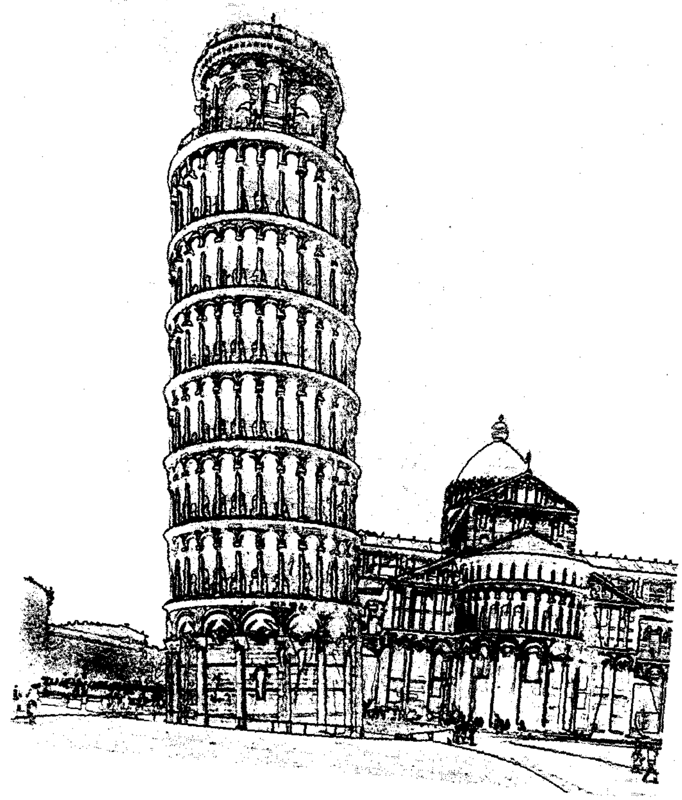
Depth (m)	N_{60}
1	12
2	13
3	18
5	15

The groundwater table is at a depth of 2.5 m. Assume a reasonable value for γ, then determine ϕ' for each test. Finally, determine a single design ϕ' value for this stratum.

4.10 A series of vane shear tests have been performed on a soft clay stratum. The results of these tests are as follows:

Depth (m)	T_f (N-m)
5.0	9.0
5.5	10.7
7.5	12.0
9.0	14.7

The vane was 60 mm in diameter and 120 mm long. The soil has a liquid limit of 100 and a plastic limit of 30. Compute the undrained shear strength for each test, then develop a plot of undrained shear strength vs. depth. This plot should have depth on the vertical axis, with zero at the top of the plot.



Part B

Shallow Foundation Analysis and Design

5

Shallow Foundations

The most important thing is to keep the most important thing the most important thing.

Shallow foundations are those that transmit structural loads to the near-surface soils. These include *spread footing foundations* and *mat foundations*. This chapter introduces both types of foundations, then Chapters 6 to 10 discuss the various geotechnical and structural design aspects.

5.1 SPREAD FOOTINGS

A spread footing (also known as a *footer* or simply a *footing*) is an enlargement at the bottom of a column or bearing wall that spreads the applied structural loads over a sufficiently large soil area. Typically, each column and each bearing wall has its own spread footing, so each structure may include dozens of individual footings.

Spread footings are by far the most common type of foundation, primarily because of their low cost and ease of construction. They are most often used in small to medium-size structures on sites with moderate to good soil conditions, and can even be used on some large structures when they are located at sites underlain by exceptionally good soil or shallow bedrock.

Spread footings may be built in different shapes and sizes to accommodate individual needs, as shown in Figure 5.1. These include the following:

- *Square spread footings* (or simply *square footings*) have plan dimensions of $B \times B$. The depth from the ground surface to the bottom of the footing is D and the thickness is T. Square footings usually support a single centrally-located column.

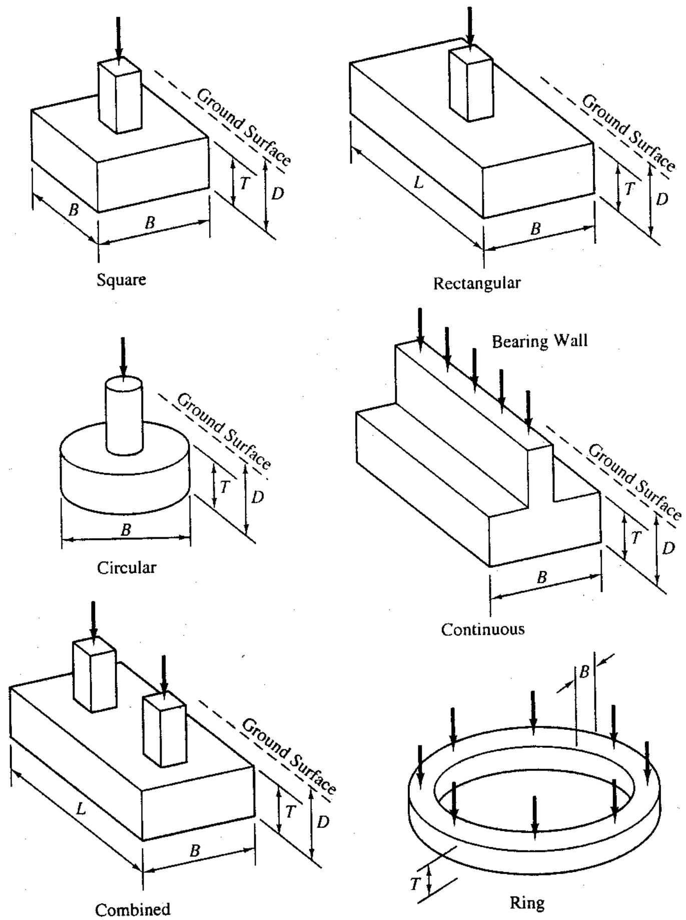


Figure 5.1 Spread footing shapes and dimensions.

- *Rectangular spread footings* have plan dimensions of $B \times L$, where L is the longest dimension. These are useful when obstructions prevent construction of a square footing with a sufficiently large base area and when large moment loads are present.
- *Circular spread footings* are round in plan view. These are most frequently used as foundations for light standards, flagpoles, and power transmission lines. If these foundations extend to a large depth (i.e., D/B greater than about 3), they may behave more like a deep foundation (see Chapter 11).

- *Continuous spread footings* (also known as *wall footings* or *strip footings*) are used to support bearing walls.
- *Combined footings* are those that support more than one column. These are useful when columns are located too close together for each to have its own footing.
- *Ring spread footings* are continuous footings that have been wrapped into a circle. This type of footing is commonly used to support the walls of above-ground circular storage tanks. However, the contents of these tanks are spread evenly across the total base area, and this weight is probably greater than that of the tank itself. Therefore, the geotechnical analyses of tanks usually treat them as circular foundations with diameters equal to the diameter of the tank.

Sometimes it is necessary to build spread footings very close to a property line, another structure, or some other place where no construction may occur beyond one or more of the exterior walls. This circumstance is shown in Figure 5.2. Because such a footing cannot be centered beneath the column, the load is eccentric. This can cause the footing to rotate and thus produce undesirable moments and displacements in the column.

One solution to this problem is to use a *strap footing* (also known as a *cantilever footing*), which consists of an eccentrically loaded footing under the exterior column connected to the first interior column using a *grade beam*. This arrangement, which is similar to a combined footing, provides the necessary moment in the exterior footing to counter the eccentric load. Sometimes we use grade beams to connect all of the spread footings in a structure to provide a more rigid foundation system.

Materials

Before the mid-nineteenth-century, almost all spread footings were made of masonry, as shown in Figure 5.3. *Dimension-stone footings* were built of stones cut and dressed to specific sizes and fit together with minimal gaps, while *rubble-stone footings* were built from random size material joined with mortar (Peck et al., 1974). These footings had very little tensile strength, so builders had to use large height-to-width ratios to keep the flexural stresses tolerably small and thus avoid tensile failures.

Although masonry footings were satisfactory for small structures, they became large and heavy when used in heavier structures, often encroaching into the basement. For example, the masonry footings beneath the nine-story Home Insurance Building in Chicago (built in 1885) had a combined weight equal to that of one of the stories (Peck, 1948). As larger structures became more common, it was necessary to develop footings that were shorter and lighter, yet still had the same base dimensions. This change required structural materials that could sustain flexural stresses.

The *steel grillage footings* used in the ten-story Montauk Block Building in Chicago in 1882, may have been the first spread footings designed to resist flexure. They included several layers of railroad tracks, as shown in Figure 5.4. The flexural strength of the steel permitted construction of a short and lightweight footing. Steel grillage footings, modified to use I-beams instead of railroad tracks, soon became the dominant design. They prevailed until the advent of reinforced concrete in the early twentieth century.

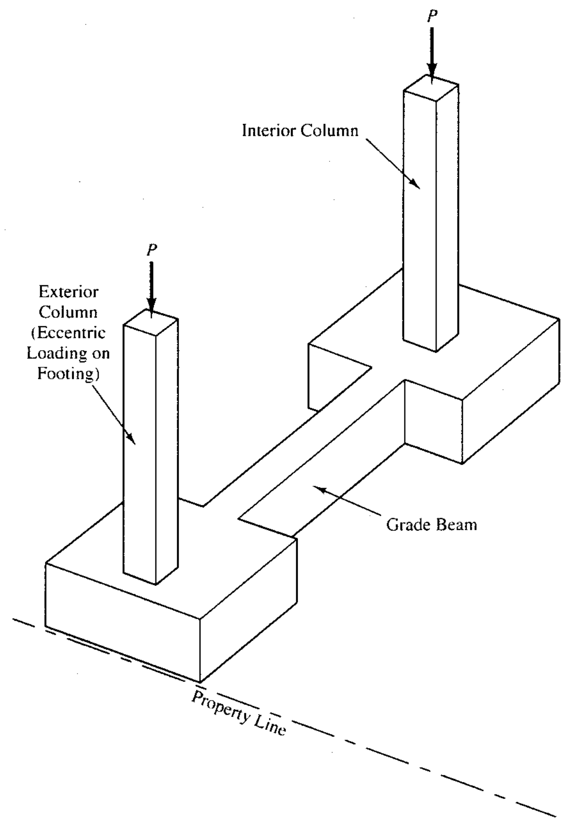


Figure 5.2 Use of a strap footing to support exterior columns when construction cannot extend beyond the property line.

Figure 5.5 shows a typical reinforced concrete footing. These are very strong, economical, durable, and easy to build. Reinforced concrete footings are much thinner than the old masonry footings, so they do not require large excavations and do not intrude into basements. Thus, nearly all spread footings are now made of reinforced concrete.

Construction Methods

Contractors usually use a *backhoe* to excavate spread footings, as shown in Figure 5.6. Typically, some hand work is also necessary to produce a clean excavation. Once the excavation is open, it is important to check the exposed soils to verify that they are comparable to those used in the design. Inspectors often check the firmness of these soils using a

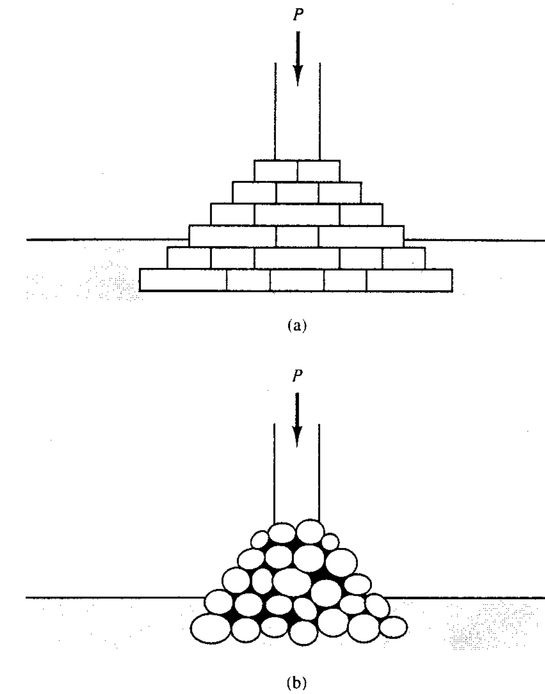


Figure 5.3 (a) Dimension-stone footing, and (b) Rubble-stone footing.

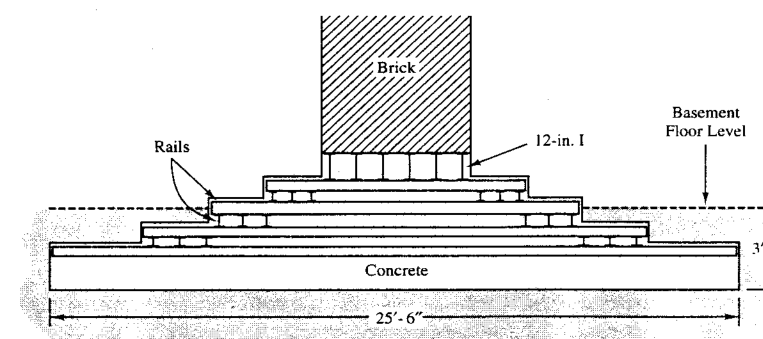


Figure 5.4 Steel grillage footing made from railroad tracks, Montauk Block Building, Chicago, 1882. The concrete that surrounded the steel was for corrosion protection only (Peck, 1948).

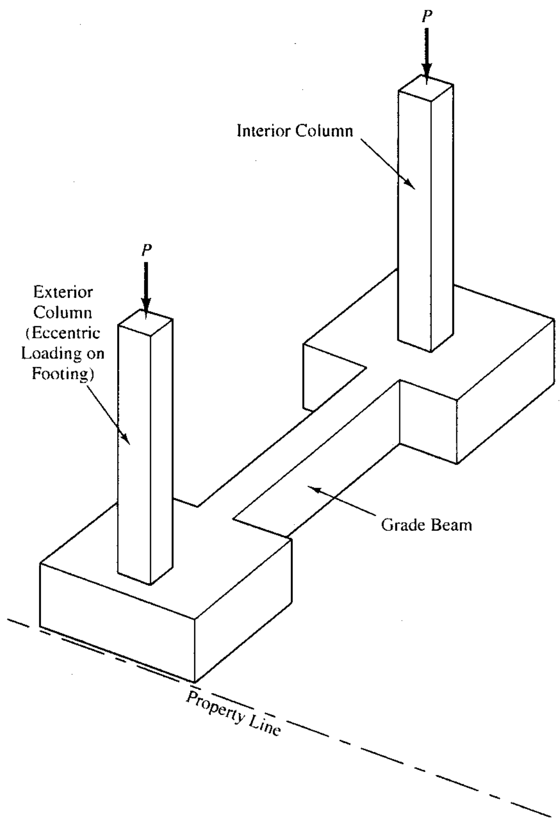


Figure 5.2 Use of a strap footing to support exterior columns when construction cannot extend beyond the property line.

Figure 5.5 shows a typical reinforced concrete footing. These are very strong, economical, durable, and easy to build. Reinforced concrete footings are much thinner than the old masonry footings, so they do not require large excavations and do not intrude into basements. Thus, nearly all spread footings are now made of reinforced concrete.

Construction Methods

Contractors usually use a *backhoe* to excavate spread footings, as shown in Figure 5.6. Typically, some hand work is also necessary to produce a clean excavation. Once the excavation is open, it is important to check the exposed soils to verify that they are comparable to those used in the design. Inspectors often check the firmness of these soils using a

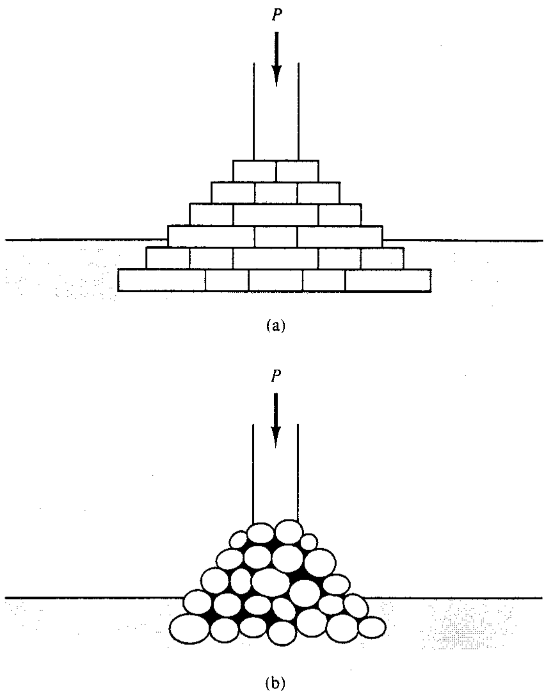


Figure 5.3 (a) Dimension-stone footing, and (b) Rubble-stone footing.

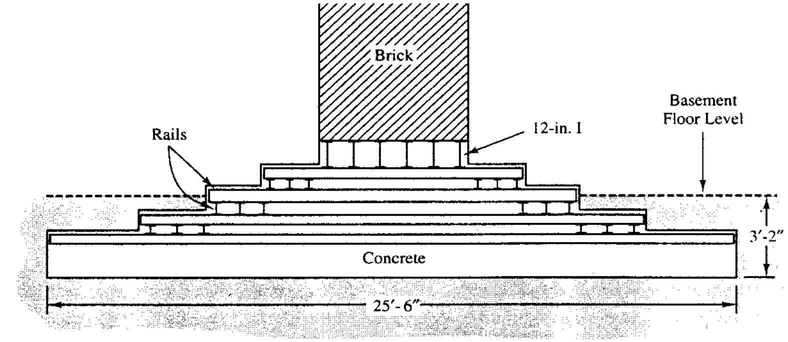


Figure 5.4 Steel grillage footing made from railroad tracks, Montauk Block Building, Chicago, 1882. The concrete that surrounded the steel was for corrosion protection only (Peck, 1948).

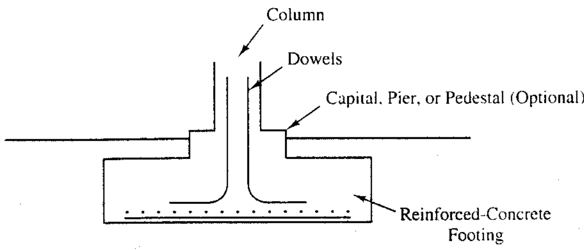


Figure 5.5 Reinforced concrete spread footing.

9-mm (3/8 in) diameter steel probe. If the soil conditions are not as anticipated, especially if they are too soft, it may be necessary to revise the design accordingly.

Most soils have sufficient strength to stand vertically until it is time to pour the concrete. This procedure of pouring the concrete directly against the soil is known as pouring a *neat footing*, as shown in Figure 5.7a. Sometimes shallow wooden forms are placed above the excavation, as shown in Figure 5.7b, so the top of the footing is at the proper elevation. If the soil will not stand vertically, such as with clean sands or gravels, it is necessary to make a larger excavation and build a full-depth wooden form, as shown in Figure 5.7c. This is known as a *formed footing*.

Once the excavation has been made and cleaned, and the forms (if needed) are in place, the contractor places the reinforcing steel. If the footing will support a wood or steel structure, threaded *anchor bolts* and/or steel brackets are embedded into the con-

Figure 5.6 A backhoe making an excavation for a spread footing.

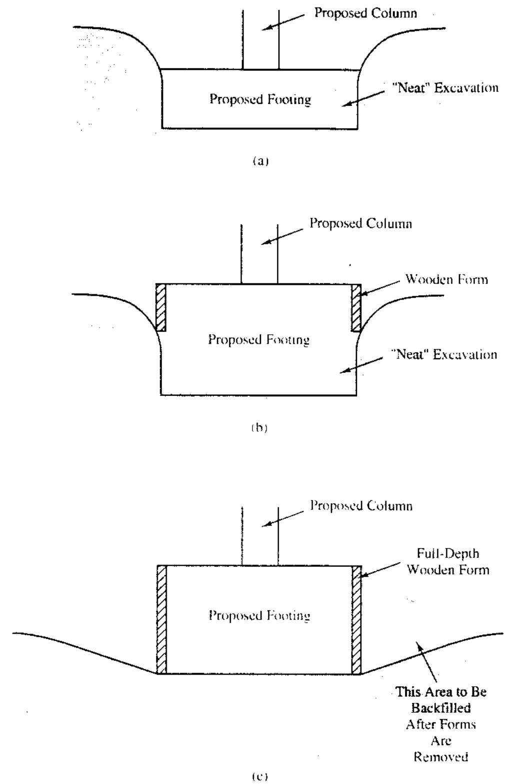


Figure 5.7 Methods of placing concrete in footings: (a) Neat excavation, (b) Neat excavation with wooden forms at the top, (c) Formed footing with full-depth wooden forms.

crete. For concrete or masonry structures, short steel rebars, called *dowels* are placed such that they extend above the completed footing, thus providing for a lap splice with the column or wall steel. Chapter 9 discusses these connections in more detail. Finally, the concrete is placed and, once it has cured, the forms are removed. Figure 5.8 shows a completed spread footing.

5.2 MATS

The second type of shallow foundation is a *mat foundation*, as shown in Figure 5.9. A mat is essentially a very large spread footing that usually encompasses the entire footprint of the structure. They also are known as *raft foundations*. They are always made of reinforced concrete.

Figure 5.8 A completed spread footing. The four bolts extending out of the footing will be connected to the base plate of a steel column.

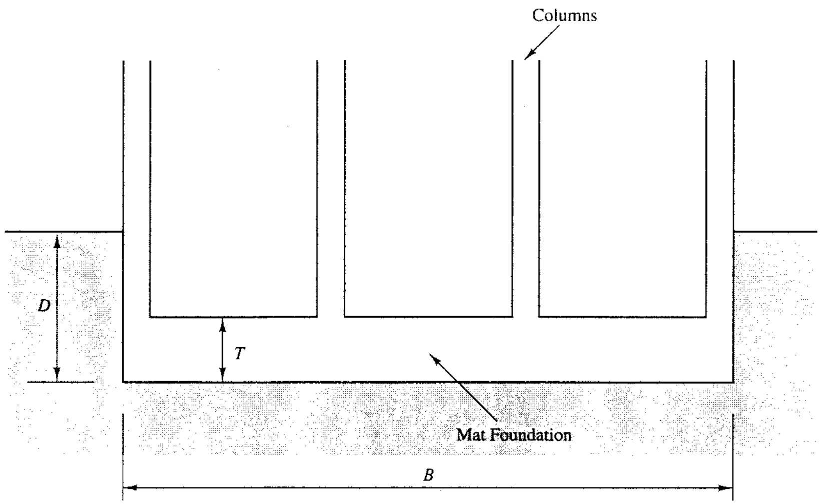


Figure 5.9 A mat foundation.

Foundation engineers often consider mats when dealing with any of the following conditions:

- The structural loads are so high or the soil conditions so poor that spread footings would be exceptionally large. As a general rule of thumb, if spread footings would cover more than 50 percent of the building footprint area, a mat or some type of deep foundation will usually be more economical.
- The soil is very erratic and prone to excessive differential settlements. The structural continuity and flexural strength of a mat will bridge over these irregularities. The same is true of mats on highly expansive soils prone to differential heaves.
- The structural loads are erratic, and thus increase the likelihood of excessive differential settlements. Again, the structural continuity and flexural strength of the mat will absorb these irregularities.
- The lateral loads are not uniformly distributed through the structure and thus may cause differential horizontal movements in spread footings or pile caps. The continuity of a mat will resist such movements.
- The uplift loads are larger than spread footings can accommodate. The greater weight and continuity of a mat may provide sufficient resistance.
- The bottom of the structure is located below the groundwater table, so waterproofing is an important concern. Because mats are monolithic, they are much easier to waterproof. The weight of the mat also helps resist hydrostatic uplift forces from the groundwater.

Many buildings are supported on mat foundations, as are silos, chimneys, and other types of tower structures. Mats are also used to support storage tanks and large machines. The seventy five story Texas Commerce Tower in Houston is one of the largest mat-supported structures in the world. Its mat is 3 m (9 ft 9 in) thick and is bottomed 19.2 m (63 ft) below the street level.

5.3 BEARING PRESSURE

The most fundamental parameter that defines the interface between a shallow foundation and the soil that supports it is the *bearing pressure*. This is the contact force per unit area along the bottom of the foundation. Engineers recognized the importance of bearing pressure during the nineteenth century, thus forming the basis for later developments in bearing capacity and settlement theories.

Distribution of Bearing Pressure

Although the integral of the bearing pressure across the base area of a shallow foundation must be equal to the force acting between the foundation and the soil, this pressure is not necessarily distributed evenly. Analytical studies and field measurements (Schultze, 1961; Dempsey and Li, 1989; and others) indicate that the actual distribution depends on several factors, including the following:

- Eccentricity, if any, of the applied load
- Magnitude of the applied moment, if any
- Structural rigidity of the foundation
- Stress-strain properties of the soil
- Roughness of the bottom of the foundation

Figure 5.10 shows the distribution of bearing pressure along the base of shallow foundations subjected to concentric vertical loads. Perfectly flexible foundations bend as necessary to maintain a uniform bearing pressure, as shown in Figures 5.10a and 5.10b, whereas perfectly rigid foundations settle uniformly but have variations in the bearing pressure, as shown in Figures 5.10c and 5.10d.

Real spread footings are close to being perfectly rigid, so the bearing pressure distribution is not uniform. However, bearing capacity and settlement analyses based on such a distribution would be very complex, so it is customary to assume that the pressure beneath concentric vertical loads is uniform across the base of the footing, as shown in Figure 5.10e. The error introduced by this simplification is not significant.

Mat foundations have a much smaller thickness-to-width ratio, and thus are more flexible than spread footings. In addition, we evaluate the flexural stresses in mats more carefully and develop more detailed reinforcing steel layouts. Therefore, we conduct more detailed analyses to determine the distribution of bearing pressure on mats. Chapter 10 discusses these analyses.

When analyzing shallow foundations, it is customary and reasonable to neglect any sliding friction along the sides of the footing and assume that the entire load is transmitted to the bottom. This is an important analytical difference between shallow and deep foundations, and will be explored in more detail in Chapter 11.

Computation of Bearing Pressure

The *bearing pressure* (or *gross bearing pressure*) along the bottom of a shallow foundation is:

$$q = \frac{P + W_f}{A} - u_D \tag{5.1}$$

Where:

q = bearing pressure

P = vertical column load

W_f = weight of foundation, including the weight of soil above the foundation, if any

A = base area of foundation (B^2 for square foundations or BL for rectangular foundations)

u_D = pore water pressure at bottom of foundation (i.e., at a depth D below the ground surface).

Virtually all shallow foundations are made of reinforced concrete, so W_f is computed using a unit weight for concrete of 150 lb/ft^3 or 23.6 kN/m^3.

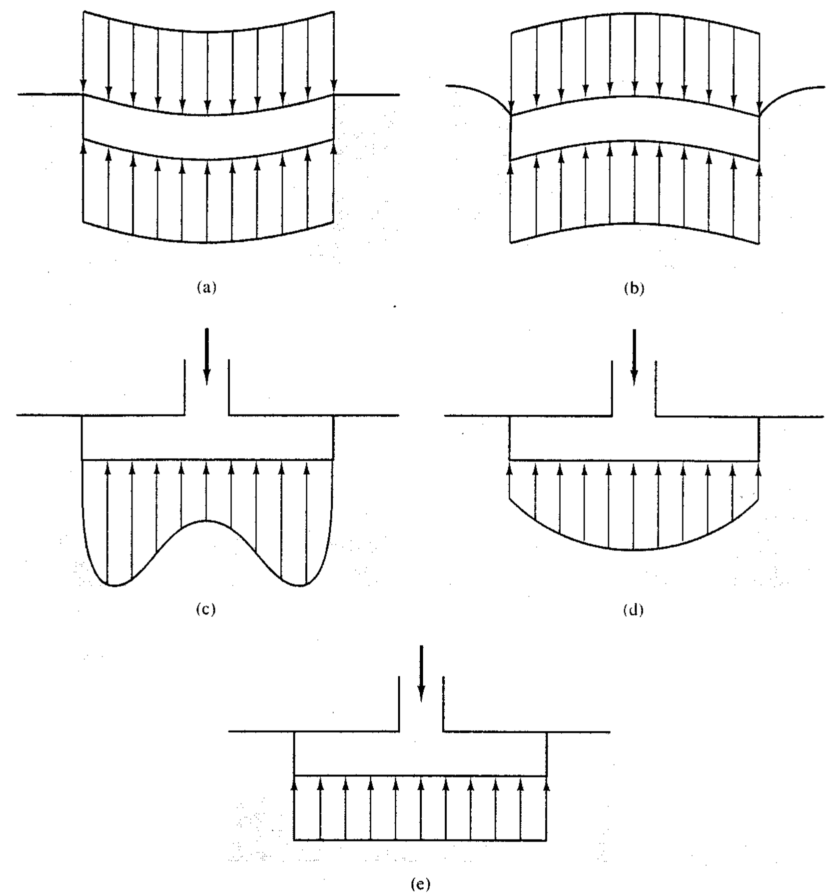


Figure 5.10 Distribution of bearing pressure along the base of shallow foundations subjected to concentric vertical loads: (a) flexible foundation on clay, (b) flexible foundation on sand, (c) rigid foundation on clay, (d) rigid foundation on sand, and (e) simplified distribution (after Taylor, 1948).

The pore water pressure term accounts for uplift pressures (buoyancy forces) that are present if a portion of the foundation is below the groundwater table. If the groundwater table is at a depth greater than D, then set $u_D = 0$.

For continuous footings, we express the applied loads as a force per unit length, such as 2000 kN/m. For ease of computation, we identify this unit length as b, which is usually 1 m or 1 ft as shown in Figure 5.11. Thus, the load is expressed using the variable P/b, and the weight using W_f/b.

The bearing pressures for continuous footings is then:

$$q = \frac{P/b + W_f/b}{B} - u_D \tag{5.2}$$

Example 5.1

The 5-ft square footing shown in Figure 5.12 supports a column load of 100 k. Compute the bearing pressure.

Solution

Use 150 lb/ft^3 for the unit weight of concrete, and compute W_f as if the concrete extends from the ground surface to a depth D (this is conservative when the footing is covered with soil because soil has a lower unit weight, but the error introduced is small).

$$W_f = (5\text{ ft})(5\text{ ft})(4\text{ ft})(150\text{ lb/ft}^3) = 15{,}000\text{ lb}$$

$$A = (5\text{ ft})(5\text{ ft}) = 25\text{ ft}^2$$

$$u_D = \gamma_w z_w = (62.4\text{ lb/ft}^3)(4\text{ ft} - 3\text{ ft}) = 62\text{ lb/ft}^2$$

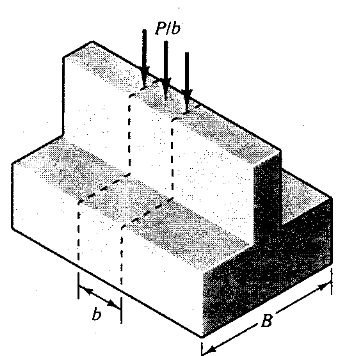


Figure 5.11 Definitions for loads on continuous footings.

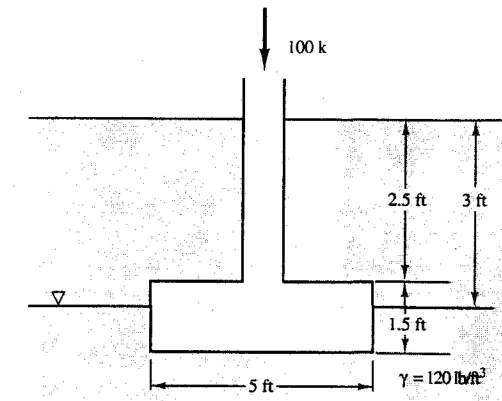


Figure 5.12 Spread footing for Example 5.1.

$$q = \frac{P + W_f}{A} - u_D$$

$$= \frac{100{,}000\text{ lb} + 15{,}000\text{ lb}}{25\text{ ft}^2} - 62\text{ lb/ft}^2$$

$$= \mathbf{4538\ lb/ft^2} \quad \Leftarrow \textit{Answer}$$

Example 5.2

A 0.70-m wide continuous footing supports a wall load of 110 kN/m. The bottom of this footing is at a depth of 0.50 m below the adjacent ground surface and the soil has a unit weight of 17.5 kN/m^3. The groundwater table is at a depth of 10 m below the ground surface. Compute the bearing pressure.

Solution

Use 23.6 kN/m^3 for the unit weight of concrete.

$$W_f/b = (0.70\text{ m})(0.50\text{ m})(23.6\text{ kN/m}^3) = 8\text{ kN/m}$$

$$u_D = 0$$

$$q = \frac{P/b + W_f/b}{B} - u_D = \frac{110\text{ kN/m} + 8\text{ kN/m}}{0.70\text{ m}} - 0 = \mathbf{169\ kPa} \quad \Leftarrow \textit{Answer}$$

Net Bearing Pressure

An alternative way to define bearing pressure is the *net bearing pressure*, q', which is the difference between the gross bearing pressure, q, and the initial vertical effective stress, σ_{z0}', at depth D. In other words, q' is a measure of the increase in vertical effective stress at depth D (Coduto, 1994).

Use of the net bearing pressure simplifies some computations, especially those associated with settlement of spread footings, but it makes others more complex. Some engineers prefer this method, while others prefer to use the gross bearing pressure. Therefore, it is important to understand which definition is being used. In this book we will use only the gross bearing pressure for designing shallow foundations because:

- This is the most commonly used method.
- The presumptive bearing pressures presented in building codes use this method [IBC 1805.4.1.1].
- It is conceptually easier.
- It simplifies the analysis of floating foundations.

Floating Foundations

Mat foundations are often placed in deep excavations, as shown in Figure 5.13. In addition to providing underground space, such as for subterranean parking, this design decreases the bearing pressure because the weight of the foundation is substantially less than the weight of the excavated soil. In other words, the weight of the structure and the foundation is partially offset by the removal of soil from the excavation. This reduction in q significantly reduces the settlement. Such designs are called *floating foundations*, and they are discussed in more detail in Chapter 18.

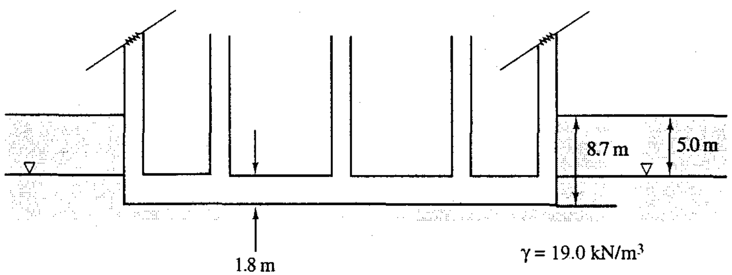


Figure 5.13 A floating foundation.

Example 5.3

The mat foundation in Figure 5.13 is to be 50 m wide, 70 m long, and 1.8 m thick. The sum of the column and wall loads is 805 MN. Compute the average bearing pressure, then compare it with the initial vertical effective stress in the soil immediately below the mat.

Solution

Use $\gamma_{conc} = 23.6 \text{ kN/m}^3$

$$W_f = (50 \text{ m})(70 \text{ m})(1.8 \text{ m})(23.6 \text{ kN/m}^3) = 149{,}000 \text{ kN}$$

$$u_D = \gamma_w z_{wD} = (9.8 \text{ kN/m}^3)(8.7 \text{ m} - 5.0 \text{ m}) = 36 \text{ kPa}$$

$$A = (50 \text{ m})(70 \text{ m}) = 3500 \text{ m}^2$$

$$q = \frac{P + W_f}{A} - u_D$$

$$= \frac{805{,}000 \text{ kN} + 149{,}000 \text{ kN}}{3500 \text{ m}^2} - 36 \text{ kPa}$$

$$= \mathbf{237 \text{ kPa}} \quad \Leftarrow \textit{Answer}$$

Note: Mat foundations are neither perfectly flexible nor perfectly rigid. Thus, the actual bearing pressure will vary across the base area. This computation produced the average value.

The vertical effective stress at a depth D before construction was:

$$\sigma'_{zD} = \Sigma \gamma H - u = (19.0 \text{ kN/m}^3)(8.7 \text{ m}) - 36 \text{ kPa} = \mathbf{129 \text{ kPa}} \quad \Leftarrow \textit{Answer}$$

Thus, the net result of making the excavation and constructing the building will result in a 237 − 129 = 108 kPa increase in the vertical effective stress immediately below the mat. ⇐ ***Answer***

Foundations with Eccentric or Moment Loads

Most foundations are built so the vertical load acts through the centroid, thus producing a fairly uniform distribution of bearing pressure. However, sometimes it becomes necessary to accommodate loads that act through other points, as shown in Figure 5.14a. These are called *eccentric loads*, and they produce a non-uniform bearing pressure distribution. The eccentricity, e, of the bearing pressure is equal to:

$$\boxed{e = \frac{Pe_1}{P + W_f}} \tag{5.3}$$

or, for continuous footings:

$$\boxed{e = \frac{(P/b)\, e_1}{P/b + W_f/b}} \tag{5.4}$$

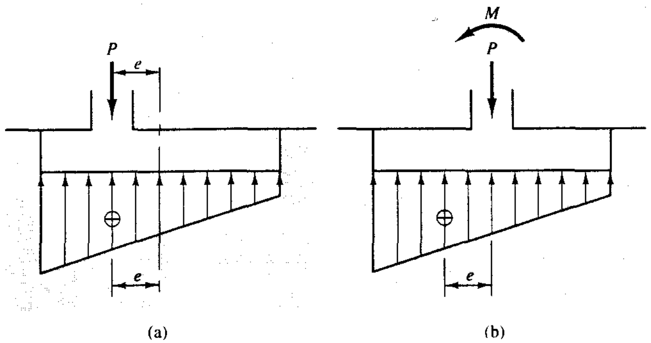


Figure 5.14 (a) Eccentric and (b) moment loads on shallow foundations.

Another similar condition occurs when moment loads are applied to foundations, as shown in Figure 5.14b. These loads also produce non-uniform bearing pressures. In this case, the eccentricity of the bearing pressure is:

$$\boxed{e = \frac{M}{P + W_f}} \tag{5.5}$$

$$\boxed{e = \frac{M/b}{P/b + W_f/b}} \tag{5.6}$$

Where:

- e = eccentricity of bearing pressure distribution
- P = applied vertical load
- P/b = applied vertical load per unit length of foundation
- M = applied moment load
- M/b = applied moment load per unit length of foundation
- e_1 = eccentricity of applied vertical load
- W_f = weight of foundation
- W_f/b = weight of unit length of foundation

In both cases, we assume the bearing pressure distribution beneath spread footings is linear, as shown in Figure 5.14. This is a simplification of the truth, but sufficiently accurate for practical design purposes. For mat foundations, we perform more detailed analyses as discussed in Chapter 10.

One-Way Loading

If the eccentric or moment loads occur only in the B direction, then the bearing pressure distribution is as shown in Figure 5.15.

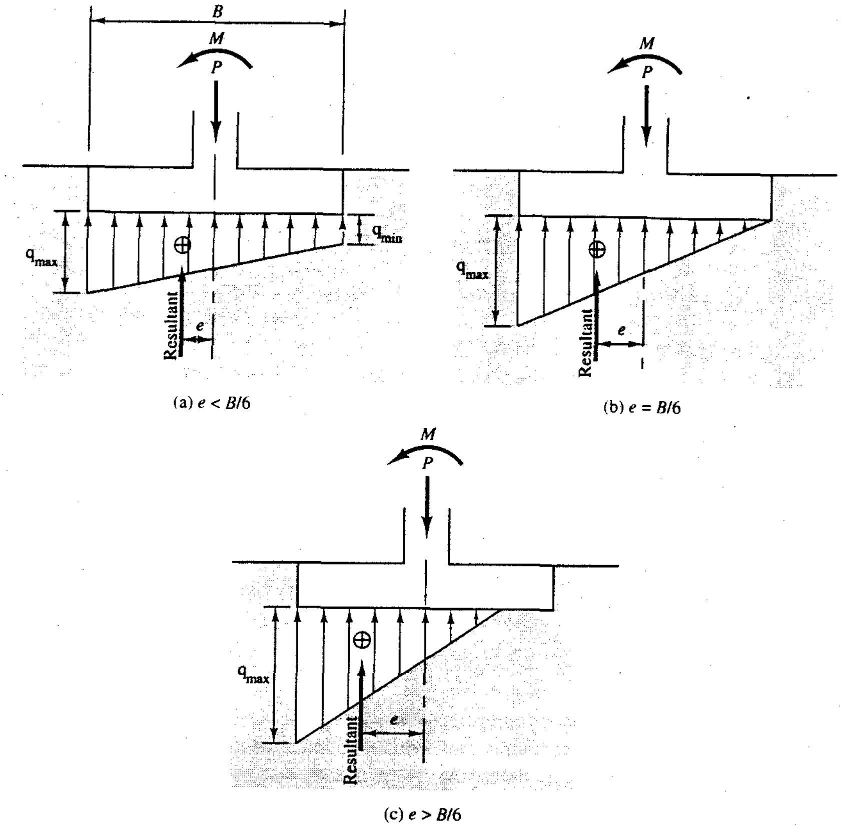


Figure 5.15 Distribution of bearing pressure beneath footings with various eccentricities: (a) $e < B/6$, (b) $e = B/6$, and (c) $e > B/6$.

If $e \leq B/6$ the bearing pressure distribution is trapezoidal, as shown in Figure 5.15a, and the minimum and maximum bearing pressures are:

$$\boxed{q_{min} = \left(\frac{P + W_f}{A} - u_D\right)\left(1 - \frac{6e}{B}\right)} \tag{5.7}$$

$$\boxed{q_{max} = \left(\frac{P + W_f}{A} - u_D\right)\left(1 + \frac{6e}{B}\right)} \tag{5.8}$$

Where:

q_{min} = minimum bearing pressure
q_{max} = maximum bearing pressure
P = column load
A = base area of foundation
u_D = pore water pressure along base of foundation
e = eccentricity of bearing pressure distribution
B = width of foundation

If the eccentric or moment load is only in the L direction, substitute L for B in Equations 5.7 and 5.8. For continuous footings, substitute P/b and M/b for P and M, respectively, and substitute B for A.

If $e = B/6$ (i.e., the resultant force acts at the third-point of the foundation), then $q_{min} = 0$ and the bearing pressure distribution is triangular as shown in Figure 5.15b. Therefore, so long as $e \leq B/6$, there will be some contact pressure along the entire base area.

However, if $e > B/6$, the resultant of the bearing pressure acts outside the third-point and the pressure distribution is as shown in Figure 5.15c. There can be no tension between the foundation and the soil, so one side of the foundation will lift off the ground. In addition, the high bearing pressure at the opposite side may cause a large settlement there. The net result is an excessive tilting of the foundation, which is not desirable. Therefore, foundations with eccentric or moment loads must satisfy the following condition:

$$\boxed{e \leq \frac{B}{6}} \tag{5.9}$$

This criterion maintains compressive stresses along the entire base area.

For rectangular foundations with the moment or eccentric load in the long direction, substitute L for B in Equation 5.9.

Example 5.4

A 5.0-ft wide continuous footing is subjected to a concentric vertical load of 12.0 k/ft and a moment load of 8.0 ft-k/ft acting laterally across the footing, as shown in Figure 5.16. The groundwater table is at a great depth. Determine whether the resultant force on the base of the footing acts within the middle third and compute the maximum and minimum bearing pressures.

Solution

$$W_f/b = (5.0\text{ ft})(1.5\text{ ft})(150\text{ lb/ft}^3) = 1125\text{ lb/ft}$$

$$e = \frac{M/b}{P/b + W_f/b} = \frac{8{,}000\text{ ft-lb/ft}}{12{,}000\text{ lb/ft} + 1125\text{ lb/ft}} = 0.610\text{ ft}$$

$$\frac{B}{6} = \frac{5\text{ ft}}{6} = 0.833\text{ ft}$$

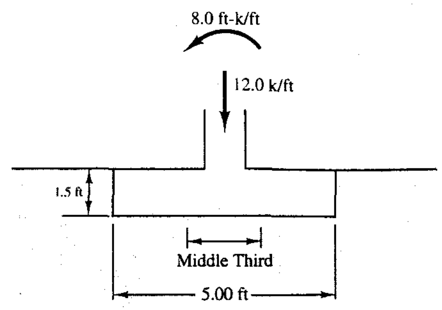


Figure 5.16 Proposed footing for Example 5.4.

$e < B/6$; therefore **the resultant is in the middle third** ⇐ ***Answer***

$$q_{min} = \left(\frac{P/b + W_f/b}{B} - u_D\right)\left(1 - \frac{6e}{B}\right)$$

$$= \left(\frac{12{,}000 + 1125}{5.0} - 0\right)\left(1 - \frac{(6)(0.610)}{5.0}\right)$$

$$= \mathbf{703\ lb/ft^2} \quad \Leftarrow \textit{Answer}$$

$$q_{max} = \left(\frac{P/b + W_f/b}{B} - u_D\right)\left(1 + \frac{6e}{B}\right)$$

$$= \left(\frac{12{,}000 + 1125}{5.0} - 0\right)\left(1 + \frac{(6)(0.610)}{5.0}\right)$$

$$= \mathbf{4546\ lb/ft^2} \quad \Leftarrow \textit{Answer}$$

When designing combined footings, try to arrange the footing dimensions and column locations so the resultant of the applied loads acts through the centroid of the footing. This produces a uniform bearing pressure distribution. Some combined footing designs accomplish this by using a trapezoidal shaped footing (as seen in plan view) with the more lightly loaded column on the narrow side of the trapezoid. When this is not possible, be sure all of the potential loading conditions produce eccentricities no more than $B/6$.

Two-Way Eccentric or Moment Loading

If the resultant load acting on the base is eccentric in both the B and L directions, it must fall within the diamond-shaped kern shown in Figure 5.17 for the contact pressure to be compressive along the entire base of the foundation. It falls within this kern only if the following condition is met:

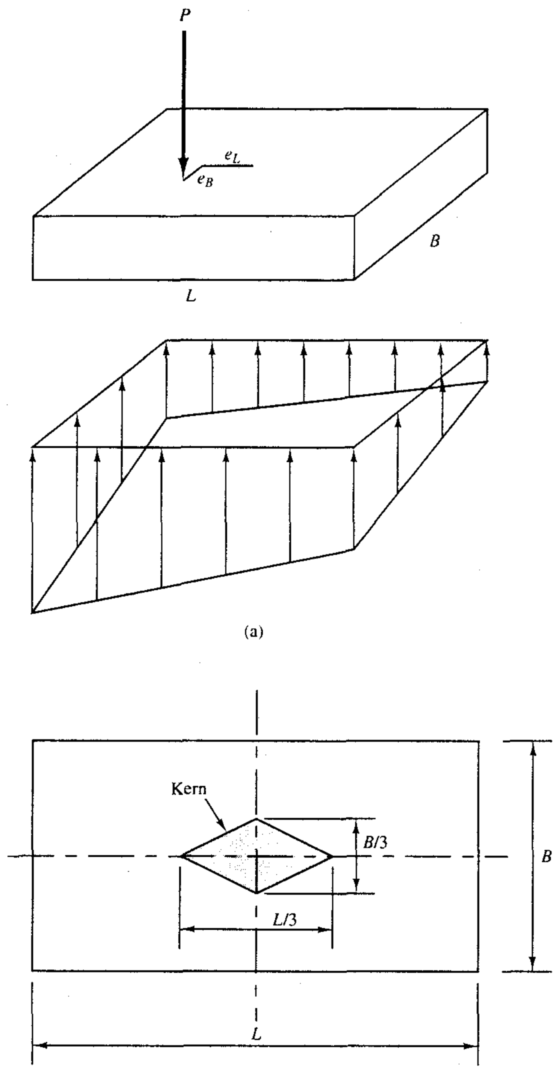


Figure 5.17 (a) Pressure distribution beneath spread footing with vertical load that is eccentric in both the B and L directions; (b) To maintain $q \geq 0$ along with the entire base of the footing, the resultant force must be located within this diamond-shaped kern.

$$\boxed{\frac{6\,e_B}{B} + \frac{6\,e_L}{L} \leq 1.0} \tag{5.10}$$

Where:

e_B = eccentricity in the B direction

e_L = eccentricity in the L direction

If Equation 5.10 is satisfied, the magnitudes of q at the four corners of a square or rectangular shallow foundation are:

$$\boxed{q = \left(\frac{P + W_f}{A} - u_D\right)\left(1 \pm \frac{6\,e_B}{B} \pm \frac{6\,e_L}{L}\right)} \tag{5.11}$$

Example 5.5

The mat foundation shown in Figure 5.18 will support four grain silos. These are cylindrical structures used to store grain. Each of the silos has an empty weight of 29 MN, and can hold up to 110 MN of grain. The mat has a weight of 60 MN. Since each silo is filled independently, the resultant load imposed on the mat does not necessarily act through the centroid. Evaluate the various loading conditions and determine whether eccentric loading requirements will be met. If these requirements are not met, determine the minimum mat width, B, needed to satisfy these requirements.

Solution

1. Check one-way eccentricity

The greatest one-way eccentricity occurs when two adjacent silos are full, and the other two are empty.

$$P = 4(29\text{ MN}) + 2(110\text{ MN}) = 336\text{ MN}$$

$$M = 2(110\text{ MN})(12\text{ m}) = 2640\text{ MN-m}$$

$$e = \frac{M}{P + W_f} = \frac{2640\text{ MN-m}}{336\text{ MN} + 60\text{ MN}} = 6.67\text{ m}$$

$$e \leq \frac{B}{6} \qquad \therefore \text{ OK for one-way eccentricity}$$

2. Check two-way eccentricity

The greatest two-way eccentricity occurs when one silos is full and the other three are empty.

$$\frac{B}{6} = \frac{50\text{ m}}{6} = 8.33\text{ m}$$

$$P = 4(29\text{ MN}) + (110\text{ MN}) = 226\text{ MN}$$

$$M_B = M_L = (110\text{ MN})(12\text{ m}) = 1320\text{ MN-m}$$

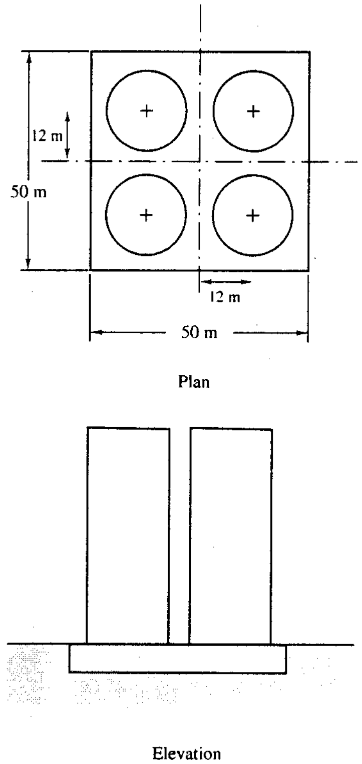


Figure 5.18 Mat foundation and grain silos for Example 5.5.

$$e_B = e_L = \frac{M}{P + W_f} = \frac{1320 \text{ MN-m}}{226 \text{ MN} + 60 \text{ MN}} = 4.62 \text{ m}$$

$$\frac{6e_B}{B} + \frac{6e_L}{L} = \frac{6(4.62 \text{ m})}{50 \text{ m}} + \frac{6(4.62 \text{ m})}{50 \text{ m}} = 1.11 > 1 \quad \text{Not acceptable}$$

3. Conclusion

Although the foundation is satisfactory for one-way eccentricity, it does not meet the criterion for two-way eccentricity because the resultant is outside the kern. This means the corner of the mat opposite the loaded silo may lift up, causing excessive tilting. Therefore, it is necessary to increase *B*. ⇐ ***Answer***

4. Revised Design

$$\frac{6e_B}{B} + \frac{6e_L}{L} = \frac{6(4.62 \text{ m})}{B} + \frac{6(4.62 \text{ m})}{L} = 1$$

minimum *B* = minimum *L* = 55.4 m ⇐ ***Answer***

Therefore, a 55.4 m × 55.4 m mat would be required to keep the resultant within the kern. However, this is only one of many design criteria for mat foundations. Other criteria, as discussed in Chapters 6 to 10, also need to be checked.

SUMMARY

Major Points

1. Shallow foundations are those that transmit structural loads to the near-surface soils. There are two kinds: spread footing foundations and mat foundations.
2. Although other materials have been used in the past, today virtually all shallow foundations are made of reinforced concrete.
3. Spread footings are most often used in small- to medium-size structures on sites with moderate to good soil conditions. Mats are most often used on larger structures, especially those with differential settlement problems and those with foundations below the groundwater table.
4. The bearing pressure is the contact pressure between the bottom of a shallow foundation and the underlying soils.
5. A floating foundation is one where the weight of the foundation is substantially less than the weight of the excavated soils. This occurs in buildings with basements and other similar structures.
6. If the loads applied to a foundation are eccentric, or if moment loads are applied, the resulting bearing pressure distribution also will be eccentric. In such cases, the foundation needs to be designed so the resultant of the bearing pressure is within the middle third of the foundation (for one-way eccentricity) or in a diamond-shaped kern (for two-way eccentricity). This requirement ensures the entire base of the foundation has compressive bearing pressures, and thus avoids problems with uplift.

Vocabulary

Backhoe	Eccentric load	Moment load
Bearing pressure	Floating foundation	Neat footing
Circular spread footing	Formed footing	Net bearing pressure
Combined spread footing	Gross bearing pressure	Raft foundation
Continuous spread footing	Kern	Rectangular spread footing
Dimension-stone footings	Mat	Ring footing

Rubble-stone footings	Spread footing	Steel grillage footing
Shallow foundation	Square spread footing	

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

5.1 What is the difference between a square footing and a continuous footing, and when would each type be used?

5.2 A 400 kN vertical downward column load acts at the centroid of a 1.5-m square footing. The bottom of this footing is 0.4 m below the ground surface and the top is flush with the ground surface. The groundwater table is at a depth of 3 m below the ground surface. Compute the bearing pressure.

5.3 A bearing wall carries a dead load of 5.0 k/ft and a live load of 3.0 k/ft. It is supported on a 3 ft wide, 2 ft deep continuous footing. The top of this footing is flush with the ground surface and the groundwater table is at a depth of 35 ft below the ground surface. Compute the bearing pressure.

5.4 The mat foundation in Figure 5.19 is 45 m wide and 90 m long. It has a weight of 140 MN. The sum of the applied structural loads is 1300 MN. Compute the average bearing pressure with the groundwater table at position A. Then repeat the computation with the groundwater table at position B. Explain why these two values of q are different.

5.5 A 5-ft square, 2-ft deep spread footing is subjected to a concentric vertical load of 60 k and an overturning moment of 30 ft-k. The overturning moment acts parallel to one of the sides of the footing, the top of the footing is flush with the ground surface, and the groundwater table is at a depth of 20 ft below the ground surface. Determine whether the resultant force acts within the middle third of the footing, compute the minimum and maximum bearing pressures, and show the distribution of bearing pressure in a sketch.

5.6 Consider the footing and loads in Problem 5.5, except that the overturning moment now acts at a 45° angle from the side of the footing (i.e., it acts diagonally across the top of the footing). Determine whether the resultant force acts within the kern. If it does, then compute the bearing pressure at each corner of the footing and show the pressure distribution in a sketch similar to Figure 5.17.

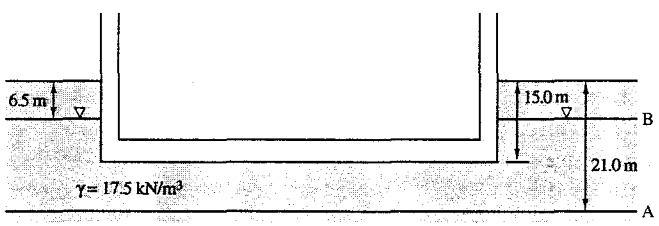


Figure 5.19 Mat foundation for Problem 5.4

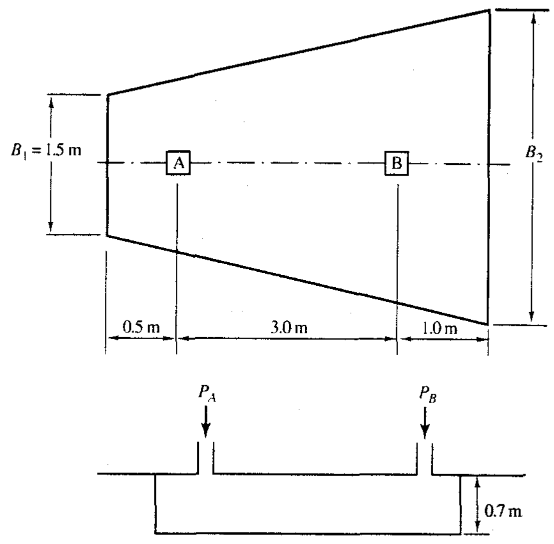


Figure 5.20 Proposed combined footing for Problems 5.7 and 5.8.

5.7 The two columns in Figure 5.20 are to be supported on a combined footing. The vertical dead loads on Columns A and B are 500 and 1400 kN, respectively. Determine the required dimension B_2 so the resultant of the column loads acts through the centroid of the footing and express your answer as a multiple of 100 mm.

5.8 In addition to the dead loads described in Problem 5.7, Columns A and B in Figure 5.20 also can carry vertical live loads of up to 800 and 1200 k, respectively. The live loads vary with time, and thus may be present some days and absent other days. In addition, the live load on each column is independent of that on the other column (i.e., one could be carrying the full live load while the other has zero live load). Using the dimensions obtained in Problem 5.7, and the worst possible combination of live loads, determine if the bearing pressure distribution always meets the eccentricity requirements described in this chapter. The groundwater table is at a depth of 10 m.

5.9 Beginning with $(P + W_f)/A \pm Mc/I$, derive Equations 5.7 and 5.8. Would these equations also apply to circular shallow foundations? Why or why not?

Rubble-stone footings	Spread footing	Steel grillage footing
Shallow foundation	Square spread footing	

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

5.1 What is the difference between a square footing and a continuous footing, and when would each type be used?

5.2 A 400 kN vertical downward column load acts at the centroid of a 1.5-m square footing. The bottom of this footing is 0.4 m below the ground surface and the top is flush with the ground surface. The groundwater table is at a depth of 3 m below the ground surface. Compute the bearing pressure.

5.3 A bearing wall carries a dead load of 5.0 k/ft and a live load of 3.0 k/ft. It is supported on a 3 ft wide, 2 ft deep continuous footing. The top of this footing is flush with the ground surface and the groundwater table is at a depth of 35 ft below the ground surface. Compute the bearing pressure.

5.4 The mat foundation in Figure 5.19 is 45 m wide and 90 m long. It has a weight of 140 MN. The sum of the applied structural loads is 1300 MN. Compute the average bearing pressure with the groundwater table at position A. Then repeat the computation with the groundwater table at position B. Explain why these two values of q are different.

5.5 A 5-ft square, 2-ft deep spread footing is subjected to a concentric vertical load of 60 k and an overturning moment of 30 ft-k. The overturning moment acts parallel to one of the sides of the footing, the top of the footing is flush with the ground surface, and the groundwater table is at a depth of 20 ft below the ground surface. Determine whether the resultant force acts within the middle third of the footing, compute the minimum and maximum bearing pressures, and show the distribution of bearing pressure in a sketch.

5.6 Consider the footing and loads in Problem 5.5, except that the overturning moment now acts at a 45° angle from the side of the footing (i.e., it acts diagonally across the top of the footing). Determine whether the resultant force acts within the kern. If it does, then compute the bear-

Figure 5.19 Mat foundation for Problem 5.4

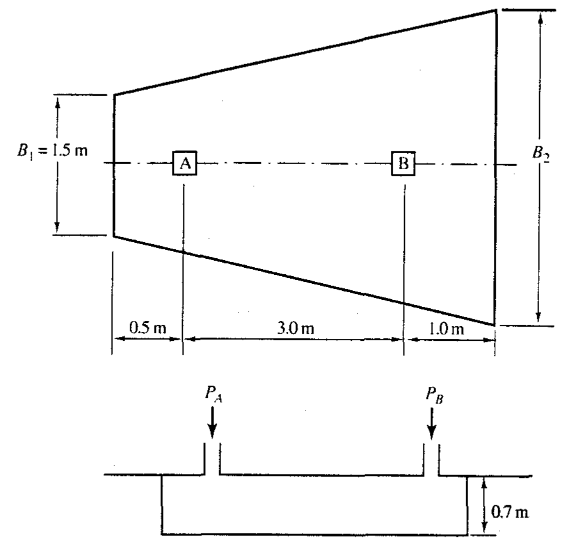


Figure 5.20 Proposed combined footing for Problems 5.7 and 5.8.

ing pressure at each corner of the footing and show the pressure distribution in a sketch similar to Figure 5.17.

5.7 The two columns in Figure 5.20 are to be supported on a combined footing. The vertical dead loads on Columns A and B are 500 and 1400 kN, respectively. Determine the required dimension B_2 so the resultant of the column loads acts through the centroid of the footing and express your answer as a multiple of 100 mm.

5.8 In addition to the dead loads described in Problem 5.7, Columns A and B in Figure 5.20 also can carry vertical live loads of up to 800 and 1200 k, respectively. The live loads vary with time, and thus may be present some days and absent other days. In addition, the live load on each column is independent of that on the other column (i.e., one could be carrying the full live load while the other has zero live load). Using the dimensions obtained in Problem 5.7, and the worst possible combination of live loads, determine if the bearing pressure distribution always meets the eccentricity requirements described in this chapter. The groundwater table is at a depth of 10 m.

5.9 Beginning with $(P + W_f)/A \pm Mc/I$, derive Equations 5.7 and 5.8. Would these equations also apply to circular shallow foundations? Why or why not?

6

Shallow Foundations—Bearing Capacity

When we are satisfied with the spot fixed on for the site of the city . . . the foundations should be carried down to a solid bottom, if such can be found, and should be built thereon of such thickness as may be necessary for the proper support of that part of the wall which stands above the natural level of the ground. They should be of the soundest workmanship and materials, and of greater thickness than the walls above. If solid ground can be come to, the foundations should go down to it and into it, according to the magnitude of the work, and the substruction to be built up as solid as possible. Above the ground of the foundation, the wall should be one-half thicker than the column it is to receive so that the lower parts which carry the greatest weight, may be stronger than the upper part . . . Nor must the mouldings of the bases of the columns project beyond the solid. Thus, also, should be regulated the thickness of all walls above ground.

Marcus Vitruvius, Roman Architect and Engineer
1st century B.C.
as translated by Morgan (1914)

Shallow foundations must satisfy various performance requirements, as discussed in Chapter 2. One of them is called *bearing capacity*, which is a geotechnical strength requirement. This chapter explores this requirement, and shows how to design shallow foundations so that they do not experience bearing capacity failures.

6.1 BEARING CAPACITY FAILURES

Shallow foundations transmit the applied structural loads to the near-surface soils. In the process of doing so, they induce both compressive and shear stresses in these soils. The magnitudes of these stresses depend largely on the bearing pressure and the size of the footing. If the bearing pressure is large enough, or the footing is small enough, the shear stresses may exceed the shear strength of the soil or rock, resulting in a *bearing capacity failure*. Researchers have identified three types of bearing capacity failures: *general shear failure*, *local shear failure*, and *punching shear failure*, as shown in Figure 6.1. A typical load-displacement curve for each mode of failure is shown in Figure 6.2.

General shear failure is the most common mode. It occurs in soils that are relatively incompressible and reasonably strong, in rock, and in saturated, normally consolidated clays that are loaded rapidly enough that the undrained condition prevails. The failure surface is well defined and failure occurs quite suddenly, as illustrated by the load-displacement curve. A clearly formed bulge appears on the ground surface adjacent to the foundation. Although bulges may appear on both sides of the foundation, ultimate failure occurs on one side only, and it is often accompanied by rotations of the foundation.

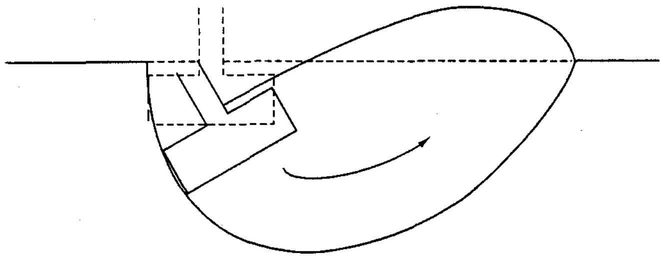
The opposite extreme is the punching shear failure. It occurs in very loose sands, in a thin crust of strong soil underlain by a very weak soil, or in weak clays loaded under slow, drained conditions. The high compressibility of such soil profiles causes large settlements and poorly defined vertical shear surfaces. Little or no bulging occurs at the ground surface and failure develops gradually, as illustrated by the ever-increasing load-settlement curve.

Local shear failure is an intermediate case. The shear surfaces are well defined under the foundation, and then become vague near the ground surface. A small bulge may occur, but considerable settlement, perhaps on the order of half the foundation width, is necessary before a clear shear surface forms near the ground. Even then, a sudden failure does not occur, as happens in the general shear case. The foundation just continues to sink ever deeper into the ground.

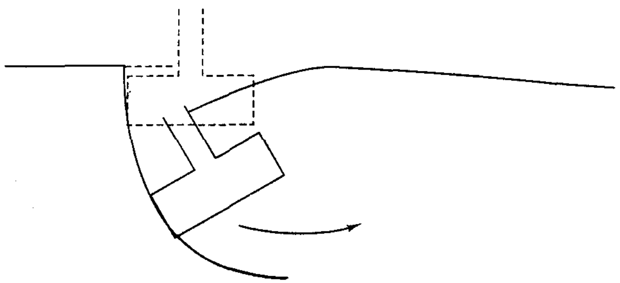
Vesić (1973) investigated these three modes of failure by conducting load tests on model circular foundations in a sand. These tests included both shallow and deep foundations. The results, shown in Figure 6.3, indicate shallow foundations (*D/B* less than about 2) can fail in any of the three modes, depending on the relative density. However, deep foundations (*D/B* greater than about 4) are always governed by punching shear. Although these test results apply only to circular foundations in Vesić's sand and cannot necessarily be generalized to other soils, it does give a general relationship between the mode of failure, relative density, and the *D/B* ratio.

Complete quantitative criteria have yet to be developed to determine which of these three modes of failure will govern in any given circumstance, but the following guidelines are helpful:

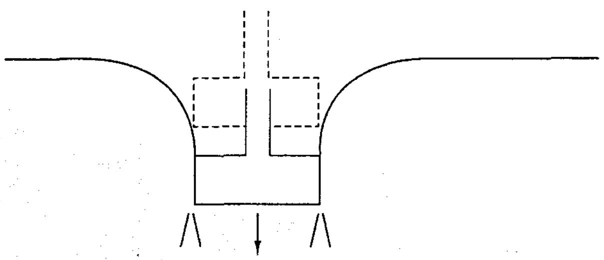
- Shallow foundations in rock and undrained clays are governed by the general shear case.



(a) General Shear Failure

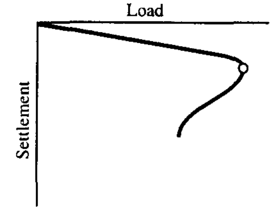


(b)

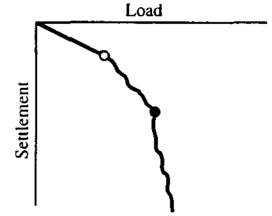


(c)

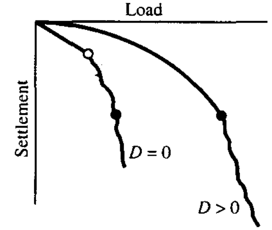
Figure 6.1 Modes of bearing capacity failure: (a) general shear failure; (b) local shear failure; (c) punching shear failure.



(a)



(b)



(c)

Figure 6.2 Typical load-displacement curves for different modes of bearing capacity failure: (a) general shear failure; (b) local shear failure; (c) punching shear failure. The circles indicate various interpretations of failure. (Adapted from Vesić, 1963).

- Shallow foundations in dense sands are governed by the general shear case. In this context, a dense sand is one with a relative density, D_r, greater than about 67%.
- Shallow foundations on loose to medium dense sands ($30\% < D_r < 67\%$) are probably governed by local shear.
- Shallow foundations on very loose sand ($D_r < 30\%$) are probably governed by punching shear.

For nearly all practical shallow foundation design problems, it is only necessary to check the general shear case, and then conduct settlement analyses to verify that the foundation will not settle excessively. These settlement analyses implicitly protect against local and punching shear failures.

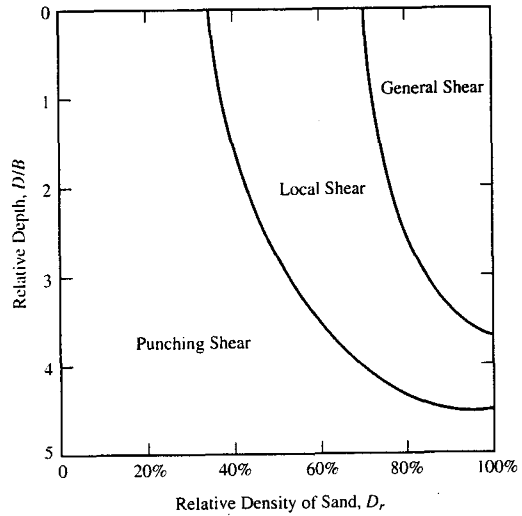


Figure 6.3 Modes of failure of model circular foundations in Chattahoochee Sand (Adapted from Vesić, 1963 and 1973).

6.2 BEARING CAPACITY ANALYSES IN SOIL—GENERAL SHEAR CASE

Methods of Analyzing Bearing Capacity

To analyze spread footings for bearing capacity failures and design them in a way to avoid such failures, we must understand the relationship between bearing capacity, load, footing dimensions, and soil properties. Various researchers have studied these relationships using a variety of techniques, including:

- Assessments of the performance of real foundations, including full-scale load tests.
- Load tests on model footings.
- Limit equilibrium analyses.
- Detailed stress analyses, such as finite element method (FEM) analyses.

Full-scale load tests, which consist of constructing real spread footings and loading them to failure, are the most precise way to evaluate bearing capacity. However, such tests are expensive, and thus are rarely, if ever, performed as a part of routine design. A few such tests have been performed for research purposes.

Model footing tests have been used quite extensively, mostly because the cost of these tests is far below that for full-scale tests. Unfortunately, model tests have their limitations, especially when conducted in sands, because of uncertainties in applying the

proper scaling factors. However, the advent of centrifuge model tests has partially overcome this problem.

Limit equilibrium analyses are the dominant way to assess bearing capacity of shallow foundations. These analyses define the shape of the failure surface, as shown in Figure 6.1, then evaluate the stresses and strengths along this surface. These methods of analysis have their roots in Prandtl's studies of the punching resistance of metals (Prandtl, 1920). He considered the ability of very thick masses of metal (i.e., not sheet metal) to resist concentrated loads. Limit equilibrium analyses usually include empirical factors developed from model tests.

Occasionally, geotechnical engineers perform more detailed bearing capacity analyses using numerical methods, such as the finite element method (FEM). These analyses are more complex, and are justified only on very critical and unusual projects.

We will consider only limit equilibrium methods of bearing capacity analyses, because these methods are used on the overwhelming majority of projects.

Simple Bearing Capacity Formula

The limit equilibrium method can be illustrated by considering the continuous footing shown in Figure 6.4. Let us assume this footing experiences a bearing capacity failure, and that this failure occurs along a circular shear surface as shown. We will further assume the soil is an undrained clay with a shear strength s_u. Finally, we will neglect the shear strength between the ground surface and a depth D, which is conservative. Thus, the soil in this zone is considered to be only a surcharge load that produces a vertical total stress of $\sigma_{zD} = \gamma D$ at a depth D.

The objective of this derivation is to obtain a formula for the *ultimate bearing capacity*, q_{ult}, which is the bearing pressure required to cause a bearing capacity failure. By

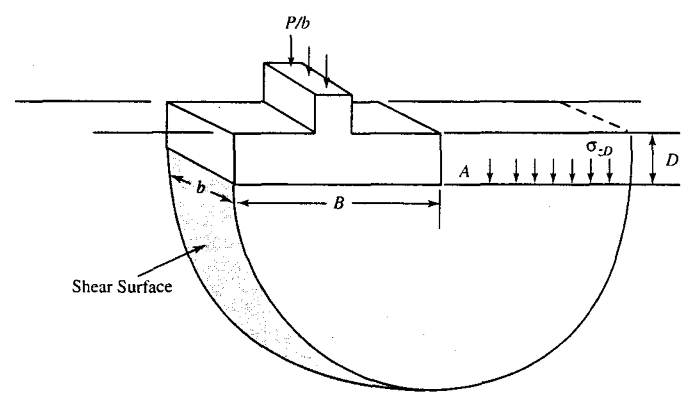


Figure 6.4 Bearing capacity analysis along a circular failure surface.

considering a slice of the foundation of length b and taking moments about Point A, we obtain the following:

$$M_A = (q_{ult}Bb)(B/2) - (s_u\pi Bb)(B) - \sigma_{zD}Bb(B/2) \quad (6.1)$$

$$q_{ult} = 2\,\pi\, s_u + \sigma_{zD} \quad (6.2)$$

It is convenient to define a new parameter, called a *bearing capacity factor*, N_c, and rewrite Equation 6.2 as:

$$\boxed{q_{ult} = N_c s_u + \sigma_{zD}} \quad (6.3)$$

Equation 6.3 is known as a *bearing capacity formula*, and could be used to evaluate the bearing capacity of a proposed foundation. According to this derivation, $N_c = 2\pi = 6.28$.

This simplified formula has only limited applicability in practice because it considers only continuous footings and undrained soil conditions ($\phi = 0$), and it assumes the foundation rotates as the bearing capacity failure occurs. However, this simple derivation illustrates the general methodology required to develop more comprehensive bearing capacity formulas.

Terzaghi's Bearing Capacity Formulas

Various limit equilibrium methods of computing bearing capacity of soils were advanced in the first half of the twentieth century, but the first one to achieve widespread acceptance was that of Terzaghi (1943). His method includes the following assumptions:

- The depth of the foundation is less than or equal to its width ($D \le B$).
- The bottom of the foundation is sufficiently rough that no sliding occurs between the foundation and the soil.
- The soil beneath the foundation is a homogeneous semi-infinite mass (i.e., the soil extends for a great distance below the foundation and the soil properties are uniform throughout).
- The shear strength of the soil is described by the formula $s = c' + \sigma' \tan \phi'$.
- The general shear mode of failure governs.
- No consolidation of the soil occurs (i.e., settlement of the foundation is due only to the shearing and lateral movement of the soil).
- The foundation is very rigid in comparison to the soil.
- The soil between the ground surface and a depth D has no shear strength, and serves only as a surcharge load.
- The applied load is compressive and applied vertically to the centroid of the foundation and no applied moment loads are present.

Terzaghi considered three zones in the soil, as shown in Figure 6.5. Immediately beneath the foundation is a *wedge zone* that remains intact and moves downward with the

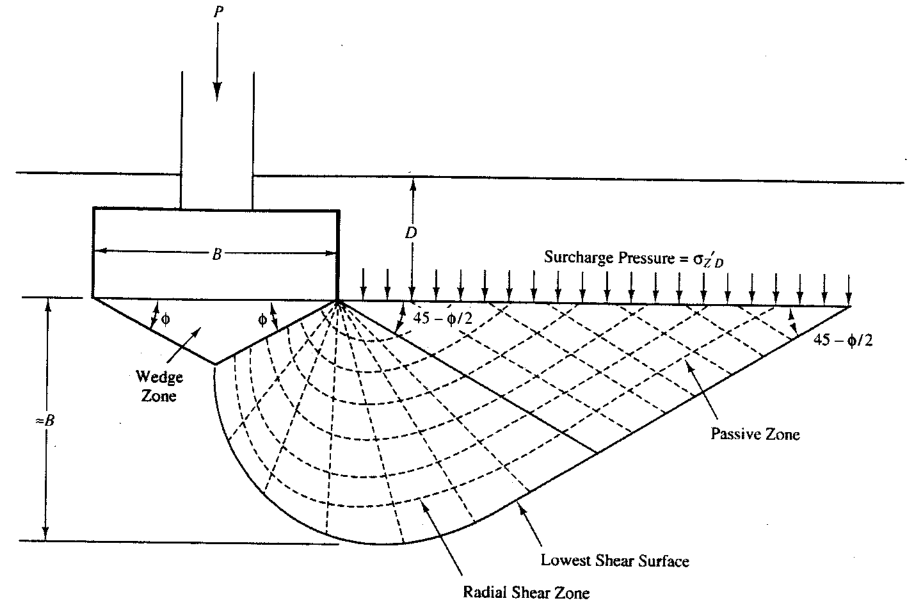


Figure 6.5 Geometry of failure surface for Terzaghi's bearing capacity formulas.

foundation. Next, a *radial shear zone* extends from each side of the wedge, where he took the shape of the shear planes to be logarithmic spirals. Finally, the outer portion is the *linear shear zone* in which the soil shears along planar surfaces.

Since Terzaghi neglected the shear strength of soils between the ground surface and a depth D, the shear surface stops at this depth and the overlying soil has been replaced with the surcharge pressure σ_{zD}'. This approach is conservative, and is part of the reason for limiting the method to relatively shallow foundations ($D \le B$).

Terzaghi developed his theory for continuous foundations (i.e., those with a very large L/B ratio). This is the simplest case because it is a two-dimensional problem. He then extended it to square and round foundations by adding empirical coefficients obtained from model tests and produced the following bearing capacity formulas:

For square foundations:

$$\boxed{q_{ult} = 1.3\,c'N_c + \sigma'_{zD}N_q + 0.4\gamma' BN_\gamma} \quad (6.4)$$

For continuous foundations:

$$\boxed{q_{ult} = c'N_c + \sigma'_{zD}N_q + 0.5\gamma' BN_\gamma} \quad (6.5)$$

For circular foundations:

$$q_{ult} = 1.3\,c'N_c + \sigma'_{zD}N_q + 0.3\gamma' BN_\gamma \quad (6.6)$$

Where:

q_{ult} = ultimate bearing capacity
c' = effective cohesion for soil beneath foundation
ϕ' = effective friction angle for soil beneath foundation
σ_{zD}' = vertical effective stress at depth D below the ground surface ($\sigma_{zD}' = \gamma D$ if depth to groundwater table is greater than D)
γ' = effective unit weight of the soil ($\gamma = \gamma'$ if groundwater table is very deep; see discussion later in this chapter for shallow groundwater conditions)
D = depth of foundation below ground surface
B = width (or diameter) of foundation
N_c, N_q, N_γ = Terzaghi's bearing capacity factors $= f(\phi')$ (See Table 6.1 or Equations 6.7–6.12.)

Because of the shape of the failure surface, the values of c' and ϕ' only need to represent the soil between the bottom of the footing and a depth B below the bottom. The soils between the ground surface and a depth D are treated simply as overburden.

Terzaghi's formulas are presented in terms of effective stresses. However, they also may be used in a total stress analyses by substituting c_T, ϕ_T, and σ_D for c', ϕ', and σ_D'. If saturated undrained conditions exist, we may conduct a total stress analysis with the shear strength defined as $c_T = s_u$ and $\phi_T = 0$. In this case, $N_c = 5.7$, $N_q = 1.0$, and $N_c = 0.0$.

The Terzaghi bearing capacity factors are:

$$N_q = \frac{a_\theta^2}{2\cos^2(45 + \phi'/2)} \quad (6.7)$$

$$a_\theta = e^{\pi(0.75 - \phi'/360)\tan\phi'} \quad (6.8)$$

$$N_c = 5.7 \quad \text{for } \phi' = 0 \quad (6.9)$$

$$N_c = \frac{N_q - 1}{\tan\phi'} \quad \text{for } \phi' > 0 \quad (6.10)$$

$$N_\gamma = \frac{\tan\phi'}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right) \quad (6.11)$$

These bearing capacity factors are also presented in tabular form in Table 6.1. Notice that Terzaghi's N_c of 5.7 is smaller than the value of 6.28 derived from the simple bearing capacity analysis. This difference the result of using a circular failure surface in the simple method and a more complex geometry in Terzaghi's method.

TABLE 6.1 BEARING CAPACITY FACTORS

ϕ' (deg)	Terzaghi (for use in Equations 6.4–6.6) N_c	N_q	N_γ	Vesić (for use in Equation 6.13) N_c	N_q	N_γ
0	5.7	1.0	0.0	5.1	1.0	0.0
1	6.0	1.1	0.1	5.4	1.1	0.1
2	6.3	1.2	0.1	5.6	1.2	0.2
3	6.6	1.3	0.2	5.9	1.3	0.2
4	7.0	1.5	0.3	6.2	1.4	0.3
5	7.3	1.6	0.4	6.5	1.6	0.4
6	7.7	1.8	0.5	6.8	1.7	0.6
7	8.2	2.0	0.6	7.2	1.9	0.7
8	8.6	2.2	0.7	7.5	2.1	0.9
9	9.1	2.4	0.9	7.9	2.3	1.0
10	9.6	2.7	1.0	8.3	2.5	1.2
11	10.2	3.0	1.2	8.8	2.7	1.4
12	10.8	3.3	1.4	9.3	3.0	1.7
13	11.4	3.6	1.6	9.8	3.3	2.0
14	12.1	4.0	1.9	10.4	3.6	2.3
15	12.9	4.4	2.2	11.0	3.9	2.6
16	13.7	4.9	2.5	11.6	4.3	3.1
17	14.6	5.5	2.9	12.3	4.8	3.5
18	15.5	6.0	3.3	13.1	5.3	4.1
19	16.6	6.7	3.8	13.9	5.8	4.7
20	17.7	7.4	4.4	14.8	6.4	5.4
21	18.9	8.3	5.1	15.8	7.1	6.2
22	20.3	9.2	5.9	16.9	7.8	7.1
23	21.7	10.2	6.8	18.0	8.7	8.2
24	23.4	11.4	7.9	19.3	9.6	9.4
25	25.1	12.7	9.2	20.7	10.7	10.9
26	27.1	14.2	10.7	22.3	11.9	12.5
27	29.2	15.9	12.5	23.9	13.2	14.5
28	31.6	17.8	14.6	25.8	14.7	16.7
29	34.2	20.0	17.1	27.9	16.4	19.3
30	37.2	22.5	20.1	30.1	18.4	22.4
31	40.4	25.3	23.7	32.7	20.6	26.0
32	44.0	28.5	28.0	35.5	23.2	30.2
33	48.1	32.2	33.3	38.6	26.1	35.2
34	52.6	36.5	39.6	42.2	29.4	41.1
35	57.8	41.4	47.3	46.1	33.3	48.0
36	63.5	47.2	56.7	50.6	37.8	56.3
37	70.1	53.8	68.1	55.6	42.9	66.2
38	77.5	61.5	82.3	61.4	48.9	78.0
39	86.0	70.6	99.8	67.9	56.0	92.2
40	95.7	81.3	121.5	75.3	64.2	109.4
41	106.8	93.8	148.5	83.9	73.9	130.2

Terzaghi used a tedious graphical method to obtain values for $K_{p\gamma}$, then used these values to compute N_γ. He also computed values of the other bearing capacity factors and presented the results in plots of N_c, N_γ, and N_γ as a function of ϕ'. These plots and tables such as Table 6.1 are still a convenient way to evaluate these parameters. However, the advent of computers and hand-held calculators has generated the need for the following simplified formula for N_γ:

$$N_\gamma \approx \frac{2\,(N_q + 1)\tan\phi'}{1 + 0.4\sin(4\,\phi')} \tag{6.12}$$

The author developed Equation 6.12 by fitting a curve to match Terzaghi's. It produces N_γ values within about 10 percent of Terzaghi's values. Alternatively, Kumbhojkar (1993) provides a more precise, but more complex, formula for N_γ.

Example 6.1

A square footing is to be constructed as shown in Figure 6.6. The groundwater table is at a depth of 50 ft below the ground surface. Compute the ultimate bearing capacity and the column load required to produce a bearing capacity failure.

Solution

For purposed of evaluating bearing capacity, ignore the slab-on-grade floor.

$$\text{For } \phi' = 30°: \quad N_c = 37.2, N_q = 22.5, N_\gamma = 20.1 \text{ (from Table 6.1)}$$

$$\sigma'_{zD} = \gamma D - u = (121\ \text{lb/ft}^3)(2\ \text{ft}) - 0 = 242\ \text{lb/ft}^2$$

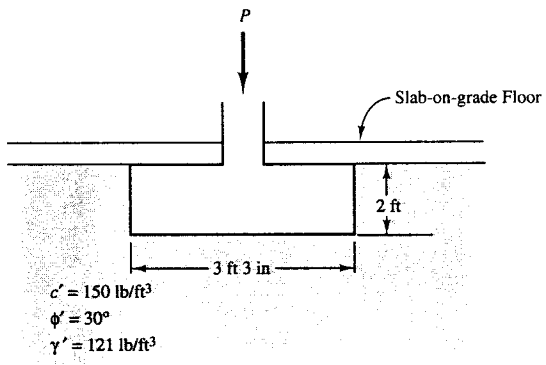


Figure 6.6 Proposed footing for Example 6.1.

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.4\gamma'BN_\gamma$$
$$= (1.3)(150\ \text{lb/ft}^2)(37.2) + (242\ \text{lb/ft}^2)(22.5) + (0.4)(121\ \text{lb/ft}^3)(3.25\ \text{ft})(20.1)$$
$$= 7254 + 5445 + 3162$$
$$= \mathbf{15{,}900\ lb/ft^2} \quad \Leftarrow \textit{Answer}$$

$$W_f = (3.25\ \text{ft})(3.25\ \text{ft})(2.0\ \text{ft})(150\ \text{lb/ft}^3) = 3169\ \text{lb}$$

Setting $q = q_{ult}$, using Equation 5.1, and solving for P gives:

$$q = \frac{P + W_f}{A} - u$$

$$15{,}900\ \text{lb/ft}^2 = \frac{P + 3169\ \text{lb}}{(3.25\ \text{ft})^2} - 0$$

$$P = 165{,}000\ \text{lb}$$
$$= \mathbf{165\ k} \quad \Leftarrow \textit{Answer}$$

According to this analysis, a column load of 165 k would cause a bearing capacity failure beneath this footing. Nearly half of this capacity comes from the first term in the bearing capacity formula and is therefore dependent on the cohesion of the soil. Since the cohesive strength is rather tenuous, it is prudent to use conservative values of c in bearing capacity analyses. In contrast, the frictional strength is more reliable and does not need to be interpreted as conservatively.

Example 6.2

The proposed continuous footing shown in Figure 6.7 will support the exterior wall of a new industrial building. The underlying soil is an undrained clay, and the groundwater table is below the bottom of the footing. Compute the ultimate bearing capacity, and compute the wall load required to cause a bearing capacity failure.

Solution

This analysis uses the undrained shear strength, s_u. Therefore, we will use Terzaghi's bearing capacity formula with $c_T = s_u = 120$ kPa and $\phi = 0$.

$$\text{For } \phi = 0: \quad N_c = 5.7, N_q = 1, N_\gamma = 0 \text{ (from Table 6.1)}$$

The depth of embedment, D, is measured from the lowest ground surface, so $D = 0.4$ m.

$$\sigma'_{zD} = \gamma D = (18.0)(0.4) = 7.2\ \text{kPa}$$

$$q_{ult} = s_uN_c + \sigma'_{zD}N_q + 0.5\gamma'BN_\gamma$$
$$= (120\ \text{kPa})(5.7) + (7.2)(1) + 0.5\gamma'B(0)$$
$$= \mathbf{691\ kPa} \quad \Leftarrow \textit{Answer}$$

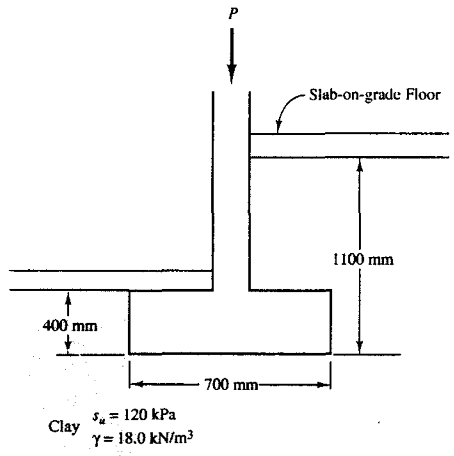


Figure 6.7 Proposed footing for Example 6.2.

The zone above the bottom of the footing is partly concrete and partly soil. The weight of this zone is small compared to the wall load, so compute it using 21 kN/m³ as the estimated weighted average for γ:

$$W_f/b = (0.7\text{ m})\left(\frac{0.4\text{ m} + 1.1\text{ m}}{2}\right)(21\text{ kN/m}^3) = 11\text{ kN/m} \qquad \Leftarrow \textit{Answer}$$

Using Equation 5.2:

$$q_{ult} = q = \frac{P/b + W_f/b}{B} - u$$

$$691\text{ kPa} = \frac{P/b + 11\text{ kN/m}}{(0.7\text{ m})} - 0$$

$$\mathbf{P = 473\text{ kN/m}} \qquad \Leftarrow \textit{Answer}$$

Terzaghi's method is still often used, primarily because it is simple and familiar. However, it does not consider special cases, such as rectangular footings, inclined loads, or footings with large depth:width ratios.

Vesić's Bearing Capacity Formulas

The topic of bearing capacity has spawned extensive research and numerous methods of analysis. Skempton (1951), Meyerhof (1953), Brinch Hansen (1961b), DeBeer and Ladanyi (1961), Meyerhof (1963), Brinch Hansen (1970), and many others have con-

tributed. The formula developed in Vesić (1973, 1975) is based on theoretical and experimental findings from these and other sources and is an excellent alternative to Terzaghi. It produces more accurate bearing values and it applies to a much broader range of loading and geometry conditions. The primary disadvantage is its added complexity.

Vesić retained Terzaghi's basic format and added the following additional factors:

s_c, s_q, s_γ = shape factors
d_c, d_q, d_γ = depth factors
i_c, i_q, i_γ = load inclination factors
b_c, b_q, b_γ = base inclination factors
g_c, g_q, g_γ = ground inclination factors

He incorporated these factors into the bearing capacity formula as follows:

$$\boxed{q_{ult} = c'N_c s_c d_c i_c b_c g_c + \sigma'_{zD} N_q s_q d_q i_q b_q g_q + 0.5\gamma' B N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma} \qquad (6.13)$$

Once again, this formula is written in terms of the effective stress parameters c' and ϕ', but also may be used in a total stress analysis by substituting c_T and ϕ_T. For undrained total stress analyses, use $c_T = s_u$ and $\phi_T = 0$.

Terzaghi's formulas consider only vertical loads acting on a footing with a horizontal base with a level ground surface, whereas Vesić's factors allow any or all of these to vary. The notation for these factors is shown in Figure 6.8.

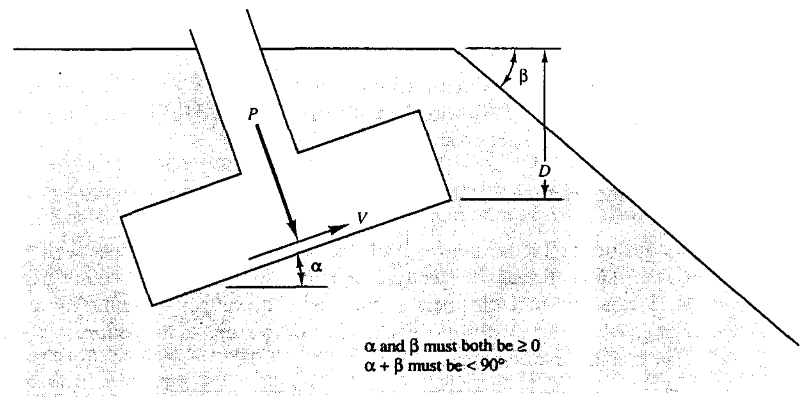


Figure 6.8 Notation for Vesic's load inclination, base inclination, and ground inclination factors. All angles are expressed in degrees.

Shape Factors

Vesić considered a broader range of footing shapes and defined them in his *s* factors:

$$s_c = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) \tag{6.14}$$

$$s_q = 1 + \left(\frac{B}{L}\right)\tan\phi' \tag{6.15}$$

$$s_\gamma = 1 - 0.4\left(\frac{B}{L}\right) \tag{6.16}$$

For continuous footings, $B/L \rightarrow 0$, so s_c, s_q, and s_γ become equal to 1. This means the *s* factors may be ignored when analyzing continuous footings.

Depth Factors

Unlike Terzaghi, Vesić has no limitations on the depth of the footing. This method might even be used for deep foundations, although other methods are probably better for reasons discussed in Chapter 14. The depth of the footing is considered in the following depth factors:

$$d_c = 1 + 0.4\,k \tag{6.17}$$

$$d_q = 1 + 2k\tan\phi'\,(1 - \sin\phi')^2 \tag{6.18}$$

$$d_\gamma = 1 \tag{6.19}$$

For relatively shallow foundations ($D/B \leq 1$), use $k = D/B$. For deeper footings ($D/B > 1$), use $k = \tan^{-1}(D/B)$ with the $\tan^{-1}$ term expressed in radians. Note that this produces a discontinuous function at $D/B = 1$.

Load Inclination Factors

The load inclination factors are for loads that do not act perpendicular to the base of the footing, but still act through its centroid (eccentric loads are discussed in Chapter 8). The variable *P* refers to the component of the load that acts perpendicular to the bottom of the footing, and *V* refers to the component that acts parallel to the bottom.

The load inclination factors are:

$$i_c = 1 - \frac{mV}{Ac'N_c} \geq 0 \tag{6.20}$$

$$i_q = \left[1 - \frac{V}{P + \dfrac{Ac'}{\tan\phi'}}\right]^m \geq 0 \tag{6.21}$$

$$i_\gamma = \left[1 - \frac{V}{P + \dfrac{Ac'}{\tan\phi'}}\right]^{m+1} \geq 0 \tag{6.22}$$

For loads inclined in the *B* direction:

$$m = \frac{2 + B/L}{1 + B/L} \tag{6.23}$$

For loads inclined in the *L* direction:

$$m = \frac{2 + L/B}{1 + L/B} \tag{6.24}$$

Where:

V = applied shear load
P = applied normal load
A = base area of footing
c' = effective cohesion (use $c = s_u$ for undrained analyses)
ϕ' = effective friction angle (use $\phi = 0$ for undrained analyses)
B = foundation width
L = foundation length

If the load acts perpendicular to the base of the footing, the *i* factors equal 1 and may be neglected. The *i* factors also equal 1 when $\phi = 0$.

See the discussion in Chapter 8 for additional information on design of spread footings subjected to applied shear loads.

Base Inclination Factors

The vast majority of footings are built with horizontal bases. However, if the applied load is inclined at a large angle from the vertical, it may be better to incline the base of the footing to the same angle so the applied load acts perpendicular to the base. However, keep in mind that such footings may be difficult to construct.

The base inclination factors are:

$$b_c = 1 - \frac{\alpha}{147°} \tag{6.25}$$

$$b_q = b_\gamma = \left(1 - \frac{\alpha \tan \phi'}{57°}\right)^2 \tag{6.26}$$

If the base of the footing is level, which is the usual case, all of the b factors become equal to 1 and may be ignored.

Ground Inclination Factors

Footings located near the top of a slope have a lower bearing capacity than those on level ground. Vesić's ground inclination factors, presented below, account for this. However, there are also other considerations when placing footings on or near slopes, as discussed in Chapter 8.

$$g_c = 1 - \frac{\beta}{147°} \tag{6.27}$$

$$g_q = g_\gamma = [1 - \tan \beta]^2 \tag{6.28}$$

If the ground surface is level ($\beta = 0$), the g factors become equal to 1 and may be ignored.

Bearing Capacity Factors

Vesić used the following formulas for computing the bearing capacity factors N_q and N_c:

$$N_q = e^{\pi \tan\phi'} \tan^2(45 + \phi'/2) \tag{6.29}$$

$$N_c = \frac{N_q - 1}{\tan \phi'} \quad \text{for } \phi' > 0 \tag{6.30}$$

$$N_c = 5.14 \quad \text{for } \phi = 0 \tag{6.31}$$

Most other authorities also accept Equations 6.29 to 6.31, or others that produce very similar results. However, there is much more disagreement regarding the proper value of N_γ. Relatively small changes in the geometry of the failure surface below the footing can create significant differences in N_γ, especially in soils with high friction angles. Vesić recommended the following formula:

$$N_\gamma = 2\,(N_q + 1) \tan \phi' \tag{6.32}$$

Vesić's bearing capacity factors also are presented in tabular form in Table 6.1. The application of Vesić's formula is illustrated in Example 6.3 later in this chapter.

QUESTIONS AND PRACTICE PROBLEMS

Note: Unless otherwise stated, all foundations have level bases, are located at sites with level ground surfaces, support vertical loads, and are oriented so the top of the foundation is flush with the ground surface.

6.1 List the three types of bearing capacity failures and explain the differences between them.

6.2 A 1.2-m square, 0.4-m deep spread footing is underlain by a soil with the following properties: γ = 19.2 kN/m^3, c' = 5 kPa, ϕ' = 30°. The groundwater table is at a great depth.

- **a.** Compute the ultimate bearing capacity using Terzaghi's method.
- **b.** Compute the ultimate bearing capacity using Vesić's method.

6.3 A 5 ft wide, 8 ft long, 2 ft deep spread footing is underlain by a soil with the following properties: γ = 120 lb/ft^3, c' = 100 lb/ft^2, ϕ' = 28°. The groundwater table is at a great depth. Using Vesić's method, compute the column load required to cause a bearing capacity failure.

6.3 GROUNDWATER EFFECTS

The presence of shallow groundwater affects shear strength in two ways: the reduction of apparent cohesion, and the increase in pore water pressure. Both of these affect bearing capacity, and thus need to be considered.

Apparent Cohesion

Sometimes soil samples obtained from the exploratory borings are not saturated, especially if the site is in an arid or semi-arid area. These soils have additional shear strength due to the presence of apparent cohesion, as discussed in Chapter 3. However, this additional strength will disappear if the moisture content increases. Water may come from landscape irrigation, rainwater infiltration, leaking pipes, rising groundwater, or other sources. Therefore, we do not rely on the strength due to apparent cohesion.

In order to remove the apparent cohesion effects and simulate the "worst case" condition, geotechnical engineers usually wet the samples in the lab prior to testing. This may be done by simply soaking the sample, or, in the case of the triaxial test, by backpressure saturation. However, even with these precautions, the cohesion measured in the laboratory test may still include some apparent cohesion. Therefore, we often perform bearing capacity computations using a cohesion value less than that measured in the laboratory.

Pore Water Pressure

If there is enough water in the soil to develop a groundwater table, and this groundwater table is within the potential shear zone, then pore water pressures will be present, the effective stress and shear strength along the failure surface will be smaller, and the ultimate bearing capacity will be reduced (Meyerhof, 1955). We must consider this effect when conducting bearing capacity computations.

When exploring the subsurface conditions, we determine the current location of the groundwater table and worst-case (highest) location that might reasonably be expected during the life of the proposed structure. We then determine which of the following three cases describes the worst-case field conditions:

- Case 1: $D_w \leq D$
- Case 2: $D < D_w < D + B$
- Case 3: $D + B \leq D_w$

All three cases are shown in Figure 6.9.

We account for the decreased effective stresses along the failure surface by adjusting the effective unit weight, γ', in the third term of Equations 6.4 to 6.6 and 6.13 (Vesić, 1973). The effective unit weight is the value that, when multiplied by the appropriate soil thickness, will give the vertical effective stress. It is the weighted average of the buoyant unit weight, γ_b, and the unit weight, γ, and depends on the position of the groundwater table. We compute γ' as follows:

For Case 1 ($D_w \leq D$):

$$\boxed{\gamma' = \gamma_b = \gamma - \gamma_w} \tag{6.33}$$

For Case 2 ($D < D_w < D + B$):

$$\boxed{\gamma' = \gamma - \gamma_w \left(1 - \left(\frac{D_w - D}{B}\right)\right)} \tag{6.34}$$

For Case 3 ($D + B \leq D_w$; no groundwater correction is necessary):

$$\boxed{\gamma' = \gamma} \tag{6.35}$$

In Case 1, the second term in the bearing capacity formulas also is affected, but the appropriate correction is implicit in the computation of σ_D'.

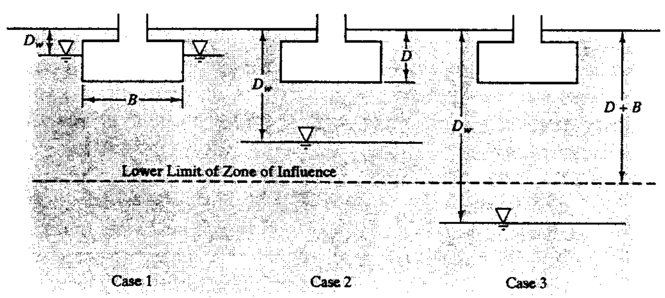


Figure 6.9 Three groundwater cases for bearing capacity analyses.

If a total stress analysis is being performed, do not apply any groundwater correction because the groundwater effects are supposedly implicit within the values of c_T and ϕ_T. In this case, simply use $\gamma' = \gamma$ in the bearing capacity equations, regardless of the groundwater table position.

Example 6.3

A 30-m by 50-m mat foundation is to be built as shown in Figure 6.10. Compute the ultimate bearing capacity.

Solution

Determine groundwater case:

$$D_w = 12 \text{ m};\ D = 10 \text{ m};\ B = 30 \text{ m} \quad D < D_w < D + B \therefore \text{Case 2}$$

Using Equation 6.34:

$$\gamma' = \gamma - \gamma_w \left(1 - \left(\frac{D_w - D}{B}\right)\right) = 18.5 - 9.8 \left(1 - \left(\frac{12 - 10}{30}\right)\right) = 9.4 \text{ kN/m}^3$$

Use Vesić's method with γ' in the third term. Since $c' = 0$, there is no need to compute any of the other factors in the first term of the bearing capacity equation.

For $\phi' = 30°$: $N_q = 18.4$, $N_\gamma = 22.4$ (from Table 6.1)

$$\sigma'_{zD} = \gamma D - u$$
$$= (18.5 \text{ kN/m}^3)(10 \text{ m}) - 0$$
$$= 185 \text{ kPa}$$

$$s_q = 1 + \left(\frac{B}{L}\right) \tan \phi = 1 + \left(\frac{30}{50}\right) \tan 30° = 1.35$$

$$k = \frac{D}{B} = \frac{10}{30} = 0.33$$

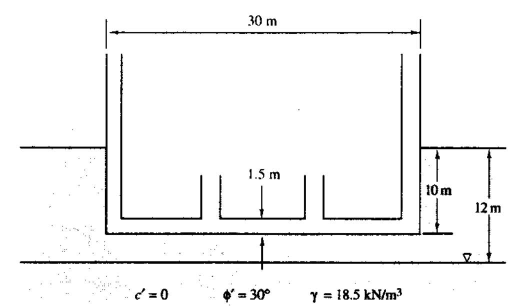


Figure 6.10 Proposed mat foundation for Example 6.3.

$$d_q = 1 + 2k \tan \phi'(1 - \sin \phi')^2$$
$$= 1 + 2(0.33) \tan 30°(1 - \sin 30°)^2$$
$$= 1.10$$

The various i, b, and g factors in Vesić's equation are all equal to 1, and thus may be ignored.

$$q_{ult} = c'N_c s_c i_c b_c g_c + \sigma'_{zD} N_q s_q d_q i_q b_q g_q + 0.5\,\gamma' B N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma$$
$$= 0 + 185\,(18.4)(1.35)(1.10) + 0.5(9.4)(30)(22.4)(0.76)(1)$$
$$= \mathbf{7455\ kPa} \quad \Leftarrow \textit{Answer}$$

Commentary

Because of the large depth and large width, this is a very large ultimate bearing capacity. It is an order of magnitude greater than the bearing pressure produced by the heaviest structures, so there is virtually no risk of a bearing capacity failure. This is always the case with mats on sandy soils. However, mats on saturated clays need to be evaluated using the undrained strength, $c = s_u$, $\phi = 0$), so q_{ult} is much smaller and bearing capacity might be a concern (see the case study of the Fargo Grain Elevator later in this chapter).

6.4 ALLOWABLE BEARING CAPACITY

Nearly all bearing capacity analyses are currently implemented using allowable stress design (ASD) methods. This is true regardless of whether or not load and resistance factor design (LRFD) methods are being used in the structural design. To use ASD, we divide the ultimate bearing capacity by a factor of safety to obtain the *allowable bearing capacity*, q_a:

$$\boxed{q_a = \frac{q_{ult}}{F}} \qquad (6.36)$$

Where:

q_a = allowable bearing capacity
q_{ult} = ultimate bearing capacity
F = factor of safety

We then design the foundation so that the bearing pressure, q, does not exceed the allowable bearing pressure, q_a:

$$\boxed{q \leq q_a} \qquad (6.37)$$

Most building codes do not specify design factors of safety. Therefore, engineers must use their own discretion and professional judgment when selecting F. Items to consider when selecting a design factor of safety include the following:

- **Soil type.** Shear strength in clays is less reliable than that in sands, and more failures have occurred in clays than in sands. Therefore, use higher factors of safety in clays.
- **Site characterization data.** Projects with minimal subsurface exploration and laboratory or in-situ tests have more uncertainty in the design soil parameters, and thus require higher factors of safety. However, when extensive site characterization data is available, there is less uncertainty so lower factors of safety may be used.
- **Soil variability.** Projects on sites with erratic soil profiles should use higher factors of safety than those with uniform soil profiles.
- **Importance of the structure and the consequences of a failure.** Important projects, such as hospitals, where foundation failure would be more catastrophic may use higher factors of safety than less important projects, such as agricultural storage buildings, where cost of construction is more important. Likewise, permanent structures justify higher factors of safety than temporary structures, such as construction falsework. Structures with large height-to-width ratios, such as chimneys or towers, could experience more catastrophic failure, and thus should be designed using higher factors of safety.
- **The likelihood of the design load ever actually occurring.** Some structures, such as grain silos, are much more likely to actually experience their design loads, and thus might be designed using a higher factor of safety. Conversely, office buildings are much less likely to experience the design load, and might use a slightly lower factor of safety.

Figure 6.11 shows ranges of these parameters and typical values of the factor of safety. Geotechnical engineers usually use factors of safety between 2.5 and 3.5 for bearing capacity analyses of shallow foundations. Occasionally we might use values as low as 2.0 or as high as 4.0.

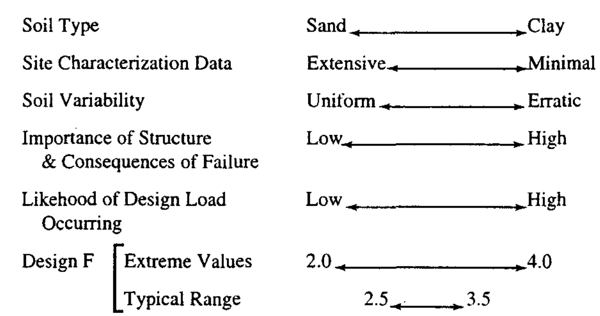


Figure 6.11 Factors affecting the design factor of safety, and typical values of F.

The true factor of safety is probably much greater than the design factor of safety, because of the following:

- The shear strength data are normally interpreted conservatively, so the design values of c and ϕ implicitly contain another factor of safety.
- The service loads are probably less than the design loads.
- Settlement, not bearing capacity, often controls the final design, so the footing will likely be larger than that required to satisfy bearing capacity criteria.
- Spread footings are commonly built somewhat larger than the plan dimensions.

Bearing capacity analyses also can be performed using LRFD, as described in Chapter 21.

Example 6.4

A column has the following design vertical loads: $P_D = 300$ k, $P_L = 140$ k, $P_W = 160$ k will be supported on a spread footing located 3 ft below the ground surface. The underlying soil has an undrained shear strength of 2000 lb/ft^2 and a unit weight of 109 lb/ft^3. The groundwater table is at a depth of 4 ft. Determine the minimum required footing width to maintain a factor of safety of 3 against a bearing capacity failure.

Solution

Determine design working load using Equations 2.1, 2.2, 2.3a, and 2.4a:

$$P_D = 300 \text{ k}$$

$$P_D + P_L = 300 \text{ k} + 140 \text{ k} = 440 \text{ k}$$

$$0.75\,(P_D + P_L + P_W) = 0.75(300 \text{ k} + 140 \text{ k} + 160 \text{ k}) = 450 \text{ k} \leftarrow \text{Controls}$$

$$0.75\,(P_D + P_W) = 0.75\,(300 \text{ k} + 160 \text{ k}) = 345 \text{ k}$$

Using Terzaghi's method:

$$\sigma'_D = \gamma D - u = (109 \text{ lb/ft}^3)(3 \text{ ft}) - 0 = 327 \text{ lb/ft}^2$$

$$\begin{aligned} q_{ult} &= 1.3 s_u N_c + \sigma'_D N_q + 0.4\gamma' B N_\gamma \\ &= 1.3(2000 \text{ lb/ft}^2)(5.7) + (327 \text{ lb/ft}^2)(1) + 0 \\ &= 15{,}147 \text{ lb/ft}^2 \end{aligned}$$

$$q_a = \frac{q_{ult}}{F} = \frac{15{,}147 \text{ lb/ft}^2}{3} = 5049 \text{ lb/ft}^2$$

$$W_f = 3\,B^2(150 \text{ lb/ft}^3) = 450\,B^2$$

$$q_a = q = \frac{P + W_f}{A} - u \rightarrow 5049 = \frac{450{,}000 + 450\,B^2}{B^2} - 0 \rightarrow B = 9.89 \text{ ft}$$

Round off to the nearest 3 in (with SI units, round off to nearest 100 mm):

B = 10 ft 0 in ⇐ ***Answer***

Note: Chapter 8 presents an alternative method of sizing footings subjected to wind or seismic loads.

6.5 SELECTION OF SOIL STRENGTH PARAMETERS

Proper selection of the soil strength parameters, c' and ϕ', can be the most difficult part of performing bearing capacity analyses. Field and laboratory test data is often incomplete and ambiguous, and thus difficult to interpret. In addition, the computed ultimate bearing capacity, q_{ult}, is very sensitive to changes in the shear strength. For example, if a bearing capacity analysis on a sandy soil is based on $\phi' = 40°$, but the true friction angle is only 35° (a 13 percent drop), the ultimate bearing capacity will be 50 to 60 percent less than expected. Thus, it is very important not to overestimate the soil strength parameters. This is why most engineers intentionally use a fairly conservative interpretation of field and laboratory test data when assessing soil strength parameters.

Degree of Saturation and Location of Groundwater Table

As discussed in Section 6.3, soils that are presently dry could become wetted sometime during the life of the structure. It is prudent to design for the worst-case conditions, so we nearly always use the saturated strength when performing bearing capacity analyses, even if the soil is not currently saturated in the field. This produces worst-case values of c' and ϕ'. We can do this by saturating, or at least soaking, the samples in the laboratory before testing them.

However, determining the location of the groundwater table is a different matter. We normally attempt to estimate the highest potential location of the groundwater table and design accordingly using the methods described in Section 6.3. The location of the groundwater table influences the bearing capacity because of its effect on the effective stress, σ'.

Drained vs. Undrained Strength

Footings located on saturated clays generate positive excess pore water pressures when they are loaded, so the most likely time for a bearing capacity failure is immediately after the load is applied. Therefore, we conduct bearing capacity analyses on these soils using the undrained shear strength, s_u.

For footings on saturated sands and gravels, any excess pore water pressures are very small and dissipate very rapidly. Therefore, evaluate such footings using the effective cohesion and effective friction angle, c' and ϕ'.

Saturated intermediate soils, such as silts, are likely to be partially drained, and engineers have varying opinions on how to evaluate them. The more conservative approach is to use the undrained strength, but many engineers use design strengths somewhere between the drained and undrained strength.

Unsaturated soils are more complex and thus more difficult to analyze. If the groundwater table will always be well below the ground surface, many engineers use total stress parameters c_T and ϕ_T based on samples that have been "soaked," but not necessarily fully saturated, in the laboratory. Another option is to treat such soils as being fully saturated and analyze them as such.

Collapse of the Fargo Grain Elevator

One of the most dramatic bearing capacity failures was the Fargo Grain Elevator collapse of 1955. This grain elevator, shown in Figure 6.12, was built near Fargo, North Dakota, in 1954. It was a reinforced concrete structure composed of twenty cylindrical bins and other appurtenant structures, all supported on a 52-ft (15.8 m) wide, 218-ft (66.4 m) long, 2-ft 4-in (0.71 m) thick mat foundation.

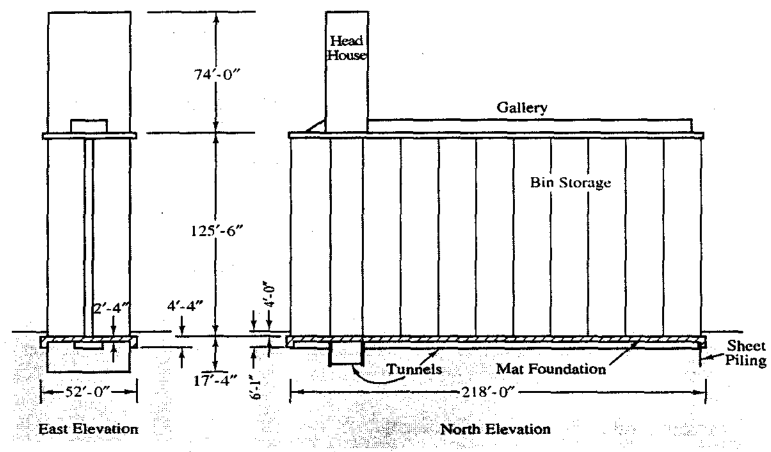


Figure 6.12 Elevation views of the elevator (Nordlund and Deere, 1970; Reprinted by permission of ASCE).

The average net bearing pressure, $q' = q - \sigma_{z0}'$, caused by the weight of the empty structure was 1590 lb/ft^2 (76.1 kPa). When the bins began to be filled with grain in April 1955, q' began to rise, as shown in Figure 6.13. In this type of structure, the live load (i.e., the grain) is much larger than the dead load; so by mid-June, the average net bearing pressure had tripled and reached 4750 lb/ft^2 (227 kPa). Unfortunately, as the bearing pressure rose, the elevator began to settle at an accelerating rate, as shown in Figure 6.14.

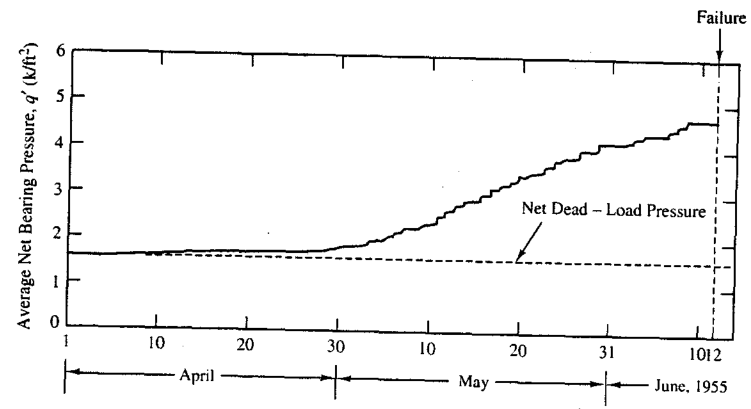


Figure 6.13 Rate of loading (Nordlund and Deere, 1970; Reprinted by permission of ASCE).

Early on the morning of June 12, 1955, the elevator collapsed and was completely destroyed. This failure was accompanied by the formation of a 6 ft (2 m) bulge, as shown in Figure 6.15.

No geotechnical investigation had been performed prior to the construction of the elevator, but Nordlund and Deere (1970) conducted an extensive after-the-fact investigation. They found that the soils were primarily saturated clays with s_u = 600–1000 lb/ft^2 (30–50 kPa). Bearing capacity analyses based on this data indicated a net ultimate bearing capacity of 4110 to 6520 lb/ft^2 (197–312 kPa) which compared well with the q' at failure of 4750 lb/ft^2 (average) and 5210 lb/ft^2 (maximum).

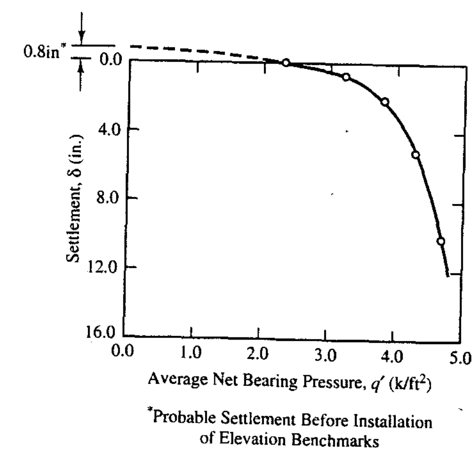


Figure 6.14 Settlement at centroid of mat (Nordlund and Deere, 1970; Reprinted by permission of ASCE).

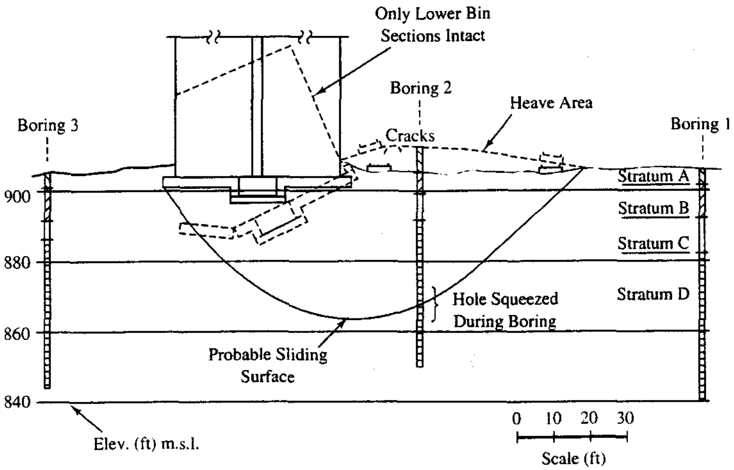


Figure 6.15 Cross section of collapsed elevator (Nordlund and Deere, 1970; Reprinted by permission of ASCE).

The investigation of the Fargo Grain Elevator failure demonstrated the reliability of bearing capacity analyses. Even a modest exploration and testing program would have produced shear strength values that would have predicted this failure. If such an investigation had been performed, and if the design had included an appropriate factor of safety, the failure would not have occurred. However, we should not be too harsh on the designers, since most engineers in the early 1950s were not performing bearing capacity analyses.

Although bearing capacity failures of this size are unusual, this failure was not without precedent. A very similar failure occurred in 1913 at a grain elevator near Winnipeg, Manitoba, approximately 200 miles (320 km) north of Fargo (Peck and Bryant, 1953; White, 1953; Skaftfeld, 1998). This elevator rotated to an inclination of 27° from the vertical when the soil below experienced a bearing capacity failure at an average q' of 4680 lb/ft^2 (224 kPa). The soil profile is very similar to the Fargo site, as is the average q' values at failure.

Geotechnical researchers from the University of Illinois investigated the Winnipeg failure in 1951. They drilled exploratory borings, performed laboratory tests, and computed a net ultimate bearing capacity of 5140 lb/ft^2 (246 kPa). Once again, a bearing capacity analysis would have predicted the failure, and a design with a suitable factor of safety would have prevented it. Curiously, the results of their study were published in 1953, only two years before the Fargo failure. This is a classic example of engineers failing to learn from the mistakes of others.

QUESTIONS AND PRACTICE PROBLEMS

Note: Unless otherwise stated, all foundations have level bases, are located at sites with level ground surfaces, support vertical loads, and are oriented so the top of the foundation is flush with the ground surface.

6.4 A column carrying a vertical downward dead load and live load of 150 k and 120 k, respectively, is to be supported on a 3-ft deep square spread footing. The soil beneath this footing is an undrained clay with s_u = 3000 lb/ft^2 and γ = 117 lb/ft^3. The groundwater table is below the bottom of the footing. Compute the width B required to obtain a factor of safety of 3 against a bearing capacity failure.

6.5 A 120-ft diameter cylindrical tank with an empty weight of 1,900,000 lb (including the weight of the cylindrical mat foundation) is to be built. The bottom of the mat will be at a depth of 2 ft below the ground surface. This tank is to be filled with water. The underlying soil is an undrained clay with s_u = 1000 lb/ft^2 and γ = 118 lb/ft^3, and the groundwater table is at a depth of 5 ft. Using Terzaghi's equations, compute the maximum allowable depth of the water in the tank that will maintain a factor of safety of 3.0 against a bearing capacity failure. Assume the weight of the water and tank is spread uniformly across the bottom of the tank.

6.6 A 1.5-m wide, 2.5-m long, 0.5-m deep spread footing is underlain by a soil with c' = 10 kPa, ϕ' = 32°, γ = 18.8 kN/m^3. The groundwater table is at a great depth. Compute the maximum load this footing can support while maintaining a factor of safety of 2.5 against a bearing capacity failure.

6.7 A bearing wall carries a dead load of 120 kN/m and a live load of 100 kN/m. It is to be supported on a 400-mm deep continuous footing. The underlying soils are medium sands with c' = 0, ϕ' = 37°, γ = 19.2 kN/m^3. The groundwater table is at a great depth. Compute the minimum footing width required to maintain a factor of safety of at least 2 against a bearing capacity failure. Express your answer to the nearest 100 mm.

6.8 After the footing in Problem 6.7 was built, the groundwater table rose to a depth of 0.5 m below the ground surface. Compute the new factor of safety against a bearing capacity failure. Compare it with the original design value of 2 and explain why it is different.

6.9 A 5-ft wide, 8-ft long, 3-ft deep footing supports a downward load of 200 k and a horizontal shear load of 25 k. The shear load acts parallel to the 8-ft dimension. The underlying soils have c_T = 500 lb/ft^2, ϕ_T = 28°, γ = 123 lb/ft^3. Using a total stress analysis, compute the factor of safety against a bearing capacity failure.

6.10 A spread footing supported on a sandy soil has been designed to support a certain column load with a factor of safety of 2.5 against a bearing capacity failure. However, there is some uncertainty in both the column load, P, and the friction angle, ϕ. Which would have the greatest impact on the actual factor of safety: An actual P that is twice the design value, or actual ϕ that is half the design value? Use bearing capacity computations with reasonable assumed values to demonstrate the reason for your response.

6.6 BEARING CAPACITY ANALYSIS IN SOIL—LOCAL AND PUNCHING SHEAR CASES

As discussed earlier, engineers rarely need to compute the local or punching shear bearing capacities because settlement analyses implicitly protect against this type of failure. In addition, a complete bearing capacity analysis would be more complex because of the following:

- These modes of failure do not have well-defined shear surfaces, such as those shown in Figure 6.1, and are therefore more difficult to evaluate.
- The soil can no longer be considered incompressible (Ismael and Vesić, 1981).
- The failure is not catastrophic (refer to Figure 6.2), so the failure load is more difficult to define.
- Scale effects make it difficult to properly interpret model footing tests.

Terzaghi (1943) suggested a simplified way to compute the local shear bearing capacity using the general shear formulas with appropriately reduced values of c' and ϕ':

$$\boxed{c'_{\text{adj}} = 0.67\,c'} \tag{6.38}$$

$$\boxed{\phi'_{\text{adj}} = \tan^{-1}(0.67 \tan \phi')} \tag{6.39}$$

Vesić (1975) expanded upon this concept and developed the following adjustment formula for sands with a relative density, D_r, less than 67%:

$$\boxed{\phi'_{\text{adj}} = \tan^{-1}[(0.67 + D_r - 0.75D_r^2) \tan \phi']} \tag{6.40}$$

Where:

c'_{adj} = adjusted effective cohesion
ϕ'_{adj} = adjusted effective friction angle
D_r = relative density of sand, expressed in decimal form ($0 \le D_r \le 67\%$)

Although Equation 6.40 was confirmed with a few model footing tests, both methods are flawed because the failure mode is not being modeled correctly. However, local or punching shear will normally only govern the final design with shallow, narrow footings on loose sands, so an approximate analysis is acceptable. The low cost of such footings does not justify a more extensive analysis, especially if it would require additional testing.

An important exception to this conclusion is the case of a footing supported by a thin crust of strong soil underlain by very weak soil. This would likely be governed by punching shear and would justify a custom analysis.

6.7 BEARING CAPACITY ON LAYERED SOILS

Thus far, the analyses in this chapter have considered only the condition where c', ϕ', and γ are constant with depth. However, many soil profiles are not that uniform. Therefore, we need to have a method of computing the bearing capacity of foundations on soils where c, ϕ, and γ vary with depth. There are three primary ways to do this:

1. Evaluate the bearing capacity using the lowest values of c', ϕ', and γ in the zone between the bottom of the foundation and a depth B below the bottom, where B = the width of the foundation. This is the zone in which bearing capacity failures occur (per Figure 6.5), and thus is the only zone in which we need to assess the soil parameters. This method is conservative, since some of the shearing occurs in the other, stronger layers. However, many design problems are controlled by settlement anyway, so a conservative bearing capacity analysis may be the simplest and easiest solution. In other words, if bearing capacity does not control the design even with a conservative analysis, there is no need to conduct a more detailed analysis.

or 2. Use weighted average values of c', ϕ', and γ based on the relative thicknesses of each stratum in the zone between the bottom of the footing and a depth B below the bottom. This method could be conservative or unconservative, but should provide acceptable results so long as the differences in the strength parameters are not too great.

or 3. Consider a series of trial failure surfaces beneath the footing and evaluate the stresses on each surface using methods similar to those employed in slope stability analyses. The surface that produces the lowest value of q_{ult} is the critical failure surface. This method is the most precise of the three, but also requires the most effort to implement. It would be appropriate only for critical projects on complex soil profiles.

Example 6.5

Using the second method described above, compute the factor of safety against a bearing capacity failure in the square footing shown in Figure 6.16.

Solution

Weighting factors
Upper stratum: 1.1/1.8 = 0.611
Lower stratum: 0.7/1.8 = 0.389

Weighted values of soil parameters:

$$c' = (0.611)(5\text{ kPa}) + (0.389)(0) = 3\text{ kPa}$$

$$\phi' = (0.611)(32°) + (0.389)(38°) = 34°$$

$$\gamma = (0.611)(18.2\text{ kN/m}^3) + (0.389)(20.1\text{ kN/m}^3) = 18.9\text{ kN/m}^3$$

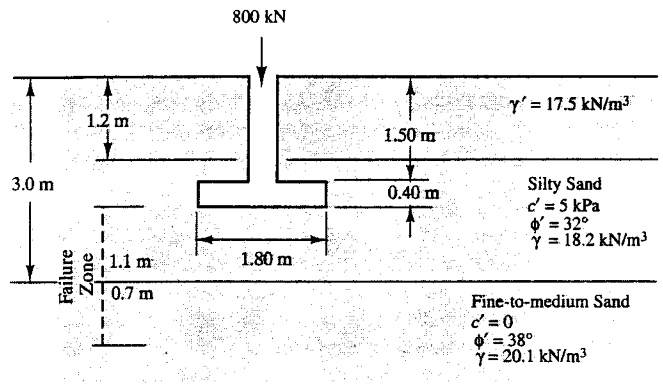


Figure 6.16 Spread footing for Example 6.6.

Groundwater case 1 ($D_w \leq D$)

$$\gamma' = \gamma - \gamma_w = 18.9 \text{ kN/m}^3 - 9.8 \text{ kN/m}^3 = 9.1 \text{ kN/m}^3$$

$$W_f = (1.8 \text{ m})^2(1.5 \text{ m})(17.5 \text{ kN/m}^3) + (1.8 \text{ m})^2(0.4 \text{ m})(23.6 \text{ kN/m}^3) = 116 \text{ kN}$$

$$\begin{aligned}\sigma'_D &= \Sigma\, \gamma H - u \\ &= (17.5 \text{ kN/m}^3)(1.2 \text{ m}) + (18.2 \text{ kN/m}^3)(0.7 \text{ m}) - (9.8 \text{ kN/m}^3)(0.7 \text{ m}) \\ &= 27 \text{ kPa}\end{aligned}$$

$$q = \frac{P + W_f}{A} - u_D = \frac{800 \text{ kN} + 116 \text{ kN}}{(1.8 \text{ m}^2)} - 27 \text{ kPa} = 256 \text{ kPa}$$

Use Terzaghi's formula
For $\phi' = 34°$, $N_c = 52.6$, $N_q = 36.5$, $N_\gamma = 39.6$

$$\begin{aligned}q_{ult} &= 1.3c'N_c + \sigma'_D N_q + 0.4\,\gamma B N_\gamma \\ &= (1.3)(3)(52.6) + (27)(36.5) + (0.4)(9.1)(1.8)(39.6) \\ &= 1450 \text{ kPa}\end{aligned}$$

$$F = \frac{q_{ult}}{q} = \frac{1450 \text{ kPa}}{256 \text{ kPa}} = 5.7 \qquad \Leftarrow \textit{Answer}$$

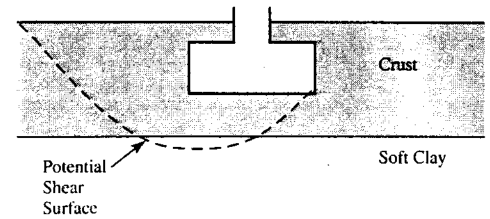


Figure 6.17 Spread footing on a hard crust underlain by softer soils.

The computed factor of safety of 5.7 is much greater than the typical minimum values of 2.5 to 3.5. Therefore, the footing is overdesigned as far as bearing capacity is concerned. However, it is necessary to check settlement (as discussed in Chapter 7) before reducing the size of this footing. ⇐ ***Answer***

Figure 6.17 shows a layered soil condition that deserves special attention: a shallow foundation constructed on a thin crust underlain by softer soils. Such crusts are common in many soft clay deposits, and can be deceiving because they appear to provide good support for foundations. However, the shear surface for a bearing capacity failure would extend into the underlying weak soils. This is especially problematic for wide foundations, such as mats, because they have correspondingly deeper shear surfaces.

This condition should be evaluated using the third method described above. In addition, the potential for a punching shear failure needs to be checked.

6.8 ACCURACY OF BEARING CAPACITY ANALYSES

Engineers have had a few opportunities to evaluate the accuracy of bearing capacity analyses by evaluating full-scale bearing capacity failures in real foundations, and by conducting experimental load tests on full-size foundations.

Bishop and Bjerrum (1960) compiled the results of fourteen case studies of failures or load tests on saturated clays, as shown in Table 6.2, and found the computed factor of safety in each case was within 10 percent of the true value of 1.0. This is excellent agreement, and indicates the bearing capacity analyses are very accurate in this kind of soil. The primary source of error is probably the design value of the undrained shear strength, s_u. In most practical designs, the uncertainty in s_u is probably greater than 10 percent, but certainly well within the typical factor of safety for bearing capacity analyses.

Shallow foundations on sands have a high ultimate bearing capacity, especially when the foundation width, B, is large, because these soils have a high friction angle. Small-model footings, such as those described in Section 6.1, can be made to fail, but it is very difficult to induce failure in large footings on sand. For example, Briaud and

TABLE 6.2 EVALUATIONS OF BEARING CAPACITY FAILURES ON SATURATED CLAYS (Bishop and Bjerrum, 1960).

	Clay Properties					
Locality	Moisture content w	Liquid limit w_L	Plastic limit w_P	Plasticity index I_P	Liquidity index I_L	Computed Factor of Safety F
Loading test, Marmorerá	10	35	15	20	−0.25	0.92
Kensal Green						1.02
Silo, Transcona	50	110	30	80	0.25	1.09
Kippen	50	70	28	42	0.52	0.95
Screw pile, Lock Ryan						1.05
Screw pile, Newport						1.07
Oil tank, Fredrikstad	45	55	25	30	0.67	1.08
Oil tank A, Shellhaven	70	87	25	62	0.73	1.03
Oil tank B, Shellhaven						1.05
Silo, US	40		20	35	1.37	0.98
Loading test, Moss	9		16	8	1.39	1.10
Loading test, Hagalund	68	55	19	18	1.44	0.93
Loading test, Torp	27	24				0.96
Loading test, Rygge	45	37				0.95

Gibbens (1994) conducted static load tests on five spread footings built on a silty fine sand. The widths of these footings ranged from 1 to 3 m, the computed ultimate bearing capacity ranged from 800 to 1400 kPa, and the load-settlement curves are shown in Figure 6.18. The smaller footings show no indiction of approaching the ultimate bearing capacity, even at bearing pressures of twice q_{ult} and settlements of about 150 mm. The larger footings appear to have an ultimate bearing capacity close to q_{ult}, but a settlement of well over 150 mm would be required to reach it. These curves also indicate the design of the larger footings would be governed by settlement, not bearing capacity, so even a conservative evaluation of bearing capacity does not adversely affect the final design. For smaller footings, the design might be controlled by the computed bearing capacity and might be conservative. However, even then the conservatism in the design should not significantly affect the construction cost.

Therefore, we have good evidence to support the claim that bearing capacity analysis methods as presented in this chapter are suitable for the practical design of shallow foundations. Assuming reliable soil strength data is available, the computed values of q_{ult} are either approximately correct or conservative. The design factors of safety discussed in Section 6.4 appear to adequately cover the uncertainties in the analysis.

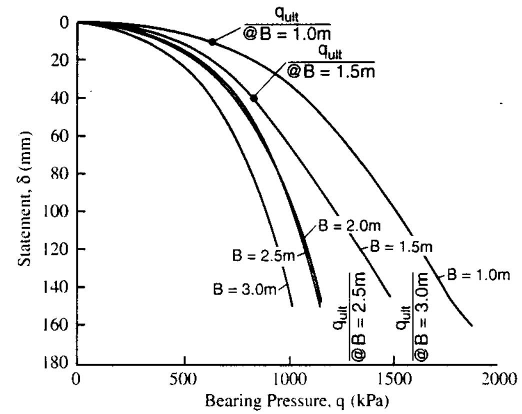


Figure 6.18 Results of static load tests on full-sized spread footings (Adapted from Briaud and Gibbens, 1994).

6.9 BEARING SPREADSHEET

Bearing capacity analyses can easily be performed using a spreadsheet, such as Microsoft Excel. These spreadsheets remove much of the tedium of performing the analyses by hand. For example, to find the required footing width, the engineer can simply input all of the other parameters and, through a rapid process of trial-and-error, find the value of B that produces the required allowable load capacity. Spreadsheets also facilitate "what-if" studies.

A Microsoft Excel spreadsheet called BEARING.XLS has been developed in conjunction with this book. It may be downloaded from the Prentice Hall web site, as described in Appendix B. Figure 6.19 shows a typical screen.

QUESTIONS AND PRACTICE PROBLEMS—SPREADSHEET ANALYSES

6.11 A certain column carries a vertical downward load of 1200 kN. It is to be supported on a 1 m deep, square footing. The soil beneath this footing has the following properties: γ = 20.5 kN/m^3, c' = 5 kPa, ϕ' =36°. The groundwater table is at a depth of 1.5 m below the ground surface. Using the BEARING.XLS spreadsheet, compute the footing width required for a factor of safety of 3.5.

6.12 **a.** Using the BEARING.XLS spreadsheet, solve Problem 6.7.

b. This footing has been built to the size determined in Part a of this problem. Sometime after construction, assume the groundwater table rises to a depth of 0.5 m below the ground surface. Use the spreadsheet to determine the new factor of safety.

6.13 A certain column carries a vertical downward load of 424 k. It is to be supported on a 3-ft deep rectangular footing. Because of a nearby property line, this footing may be no more than

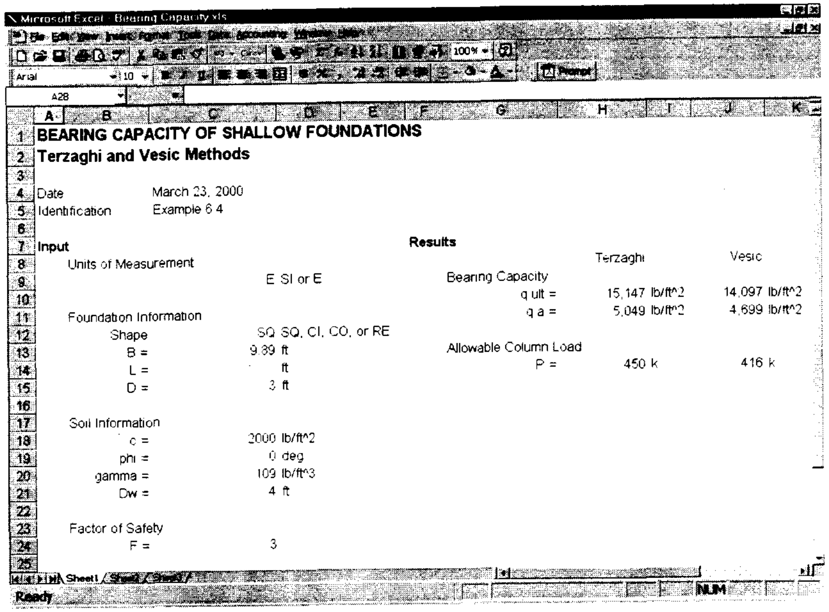


Figure 6.19 Typical screen from BEARING.XLS spreadsheet.

5 ft wide. The soil beneath this footing has the following properties: γ = 124 lb/ft^3, c' = 50 lb/ft^2, ϕ' =34°. The groundwater table is at a depth of 6 ft below the ground surface. Using the BEARING.XLS spreadsheet, compute the footing length required for a factor of safety of 3.0.

SUMMARY

Major Points

1. A *bearing capacity failure* occurs when the soil beneath the footing fails in shear. There are three types of bearing capacity failures: general shear, local shear, and punching shear.
2. Most bearing capacity analyses for shallow foundations consider only the general shear case.
3. A variety of formulas have been developed to compute the *ultimate bearing capacity*, q_{ult}. These include Terzaghi's formulas and Vesić's formulas. Terzaghi's are most appropriate for quick hand calculations, whereas Brinch Hansen's are more useful when greater precision is needed or special loading or geometry conditions must be considered.

4. Shallow groundwater tables reduce the effective stress in the near-surface soils and can therefore adversely affect bearing capacity. Adjustment factors are available to account for this effect.
5. The *allowable bearing capacity*, q_a, is the ultimate bearing capacity divided by a factor of safety. The bearing pressure, q, produced by the unfactored structural load must not exceed q_a.
6. Bearing capacity analyses should be based on the worst-case soil conditions that are likely to occur during the life of the structure. Thus, we typically wet the soil samples in the lab, even if they were not saturated in the field.
7. Bearing capacity analyses on sands and gravels are normally based on the effective stress parameters, c' and ϕ'. However, those on saturated clays are normally based on the undrained strength, s_u.
8. Bearing capacity computations also may be performed for the local and punching shear cases. These analyses use reduced values of c' and ϕ'.
9. Bearing capacity analyses on layered soils are more complex because they need to consider the c' and ϕ' values for each layer.
10. Evaluations of foundation failures and static load tests indicate the bearing capacity analysis methods presented in this chapter are suitable for the practical design of shallow foundations.

Vocabulary

Allowable bearing capacity	Bearing capacity formula	Local shear failure
Apparent cohesion	Bearing capacity failure	Punching shear failure
Bearing capacity factors	General shear failure	Ultimate bearing capacity

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

6.14 Conduct a bearing capacity analysis on the Fargo Grain Elevator (see sidebar) and back-calculate the average undrained shear strength of the soil. The groundwater table is at a depth of 6 ft below the ground surface. Soil strata A and B have unit weights of 110 lb/ft^3; stratum D has 95 lb/ft^3. The unit weight of stratum C is unknown. Assume that the load on the foundation acted through the centroid of the mat.

6.15 Three columns, A, B, and C, are colinear, 500 mm in diameter, and 2.0 m on-center. They have vertical downward loads of 1000, 550, and 700 kN, respectively, and are to be supported on a single, 1.0 m deep rectangular combined footing. The soil beneath this proposed footing has the following properties: γ = 19.5 kN/m^3, c' = 10 kPa, and ϕ' = 31°. The groundwater table is at a depth of 25 m below the ground surface.

a. Determine the minimum footing length, L, and the placement of the columns on the footing that will place the resultant load at the centroid of the footing. The footing must extend at least 500 mm beyond the edges of columns A and C.

b. Using the results from part a, determine the minimum footing width, B, that will maintain a factor of safety of 2.5 against a bearing capacity failure. Show the final design in a sketch.

Hint: Assume a value for B, compute the allowable bearing capacity, then solve for B. Repeat this process until the computed B is approximately equal to the assumed B.

6.16 Two columns, A and B, are to be built 6 ft 0 in apart (measured from their centerlines). Column A has a vertical downward dead load and live loads of 90 k and 80 k, respectively. Column B has corresponding loads of 250 k and 175 k. The dead loads are always present, but the live loads may or may not be present at various times during the life of the structure. It is also possible that the live load would be present on one column, but not the other.

These two columns are to be supported on a 4 ft 0 in deep rectangular spread footing founded on a soil with the following parameters: unit weight = 122 lb/ft^3, effective friction angle = 37°, and effective cohesion = 100 lb/ft^2. The groundwater table is at a very great depth.

a. The location of the resultant of the loads from columns A and B depends on the amount of live load acting on each at any particular time. Considering all of the possible loading conditions, how close could it be to column A? To column B?

b. Using the results of part a, determine the minimum footing length, L, and the location of the columns on the footing necessary to keep the resultant force within the middle third of the footing under all possible loading conditions. The footing does not need to be symmetrical. The footing must extend at least 24 in beyond the centerline of each column.

c. Determine the minimum required footing width, B, to maintain a factor of safety of at least 2.5 against a bearing capacity failure under all possible loading conditions.

d. If the B computed in part c is less than the L computed in part b, then use a rectangular footing with dimensions $B \times L$. If not, then redesign using a square footing. Show your final design in a sketch.

6.17 In May 1970, a 70 ft tall, 20 ft diameter concrete grain silo was constructed at a site in Eastern Canada (Bozozuk, 1972b). This cylindrical silo, which had a weight of 183 tons, was supported on a 3 ft wide, 4 ft deep ring foundation. The outside diameter of this foundation was 23.6 ft, and its weight was about 54 tons. There was no structural floor (in other words, the contents of the silo rested directly on the ground).

The silo was then filled with grain. The exact weight of this grain is not known, but was probably about 533 tons. Unfortunately, the silo collapsed on September 30, 1970 as a result of a bearing capacity failure.

The soils beneath the silo are primarily marine silty clays. Using an average undrained shear strength of 500 lb/ft^2, a unit weight of 80 lb/ft^3, and a groundwater table 2 ft below the ground surface, compute the factor of safety against a bearing capacity failure, then comment on the accuracy of the analysis, considering the fact that a failure did occur.

7

Shallow Foundations–Settlement

From decayed fortunes every flatterer shrinks,
Men cease to build where the foundation sinks.

From the seventeenth-century British opera
The Duchess of Malfi by John Webster (1624)

By the 1950s, engineers were performing bearing capacity analyses as a part of many routine design projects. However, during that period many engineers seemed to have the misconception that any footing designed with an adequate factor of safety against a bearing capacity failure would not settle excessively. Although settlement analysis methods were available, Hough (1959) observed that these analyses, if conducted at all, were considered to be secondary. Fortunately, Hough and others emphasized that bearing capacity and settlement do not go hand-in-hand, and that independent settlement analyses also need to be performed. We now know that settlement frequently controls the design of spread footings, especially when B is large, and that the bearing capacity analysis is, in fact, often secondary.

Although this chapter concentrates on settlements caused by the application of structural loads on the footing, other sources of settlement also may be important. These include the following:

- Settlements caused by the weight of a recently placed fill
- Settlements caused by a falling groundwater table
- Settlements caused by underground mining or tunneling
- Settlements caused by the formation of sinkholes
- Settlements caused by secondary compression of the underlying soils
- Lateral movements resulting from nearby excavations that indirectly cause settlements

a. Determine the minimum footing length, L, and the placement of the columns on the footing that will place the resultant load at the centroid of the footing. The footing must extend at least 500 mm beyond the edges of columns A and C.

b. Using the results from part a, determine the minimum footing width, B, that will maintain a factor of safety of 2.5 against a bearing capacity failure. Show the final design in a sketch.

Hint: Assume a value for B, compute the allowable bearing capacity, then solve for B. Repeat this process until the computed B is approximately equal to the assumed B.

6.16 Two columns, A and B, are to be built 6 ft 0 in apart (measured from their centerlines). Column A has a vertical downward dead load and live loads of 90 k and 80 k, respectively. Column B has corresponding loads of 250 k and 175 k. The dead loads are always present, but the live loads may or may not be present at various times during the life of the structure. It is also possible that the live load would be present on one column, but not the other.

These two columns are to be supported on a 4 ft 0 in deep rectangular spread footing founded on a soil with the following parameters: unit weight = 122 lb/ft^3, effective friction angle = 37°, and effective cohesion = 100 lb/ft^2. The groundwater table is at a very great depth.

a. The location of the resultant of the loads from columns A and B depends on the amount of live load acting on each at any particular time. Considering all of the possible loading conditions, how close could it be to column A? To column B?

b. Using the results of part a, determine the minimum footing length, L, and the location of the columns on the footing necessary to keep the resultant force within the middle third of the footing under all possible loading conditions. The footing does not need to be symmetrical. The footing must extend at least 24 in beyond the centerline of each column.

c. Determine the minimum required footing width, B, to maintain a factor of safety of at least 2.5 against a bearing capacity failure under all possible loading conditions.

d. If the B computed in part c is less than the L computed in part b, then use a rectangular footing with dimensions $B \times L$. If not, then redesign using a square footing. Show your final design in a sketch.

6.17 In May 1970, a 70 ft tall, 20 ft diameter concrete grain silo was constructed at a site in Eastern Canada (Bozozuk, 1972b). This cylindrical silo, which had a weight of 183 tons, was supported on a 3 ft wide, 4 ft deep ring foundation. The outside diameter of this foundation was 23.6 ft, and its weight was about 54 tons. There was no structural floor (in other words, the contents of the silo rested directly on the ground).

The silo was then filled with grain. The exact weight of this grain is not known, but was probably about 533 tons. Unfortunately, the silo collapsed on September 30, 1970 as a result of a bearing capacity failure.

The soils beneath the silo are primarily marine silty clays. Using an average undrained shear strength of 500 lb/ft^2, a unit weight of 80 lb/ft^3, and a groundwater table 2 ft below the ground surface, compute the factor of safety against a bearing capacity failure, then comment on the accuracy of the analysis, considering the fact that a failure did occur.

7

Shallow Foundations–Settlement

From decayed fortunes every flatterer shrinks,
Men cease to build where the foundation sinks.

From the seventeenth-century British opera
The Duchess of Malfi by John Webster (1624)

By the 1950s, engineers were performing bearing capacity analyses as a part of many routine design projects. However, during that period many engineers seemed to have the misconception that any footing designed with an adequate factor of safety against a bearing capacity failure would not settle excessively. Although settlement analysis methods were available, Hough (1959) observed that these analyses, if conducted at all, were considered to be secondary. Fortunately, Hough and others emphasized that bearing capacity and settlement do not go hand-in-hand, and that independent settlement analyses also need to be performed. We now know that settlement frequently controls the design of spread footings, especially when B is large, and that the bearing capacity analysis is, in fact, often secondary.

Although this chapter concentrates on settlements caused by the application of structural loads on the footing, other sources of settlement also may be important. These include the following:

- Settlements caused by the weight of a recently placed fill
- Settlements caused by a falling groundwater table
- Settlements caused by underground mining or tunneling
- Settlements caused by the formation of sinkholes
- Settlements caused by secondary compression of the underlying soils
- Lateral movements resulting from nearby excavations that indirectly cause settlements

7.1 DESIGN REQUIREMENTS

The design of most foundations must satisfy certain settlement requirements, as discussed in Chapter 2. These requirements are usually stated in terms of the allowable total settlement, δ_a, and the allowable differential settlement, δ_{Da}, as follows:

$$\boxed{\delta \leq \delta_a} \quad (7.1)$$

$$\boxed{\delta_D \leq \delta_{Da}} \quad (7.2)$$

Where:

δ = settlement (or total settlement)

δ_a = allowable settlement (or allowable total settlement)

δ_D = differential settlement

δ_{Da} = allowable differential settlement

The design must satisfy both of these requirements.

Note that there is no factor of safety in either equation, because the factor of safety is already included in δ_a and δ_{Da}. The adjective "allowable" always indicates a factor of safety has already been applied. The values of δ_a and δ_{Da} are obtained using the techniques described in Chapter 2. They depend on the type of structure being supported by the foundation, and its tolerance of total and differential settlements. This chapter describes how to compute δ and δ_D for shallow foundations.

Both δ and δ_D must be computed using the unfactored downward load as computed using Equations 2.1 to 2.4. Chapter 8 discusses design loads in more detail.

7.2 OVERVIEW OF SETTLEMENT ANALYSIS METHODS

Analyses Based on Plate Load Tests

Some of the earliest attempts to assess settlement potential in shallow foundations consisted of conducting *plate load tests*. This approach consisted of making an excavation to the depth of the proposed footings, temporarily placing a 1-ft (305 mm) square steel plate on the base of the excavation, and loading it to obtain in-situ load-settlement data. Usually the test continued until a certain settlement was reached, then the foundations were designed using the bearing pressure that corresponded to some specific settlement of the plate, such as 0.5 in or 1.0 cm.

Although plate load tests may seem to be a reasonable approach, experience has proven otherwise. This is primarily because the plate is so much smaller than the foundation, and we cannot always extrapolate the data accurately.

The depth of influence of the plate (about twice the plate width) is much shallower than that of the real footing, as shown in Figure 7.1, so the test reflects only the properties

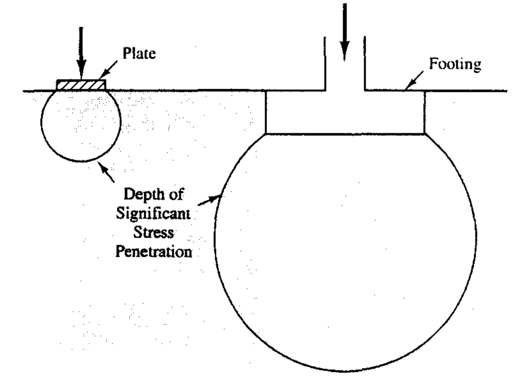


Figure 7.1 The stresses induced by a plate load test do not penetrate very deep into the soil, so its load-settlement behavior is not the same as that of a full-sized footing.

of the near-surface soils. This can introduce large errors, and several complete foundation failures occurred in spite of the use of plate load tests (Terzaghi and Peck, 1967).

In addition, because of the small size of the plate, the test reflects only the properties of the uppermost soils and thus can be very misleading, especially when the soil properties vary with depth. For example, D'Appolonia, et al. (1968) conducted a series of plate load tests in northern Indiana and found that, even after adjusting the test results for scale effects, the plate load tests underestimated the actual settlements by an average of a factor of 2. The test for a certain 12-ft wide footing at the site was in error by a factor of 3.2.

Because of these problems, and because of the development of better methods of testing and analysis, current engineering practice rarely uses plate load tests for foundation design problems. However, these tests are still useful for other design problems, such as those involving wheel loads on pavement subgrades, where the service loads act over smaller areas.

Analyses Based on Laboratory or In-Situ Tests

Today, nearly all settlement analyses are based on the results of laboratory or in-situ tests. The laboratory methods are based on the results of consolidation tests, and thus are primarily applicable to soils that can be sampled and tested without excessive disturbance. This is usually the preferred method for foundations underlain by clayey soils.

In-situ methods are based on standard penetration tests, cone penetration tests, or other in-situ tests. In principle, these methods are applicable to all soil types, but have been most often applied to sandy soils because they are difficult to sample and thus are not well suited to consolidation testing.

This chapter discusses both laboratory and in-situ methods, both of which produce predictions of the total settlement, δ. It also discusses methods of computing the differential settlement, δ_D.

7.3 INDUCED STRESSES BENEATH SHALLOW FOUNDATIONS

The bearing pressure from shallow foundations induces a vertical compressive stress in the underlying soils. We call this stress $\Delta\sigma_z$, because it is the change in stress that is superimposed on the initial vertical stress:

$$\boxed{\Delta\sigma_z = I_\sigma(q - \sigma'_{zD})} \tag{7.3}$$

Where:

$\Delta\sigma_z$ = induced vertical stress due to load from foundation

I_σ = stress influence factor

q = bearing pressure along bottom of foundation

σ_{zD}' = vertical effective stress at a depth D below the ground surface

The q term reflects the increase in vertical stress caused by the applied structural load and the weight of the foundation, while the σ_{zD}' term reflects the reduction in vertical stress caused by excavation of soil to build the foundation. Thus, $\Delta\sigma_z$ reflects the net result of these two effects.

Immediately beneath the foundation, the applied load is distributed across the base area of the foundation, so $I_\sigma = 1$. However, as the load propagates through the ground, it is spread over an increasingly larger area, so $\Delta\sigma_z$ and I_σ decrease with depth, as shown in Figure 7.2.

Boussinesq's Method

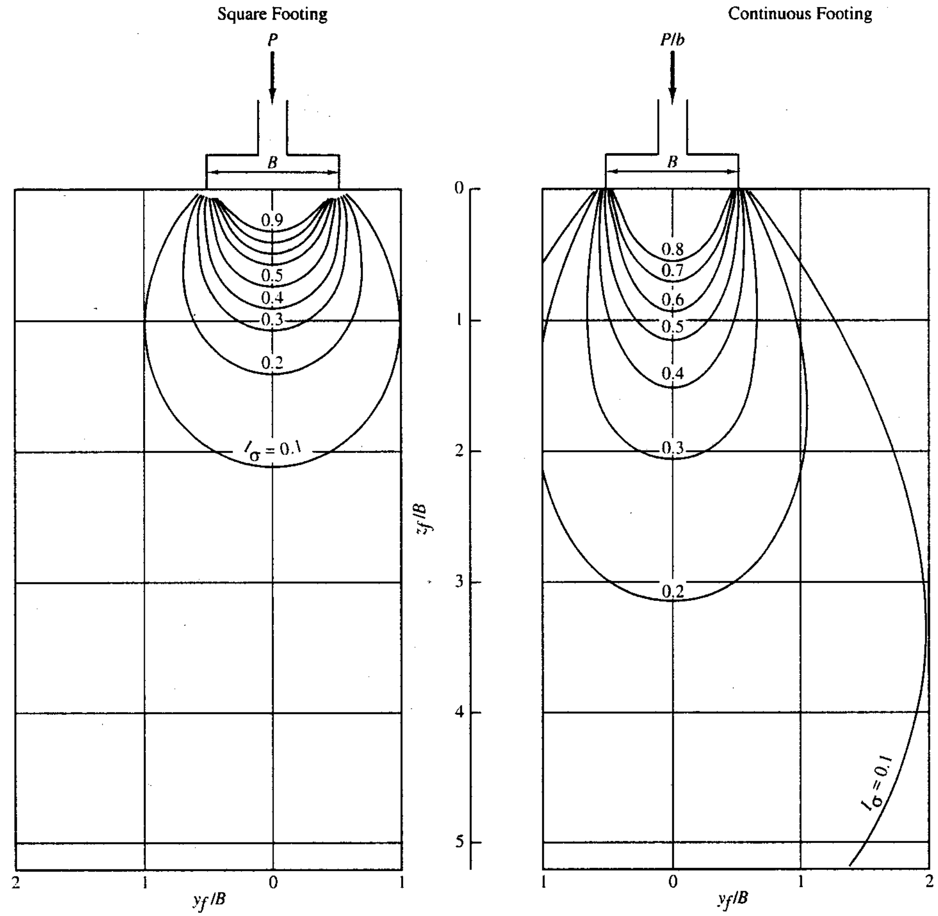
Boussinesq (1885) developed the classic solution for induced stresses in an elastic material due to an applied point load. Newmark (1935) then integrated the Boussinesq equation to produce a solution for I_σ at a depth z_f beneath the corner of a rectangular foundation of width B and length L, as shown in Figure 7.3. This solution produces the following two equations:

If $B^2 + L^2 + z_f^2 < B^2L^2/z_f^2$:

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right)\left(\frac{B^2+L^2+2z_f^2}{B^2+L^2+z_f^2}\right) + \pi - \sin^{-1}\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right] \tag{7.4}$$

Otherwise:

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right)\left(\frac{B^2+L^2+2z_f^2}{B^2+L^2+z_f^2}\right) + \sin^{-1}\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right] \tag{7.5}$$



y_f = horizontal distance from centerline of footing

z_f = depth below bottom of footing

B = footing width

I_σ = stress inflence factor

Figure 7.2 Stress bulbs based on Newmark's solution of Boussinesq's equation for square and continuous footings.

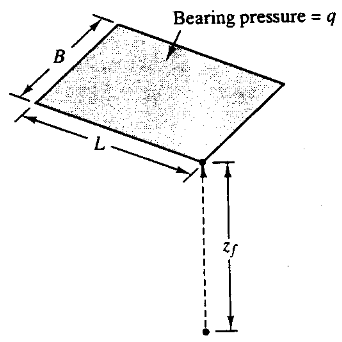


Figure 7.3 Newmark's solution for induced vertical stress beneath the corner of a rectangular footing.

where:

I_σ = strain influence factor at a point beneath the corner of a rectangular foundation

B = width of the foundation

L = length of the foundation

z_f = vertical distance from the bottom of the foundation to the point (always > 0)

q = bearing pressure

Notes:

1. The $\sin^{-1}$ term must be expressed in radians.
2. Newmark's solution is often presented as a single equation with a $\tan^{-1}$ term, but that equation is incorrect when $B^2 + L^2 + z_f^2 < B^2L^2/z_f^2$.
3. It is customary to use B as the shorter dimension and L as the longer dimension, as shown in Figure 7.3.

Example 7.1

A 1.2 m × 1.2 m square footing supports a column load of 250 kN. The bottom of this footing is 0.3 m below the ground surface, the groundwater table is at a great depth, and the unit weight of the soil is 19.0 kN/m³. Compute the induced vertical stress, $\Delta\sigma_z$, at a point 1.5 m below the corner of this footing.

Solution

Unless stated otherwise, we can assume the top of this footing is essentially flush with the ground surface.

$$\sigma'_{zD} = \gamma D - u = (19.0\ \text{kN/m}^3)(0.3\ \text{m}) - 0 = 6\ \text{kPa}$$

$$W_f = (1.2\ \text{m})(1.2\ \text{m})(0.3\ \text{m})(23.6\ \text{kN/m}^3) = 10\ \text{kN}$$

$$q = \frac{P + W_f}{A} - u_D = \frac{250\ \text{kN} + 10\ \text{kN}}{(1.2\ \text{m})^2} - 0 = 181\ \text{kPa}$$

$B^2 + L^2 + z_f^2 = 1.2^2 + 1.2^2 + 1.5^2 = 5.130$
$B^2L^2/z_f^2 = (1.2)^2(1.2)^2/(1.5)^2 = 0.9216$
$B^2 + L^2 + z_f^2 > B^2L^2/z_f^2$. Therefore, use Equation 7.5

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right)\left(\frac{B^2+L^2+2z_f^2}{B^2+L^2+z_f^2}\right) + \sin^{-1}\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right]$$

$$= \frac{1}{4\pi}\left[\left(\frac{2(1.2)(1.2)(1.5)\sqrt{5.130}}{(1.5)^2(5.130)+(1.2)^2(1.2)^2}\right)\left(\frac{(1.2)^2+(1.2)^2+2(1.5)^2}{5.130}\right) + \sin^{-1}\frac{2(1.2)(1.2)(1.5)\sqrt{5.130}}{(1.5)^2(5.130)+(1.2)^2(1.2)^2}\right]$$

$$= 0.146$$

$$\Delta\sigma_z = I_\sigma(q - \sigma'_{zD}) = (0.146)(181 - 6) = \mathbf{26\ kPa} \quad \Leftarrow \textit{Answer}$$

Using superposition, Newmark's solution of Boussinesq's method also can be used to compute $\Delta\sigma_z$ at other locations, both beneath and beyond the footing. This technique is shown in Figure 7.4, and illustrated in Example 7.2.

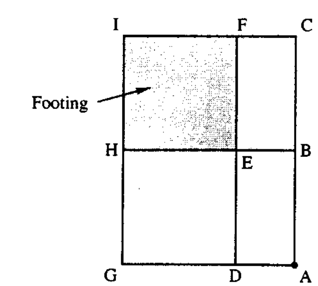


Figure 7.4 Using Newmark's solution and superposition to find the induced vertical stress at any point beneath a rectangular footing.

To Compute Stress at Point A Due to Load from Footing EFHI:
$(\Delta\sigma_v')_A = (\Delta\sigma_v')_{ACGI} - (\Delta\sigma_v')_{ACDF} - (\Delta\sigma_v')_{ABGH} + (\Delta\sigma_v')_{ABDE}$

Example 7.2

Compute the induced vertical stress, $\Delta\sigma_z$, at a point 1.5 m below the center of the footing described in Example 7.1.

Solution

Since Newmark's solution of Boussinesq's equation considers only stresses beneath the corner of a rectangular footing, we must divide the real footing into four equal quadrants. These quadrants meet at the center of the footing, which is where we wish to compute $\Delta\sigma_z$. Since each quadrant imparts one-quarter of the total load on one-quarter of the total base area, the bearing pressure is the same as computed in Example 7.1. However, the remaining computations must be redone using $B = L = 1.2$ m/2 = 0.6 m.

$B^2 + L^2 + z_f^2 = 0.6^2 + 0.6^2 + 1.5^2 = 2.970$
$B^2L^2/z_f^2 = (0.6)^2(0.6)^2/(1.5)^2 = 0.0576$
$B^2 + L^2 + z_f^2 > B^2L^2/z_f^2$. Therefore, use Equation 7.5

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right)\left(\frac{B^2+L^2+2z_f^2}{B^2+L^2+z_f^2}\right) + \sin^{-1}\frac{2BLz_f\sqrt{B^2+L^2+z_f^2}}{z_f^2(B^2+L^2+z_f^2)+B^2L^2}\right]$$

$$= \frac{1}{4\pi}\left[\left(\frac{2(0.6)(0.6)(1.5)\sqrt{2.970}}{(1.5)^2(2.970)+(0.6)^2(0.6)^2}\right)\left(\frac{(0.6)^2+(0.6)^2+2(1.5)^2}{2.970}\right) + \sin^{-1}\frac{2(0.6)(0.6)(1.5)\sqrt{2.970}}{(1.5)^2(2.970)+(0.6)^2(0.6)^2}\right]$$

$$= 0.602$$

Since there are four identical "sub-footings," we must multiply the computed stress by four.

$$\Delta\sigma_z = 4\,I_\sigma(q - \sigma'_{zD})$$
$$= 4(0.602)(181 - 6)$$
$$= \mathbf{42\ kPa} \qquad \Leftarrow \textit{Answer}$$

Westergaard's Method

Westergaard (1938) solved the same problem Boussinesq addressed, but with slightly different assumptions. Instead of using a perfectly elastic material, he assumed one that contained closely spaced horizontal reinforcement members of infinitesimal thickness, such that the horizontal strain is zero at all points. His equation also can be integrated over an area and thus may be used to compute $\Delta\sigma_z$ beneath shallow foundations (Taylor, 1948).

The Westergaard solution produces $\Delta\sigma_z$ values equal to or less than the Boussinesq values. As Poisson's ratio, ν, increases, the computed stress becomes smaller, eventually reaching zero at $\nu = 0.5$. Although some geotechnical engineers prefer Westergaard, at least for certain soil profiles, Boussinesq is more conservative, and probably more appropriate for most problems.

Simplified Method

The Boussinesq equations are tedious to solve by hand, so it is useful to have simple approximate methods of computing stresses in soil for use when a quick answer is needed, or when a computer is not available.

The following approximate formulas compute the induced vertical stress, σ_z, beneath the center of a shallow foundation.[1] They produce answers that are within 5 percent of the Boussinesq values, which is more than sufficient for virtually all practical problems.

For circular foundations (adapted from Poulos and Davis, 1974):

$$\Delta\sigma_z = \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z_f}\right)^2}\right)^{1.50}\right](q - \sigma'_{zD}) \tag{7.6}$$

For square foundations:

$$\Delta\sigma_z = \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z_f}\right)^2}\right)^{1.76}\right](q - \sigma'_{zD}) \tag{7.7}$$

For continuous foundations of width B:

$$\Delta\sigma_z = \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z_f}\right)^2}\right)^{2.60}\right](q - \sigma'_{zD}) \tag{7.8}$$

For rectangular foundations of width B and length L:

$$\Delta\sigma_z = \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z_f}\right)^{1.38+0.62B/L}}\right)^{2.60-0.84B/L}\right](q - \sigma'_{zD}) \tag{7.9}$$

[1] Equations 7.4 and 7.5 compute the induced vertical stress beneath the *corner* of the loaded area, while Equations 7.6 to 7.9 compute it beneath the *center* of the loaded area.

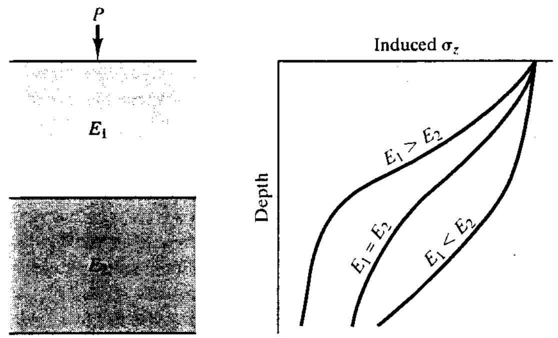


Figure 7.5 Distribution of induced stress, $\Delta\sigma_z$, in layered strata.

Where:

$\Delta\sigma_z$ = induced vertical stress beneath the center of a foundation
B = width or diameter of foundation
L = length of foundation
z_f = depth from bottom of foundation to point
q = bearing pressure
σ_{zD}' = vertical effective stress at a depth D below the ground surface

Stresses in Layered Strata

Thus far, our computations have assumed the soil beneath the foundation is homogeneous, which in this context means the modulus of elasticity, E, shear modulus, G, and Poisson's ratio, ν, are constants. This is an acceptable assumption for many soil profiles, even when there are only slight variations in the soil. However, when the strata beneath the foundation are distinctly stratified, the stress distribution changes.

One common condition consists of a soil layer underlain by a much stiffer bedrock ($E_1 < E_2$), as shown in Figure 7.5. In this case, there is less spreading of the load, so the induced stresses in the soil are greater than those computed by Boussinesq. Conversely, if we have a stiff stratum underlain by one that is softer ($E_1 > E_2$), the load spreading is enhanced and the induced stresses are less than the Boussinesq values.

Usually, engineers do not explicitly consider these effects, but we must be mindful of them to properly interpret settlement analyses. Poulos and Davis (1974) provide methods for computing these stresses in situations where an explicit analysis is warranted. Alternatively, a finite element analysis could be used.

QUESTIONS AND PRACTICE PROBLEMS

7.1 The consolidation settlement computations described in Chapter 3 considered $\Delta\sigma_z$ to be constant with depth. However, in this chapter, $\Delta\sigma_z$ decreases with depth. Why?

7.2 Examine the stress bulbs for square and continuous footings shown in Figure 7.2. Why do those for continuous footings extend deeper than those for square footings?

7.3 A 1500-mm square, 400-mm deep square footing supports a column load of 350 kN. The underlying soil has a unit weight of 18.0 kN/m^3 and the groundwater table is at a depth of 2 m below the ground surface. Compute the change in vertical stress, $\Delta\sigma_z$, beneath the center of this footing at a point 500 mm below the bottom of the footing:

a. Using the simplified method.

b. Using Newmark's integration of Boussinesq's method.

7.4 A column that carries a vertical downward load of 120 k is supported on a 5-ft square, 2-ft deep spread footing. The soil below has a unit weight of 124 lb/ft^3 above the groundwater table and 127 lb/ft^3 below. The groundwater table is at a depth of 8 ft below the ground surface.

a. Develop a plot of the initial vertical stress, σ_{z0}', (i.e., the stress present before the footing was built) vs. depth from the ground surface to a depth of 15 ft below the ground surface.

b. Using the simplified method, develop a plot of $\Delta\sigma_z$ vs. depth below the center of the bottom of the footing and plot it on the diagram developed in part a.

c. Using Newmark's integration of Boussinesq's method, compute $\Delta\sigma_z$ at depths of 2 ft and 5 ft below the center of the bottom of the footing and plot them on the diagram.

7.4 SETTLEMENT ANALYSES BASED ON LABORATORY TESTS

Many different physical processes can contribute to settlement of shallow foundations. Some of these, as listed on the first page of this chapter, are beyond the scope of our discussion. However, the most common source of settlement, and usually the only significant source, is *consolidation*. In Chapter 3 we reviewed the process of consolidation, and noted that it is caused by shifting of the solid particles in response to an increase in the vertical effective stress.

To evaluate consolidation settlement, we begin by drilling exploratory borings into the ground and retrieving undisturbed soil samples. We then bring these samples to a soil mechanics laboratory and conduct *consolidation tests*, which measure the stress–strain properties of the soil. The test results are presented in terms of C_c, C_r, e_0, and σ_m', as discussed in Chapter 3. Finally, we perform settlement analyses based on these parameters.

This approach is usable only if we can obtain good-quality samples suitable for consolidation testing. Such samples can easily be obtained in most clayey or silty soils. However, they are very difficult to obtain in clean sands. Therefore, the methods discussed in this section are most applicable to foundations to be supported on clays or silts. Settlement of foundations on sands is most often evaluated using in-situ tests, as discussed in the next section.

We will cover two methods of using consolidation test data to compute total settlement: The *classical method* and the *Skempton and Bjerrum method.*

Classical Method

The *classical method* of computing total settlement of shallow foundations is based on Terzaghi's theory of consolidation. It assumes settlement is a one-dimensional process, in which all of the strains are vertical.

Computation of Effective Stresses

To apply Terzaghi's theory of consolidation, we need to know both the initial vertical effective stress, σ_{z0}', and the final vertical effective stress, σ_{zf}', at various depths beneath the foundation. The values of σ_{z0}' are computed using the techniques described in Section 3.4, and reflect the pre-construction conditions (i.e., without the proposed foundation). We then compute σ_{zf}' using the following equation:

$$\boxed{\sigma_{zf}' = \sigma_{z0}' + \Delta\sigma_z} \tag{7.10}$$

Foundation Rigidity Effects

According to Figure 7.2, the value of $\Delta\sigma_z$ is greater under the center of a foundation than it is at the same depth under the edge. Therefore, the computed consolidation settlement will be greater at the center.

For example, consider the cylindrical steel water tank in Figure 7.6. The water inside the tank weighs much more than the tank itself, and this weight is supported directly on the plate-steel floor. In addition, the floor is relatively thin, and could be considered to be perfectly flexible. We could compute the settlement beneath both the center and the edge, using the respective values of $\Delta\sigma_z$. The difference between these two is the differential settlement, δ_D, which could then be compared to the allowable differential settlement, δ_{Da}.

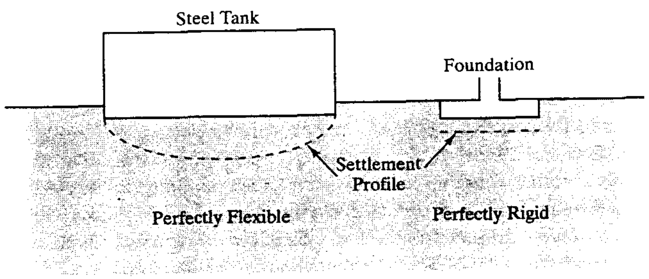


Figure 7.6 Influence of foundation rigidity on settlement. The steel tank on the left is very flexible, so the center settles more than the edge. Conversely, the reinforced concrete footing on the right is very rigid, and thus settles uniformly.

However, such an analysis would not apply to square spread footings, such as the one shown in Figure 7.6, because footings are much more rigid than plate steel tank floors. Although the center of the footing "wants" to settle more than the edge, the rigidity of the footing forces the settlement to be the same everywhere.

A third possibility would be a mat foundation, which is more rigid than the tank, but less rigid than the footing. Thus, there will be some differential settlement between the center and the edge, but not as much as with a comparably-loaded steel tank. Chapter 10 discusses methods of computing differential settlements in mat foundations, and the corresponding flexural stresses in the mat.

When performing settlement analyses on spread footings, we account for this rigidity effect by computing the settlement using $\Delta\sigma_z$ values beneath the center of the footing, then multiplying the result by a *rigidity factor, r.* Table 7.1 presents r-values for various conditions.

Many engineers choose to ignore the rigidity effect (i.e., they use $r = 1$ for all conditions), which is conservative. This practice is acceptable, especially on small or moderate-size structures, and usually has a small impact on construction costs. The use of $r < 1$ is most appropriate when the subsurface conditions have been well defined by extensive subsurface investigation and laboratory testing, which provides the needed data for a more "precise" analysis.

Settlement Computation

We compute the consolidation settlement by dividing the soil beneath the foundation into layers, computing the settlement of each layer, and summing. The top of first layer should be at the bottom of the foundation, and the bottom of the last layer should be at a depth such that $\Delta\sigma_z < 0.10\ \sigma_{z0}'$, as shown in Figure 7.7. Unless the soil is exceptionally soft, the strain below this depth is negligible, and thus may be ignored.

TABLE 7.1 r-VALUES FOR COMPUTATION OF TOTAL SETTLEMENT AT THE CENTER OF A SHALLOW FOUNDATION, AND METHODOLOGY FOR COMPUTING DIFFERENTIAL SETTLEMENT

Foundation rigidity	r for computation of δ at center of foundation	Methodology for computing δ_D
Perfectly flexible (i.e., steel tanks)	1.00	Compute $\Delta\sigma_z$ below edge and use $r = 1$.
Intermediate (i.e., mat foundations)	0.85–1.00, typically about 0.90	Use method described in Chapter 10.
Perfectly rigid (i.e., spread footings)	0.85	Entire footing settles uniformly, so long as bearing pressure is uniform. Compute differential settlement between footings or along length of continuous footing using method described in Section 7.7.

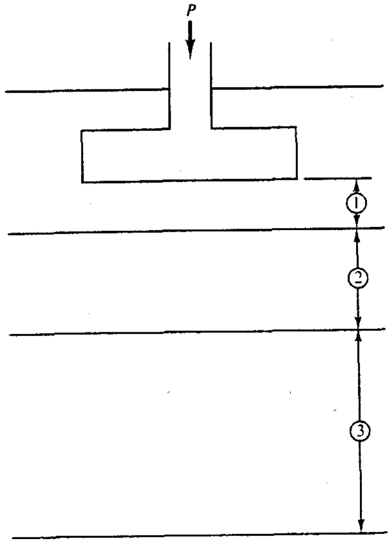


Figure 7.7 The classical method divides the soil beneath the footing into layers. The best precision is obtained when the uppermost layer is thin, and they become progressively thicker with depth.

Since strain varies nonlinearly with depth, analyses that use a large number of thin layers produce a more precise results than those that use a few thick layers. Thus, computer analyses generally use a large number of thin layers. However, this would be too tedious to do by hand, so manual computations normally use fewer layers. For most soils, the guidelines in Table 7.2 should produce reasonable results.

The consolidation settlement equations in Chapter 3 (Equations 3.25–3.27) must be modified by incorporating the r factor, as follows:

For normally consolidated soils ($\sigma_{z0}' \approx \sigma_c'$):

$$\delta_c = r \sum \frac{C_c}{1+e_0} H \log\left(\frac{\sigma_{zf}'}{\sigma_{z0}'}\right) \tag{7.11}$$

For overconsolidated soils—Case I ($\sigma_{zf}' < \sigma_c'$):

$$\delta_c = r \sum \frac{C_r}{1+e_0} H \log\left(\frac{\sigma_{zf}'}{\sigma_{z0}'}\right) \tag{7.12}$$

For overconsolidated soils—Case II ($\sigma_{z0}' < \sigma_c' < \sigma_{zf}'$):

$$\delta_c = r \sum \left[\frac{C_r}{1+e_0} H \log\left(\frac{\sigma_c'}{\sigma_{z0}'}\right) + \frac{C_c}{1+e_0} H \log\left(\frac{\sigma_{zf}'}{\sigma_c'}\right)\right] \tag{7.13}$$

TABLE 7.2 APPROXIMATE THICKNESSES OF SOIL LAYERS FOR MANUAL COMPUTATION OF CONSOLIDATION SETTLEMENT OF SHALLOW FOUNDATIONS

	Approximate Layer Thickness	
Layer Number	Square Footing	Continuous Footing
1	$B/2$	B
2	B	$2B$
3	$2B$	$4B$

1. Adjust the number and thickness of the layers to account for changes in soil properties. Locate each layer entirely within one soil stratum.
2. For rectangular footings, use layer thicknesses between those given for square and continuous footings.
3. Use somewhat thicker layers (perhaps up to 1.5 times the thicknesses shown) if the groundwater table is very shallow.
4. For quick, but less precise, analyses, use a single layer with a thickness of about $3B$ (square footings) or $6B$ (continuous footings).

Where:

δ_c = ultimate consolidation settlement
r = rigidity factor (see Table 7.1)
C_c = compression index
C_r = recompression index
e_0 = initial void ratio
H = thickness of soil layer
σ_{z0}' = initial vertical effective stress at midpoint of soil layer
σ_{zf}' = final vertical effective stress at midpoint of soil layer
σ_c' = preconsolidation stress at midpoint of soil layer

As discussed earlier, many engineers choose to ignore the rigidity effect, which means the r factor drops out of these equations.

Example 7.3

The allowable settlement for the proposed square footing in Figure 7.8 is 1.0 in. Using the classical method, compute its settlement and determine if it satisfies this criterion.

Solution

$$W_f = (6\text{ ft})^2(2\text{ ft})(150\text{ lb/ft}^3) = 10{,}800\text{ lb}$$

$$q = \frac{P+W_f}{A} - u_D = \frac{100{,}000\text{ lb} + 10{,}800\text{ lb}}{(6\text{ ft})^2} - 0 = 3078\text{ lb/ft}^2$$

$$\sigma_{zD}' = (115\text{ lb/ft}^3)(2\text{ ft}) = 230\text{ lb/ft}^2$$

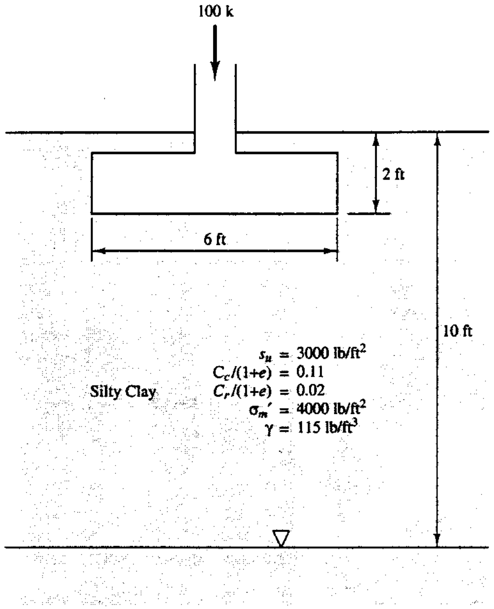


Figure 7.8 Proposed spread footing for Example 7.3.

Using Equations 3.13, 7.7, 7.10, and 7.12 with $r = 0.85$,

		At midpoint of soil layer								
Layer No.	H (ft)	z_f (ft)	σ'_{z0} (lb/ft²)	$\Delta\sigma_z$ (lb/ft²)	σ'_{zf} (lb/ft²)	σ'_c (lb/ft²)	Case	$\frac{C_c}{1+e_0}$	$\frac{C_r}{1+e_0}$	δ_c (in)
1	3.0	1.5	402	2680	3082	4402	OC-I	0.11	0.02	0.54
2	6.0	6.0	920	925	1845	4920	OC-I	0.11	0.02	0.37
3	12.0	15.0	1518	190	1708	5518	OC-I	0.11	0.02	0.13
									$\Sigma =$	1.04

Round off to **δ = 1.0 in** $\Leftarrow$ ***Answer***

$\delta \le \delta_a$, so the settlement criterion has been satisfied $\Leftarrow$ ***Answer***

Note: In this case, $\sigma_m' > q$, so the soil must be overconsolidated case I. Therefore, there is no need to compute σ_c', or to list the $C_c/(1+e_0)$ values.

Example 7.4

The allowable settlement for the proposed continuous footing in Figure 7.9 is 25 mm. Using the classical method, compute its settlement and determine if it satisfies this criterion.

Solution

$$P = P_D + P_L = 40 \text{ kN/m} + 25 \text{ kN/m} = 65 \text{ kN/m}$$

$$W_f/b = (1.2 \text{ m})(0.5 \text{ m})(23.6 \text{ kN/m}^3) = 14 \text{ kN/m}$$

$$q = \frac{P/b + W_f/b}{B} - u_D = \frac{65 \text{ kN/m} + 14 \text{ kN/m}}{1.2 \text{ m}} - 0 = 66 \text{ kPa}$$

$$\sigma'_{zD} = (18.0 \text{ kN/m}^3)(0.5 \text{ m}) = 9 \text{ kPa}$$

Using Equations 3.13, 3.23, 7.7, 7.10, 7.12, and 7.13 with $r = 0.85$,

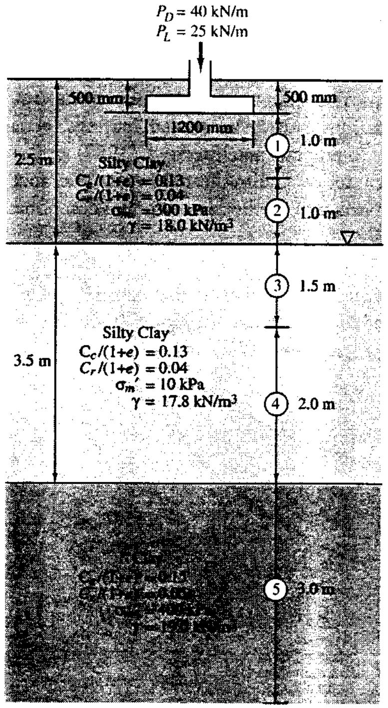


Figure 7.9 Proposed footing for Example 7.4.

Layer No.	H (m)	At midpoint of soil layer z_f (m)	σ'_{z0} (kPa)	$\Delta\sigma_z$ (kPa)	σ'_{zf} (kPa)	σ'_c (kPa)	Case	$\frac{C_c}{1+e_0}$	$\frac{C_r}{1+e_0}$	δ_c (mm)
1	1.0	0.50	18	50	68	318	OC-I	0.13	0.04	20
2	1.0	1.50	36	27	63	336	OC-I	0.13	0.04	8
3	1.5	2.75	51	15	66	61	OC-II	0.13	0.04	10
4	2.0	4.50	65	8	73	75	OC-I	0.13	0.04	3
5	3.0	7.00	87	5	92	487	OC-I	0.16	0.05	3
									$\Sigma =$	44

$\delta = 44$ mm ⇐ ***Answer***

$\delta > \delta_a$, so the settlement criterion has not been satisfied ⇐ ***Answer***

Skempton and Bjerrum Method

The classical method is based on the assumption that settlement is a one-dimensional process in which all of the strains are vertical. This assumption is accurate when evaluating settlement beneath the center of wide fills, but it is less accurate when applied to shallow foundations, especially spread footings, because their loaded area is much smaller. Therefore, Skempton and Bjerrum (1957) presented another method of computing the total settlement of shallow foundations. This method accounts for three-dimensional effects by dividing the settlement into two components:

- *Distortion settlement*, δ_d, (also called *immediate settlement*, *initial settlement*, or *undrained settlement*) is that caused by the lateral distortion of the soil beneath the foundation, as shown in Figure 7.10. This settlement is similar to that which occurs

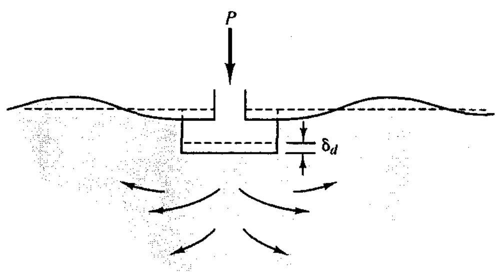


Figure 7.10 Distortion settlement beneath a spread footing.

when a load is placed on a bowl of Jello®, and occurs immediately after application of the load.

- *Consolidation settlement*, δ_c (also known as *primary consolidation* settlement), is that caused by the change in volume of the soil that results from changes in the effective stress.

In addition, Skempton and Bjerrum accounted for differences in the way excess pore water pressures are generated when the soil experiences lateral strain. This is reflected in the parameter ψ, as shown in Figure 7.11.

According to Skempton and Bjerrum's method, the settlement of a shallow foundation is computed as:

$$\boxed{\delta = \delta_d + \psi\delta_c} \tag{7.14}$$

Where:

δ = settlement

δ_d = distortion settlement (per Equation 7.15)

ψ = three-dimensional adjustment factor (from Figure 7.11)

δ_c = consolidation settlement (per Equations 7.4–7.13)

Based on elastic theory, the distortion settlement is:

$$\boxed{\delta_d = \frac{(q - \sigma'_{zD})B}{E_u} I_1 I_2} \tag{7.15}$$

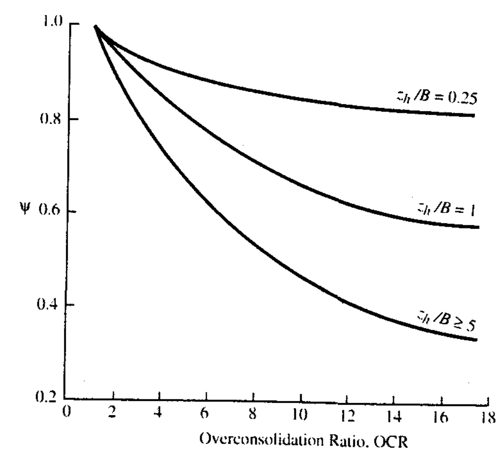


Figure 7.11 ψ factors for Skempton and Bjerrum method (Adapted from Leonards, 1976).

Where:

δ_d = distortion settlement

q = bearing pressure

σ_{zD}' = vertical effective stress at a depth D below the ground surface

B = foundation width

I_1, I_2 = influence factors (per Figure 7.12)

E_u = undrained modulus of elasticity of soil

Janbu, Bjerrum, and Kjaernsli (1956) first proposed this formula. Since then, Christian and Carrier (1978) revised the procedure and Taylor and Matyas (1983) shed additional light on its theoretical basis. The updated influence factors are shown in Figure 7.12.

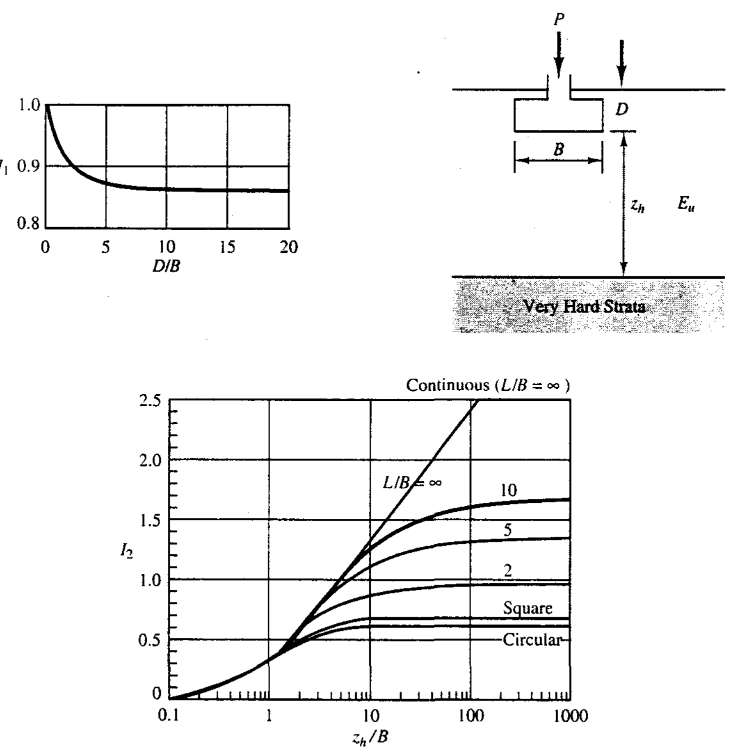


Figure 7.12 Influence factors I_1 and I_2 for use in Equation 7.15 (Adapted from Christian and Carrier, 1978; used with permission). The recommended function for continuous footings is the author's interpretation. z_h is the distance from the bottom of the footing to some very hard strata, such as bedrock. Usually, the z_h/B ratio is very large.

Equation 7.15 implicitly uses a Poisson's ratio of 0.5, which is the usual design value for saturated soils.

The undrained modulus of elasticity, E_u, is the most difficult factor to assess. Soil does not have linear stress-strain properties, so E_u must represent an equivalent linear material. One method of measuring it is to apply incremental loads on an undisturbed sample in a triaxial compression machine and measure the corresponding deformations. Unfortunately, this method tends to underestimate the modulus, sometimes by a large margin (Simons, 1987). It appears that measurements of the modulus are exceptionally sensitive to sample disturbance and the test results can be in error by as much as a factor of 3. Although careful sampling and special laboratory test techniques can reduce this error, direct laboratory testing is generally not a reliable method of measuring the modulus of elasticity.

Normally, geotechnical engineers obtain E_u for this analysis using empirical correlations with the undrained shear strength, s_u. This is convenient because we already have s_u from the bearing capacity analysis, and thus don't need to spend any extra money on additional tests. This correlation is very rough, and the ratio E_u/s_u varies between about 100 and 1500 (Duncan and Buchignani, 1976). However, the following equation should produce δ_d values that are accurate to within ±5 mm, which should be sufficient for nearly all design problems:

$$\boxed{E_u = 300\, s_u} \qquad (7.16)$$

If the soil has a large organic content, the modulus may be smaller than suggested by Equation 7.16 and the distortion settlement will be correspondingly higher. For example, Foott and Ladd (1981) reported $E_u \approx 100\, s_u$ for normally consolidated Taylor River Peat at shear stress levels comparable to those that might be found beneath a spread footing.

If the computed distortion settlement is large, it may be necessary to obtain a more precise assessment of E_u, perhaps using pressuremeter or dilatometer tests. In that case, a more sophisticated analysis, such as that proposed by D'Appolonia, Poulos, and Ladd (1971) may be justified.

Example 7.5

Solve Example 7.3 using the Skempton and Bjerrum method.

Solution

The thickness of the silty clay stratum is not given, but it appears to extend to a great depth. Therefore, use a large value of z_h.

Distortion settlement

$$E_u = 300\, s_u = (300)(3000 \text{ lb/ft}^2) = 900{,}000 \text{ lb/ft}^2$$

$$D/B = 2/6 = 0.3 \quad \therefore I_1 = 0.98$$

$$L/B = 1, z_h/B = \infty \quad \therefore I_2 = 0.7$$

$$\delta_d = \frac{(q - \sigma'_{zD})B}{E_u} I_1 I_2$$

$$= \frac{(3078 \text{ lb/ft}^2 - 230 \text{ lb/ft}^2)(6 \text{ ft})}{900{,}000 \text{ lb/ft}^2}(0.98)(0.7)$$

$$= 0.013 \text{ ft}$$

$$= 0.2 \text{ in}$$

Consolidation settlement

Per Figure 7.11, use $\psi = 0.9$

Total settlement

$$\delta = \delta_d + \psi\delta_c = 0.2 \text{ in} + (0.9)(1.0 \text{ in}) = \mathbf{1.1 \text{ in}} \qquad \Leftarrow \textit{Answer}$$

General Methodology

In summary, the general methodology for computing total settlement of shallow foundations based on laboratory tests is as follows:

1. Drill exploratory borings at the site of the proposed foundations and obtain undisturbed samples of each soil strata. Also use these borings to develop a design soil profile.
2. Perform one or more consolidation tests for each of the soil strata encountered beneath the foundation, and determine the parameters $C_c/(1 + e_0)$, $C_r/(1 + e_0)$, and σ_m' for each strata using the techniques describe in Chapter 3. In many cases, all of the soil may be considered to be a single stratum, so only one set of these parameters is needed. However, if multiple clearly defined strata are present, then each must have its own set of parameters.
3. Divide the soil below the foundation into layers. Usually, about three layers provide sufficient accuracy, but more layers may be necessary if multiple strata are present or if additional precision is required. For square foundations, the bottom of the lowest layer should be about $3B$ to $5B$ below the bottom of the foundation; for continuous foundations it should be about $6B$ to $9B$ below the bottom of the foundation. When using three layers, choose their thicknesses approximately as shown in Table 7.2.
4. Compute σ_{z0}' at the midpoint of each layer.
5. Using any of the methods described in Section 7.3, compute $\Delta\sigma_z$ at the midpoint of each layer. For hand computations, it is usually best to used Equations 7.6 to 7.9.
6. Using Equation 7.10, compute σ_{zf}' at the midpoint of each layer.
7. If the soil might be overconsolidated case II, use Equation 3.23 to compute σ_c' at the midpoint of each layer.

8. Using Equation 7.11, 7.12, or 7.13, compute δ_c for each layer, then sum. Note that the some layers may require the use of one of these equations, while other layers may require another. If using the classical method, this is the computed settlement.

If the analysis is being performed using the Skempton and Bjerrum method, continue with the following steps:

9. Determine the average undrained shear strength, s_u, in the soils between the bottom of the footing and a depth B below the bottom, then use Equation 7.16 to estimate the undrained modulus, E_u.
10. Use Equation 7.15 to compute the distortion settlement, δ_d.
11. Use Figure 7.11 to determine the three-dimensional adjustment coefficient, ψ.
12. Compute the settlement using Equation 7.14.

QUESTIONS AND PRACTICE PROBLEMS

7.5 A proposed office building will include an 8-ft 6-in square, 3-ft deep spread footing that will support a vertical downward load of 160 k. The soil below this footing is an overconsolidated clay (OC case I) with the following engineering properties: $C_c/(1 + e_0) = 0.10$, $C_r/(1 + e_0) = 0.022$, and $\gamma = 113$ lb/ft^3. This soil strata extends to a great depth and the groundwater table is at a depth of 50 ft below the ground surface. Using the classical method with hand computations, determine the total settlement of this footing.

7.6 A 1.0-m square, 0.5-m deep footing carries a downward load of 200 kN. It is underlain by an overconsolidated clay (OC case I) with the following engineering properties: $C_c = 0.20$, $C_r = 0.05$, $e = 0.7$, and $\gamma = 15.0$ kN/m^3 above the groundwater table and 16.0 kN/m^3 below. The groundwater table is at a depth of 1.0 m below the ground surface. The secondary compression settlement is negligible. Using the classical method with hand computations, determine the total settlement of this footing.

7.7 Solve Problem 7.5 using Skempton and Bjerrum's method with $s_u = 3500$ lb/ft^2 and OCR = 3.

7.8 Solve Problem 7.6 using Skempton and Bjerrum's method with $s_u = 200$ kPa and OCR = 2.

7.5 SETTLEMENT SPREADSHEET

Settlement analyses of shallow foundations, as presented in Section 7.4, can be performed using a spreadsheet such as Microsoft® Excel. Such spreadsheet solutions allow the use of much thinner layers, which improves accuracy and flexibility, and allows the analyses to be performed much more quickly. Spreadsheets are especially useful when the engineer wishes to size a footing to satisfy a particular settlement criterion, which is the most common design problem. Such analyses may be performed by quickly trying various values of footing width, B, until the required settlement is obtained.

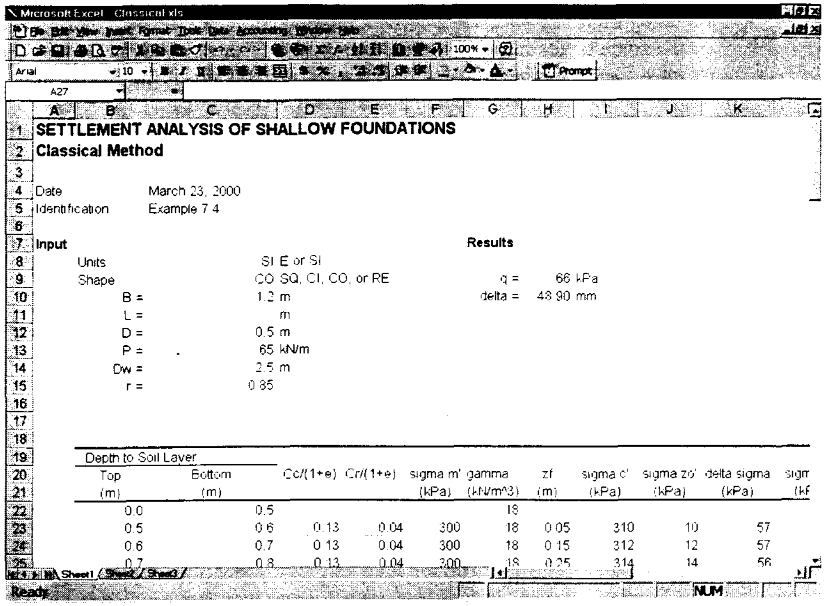


Figure 7.13 Typical screen from SETTLEMENT.XLS spreadsheet.

A Microsoft Excel spreadsheet SETTLEMENT.XLS has been developed in conjunction with this book and may be downloaded from the Prentice Hall website. Downloading instructions are presented in Appendix B. A typical screen is shown in Figure 7.13.

QUESTIONS AND PRACTICE PROBLEMS—SPREADSHEET ANALYSES

7.9 Using the SETTLEMENT.XLS spreadsheet, solve Problem 7.5. Since the soil is stated as being "overconsolidated case I," you should use a large value for σ_m'.

7.10 Using the SETTLEMENT.XLS spreadsheet and the data in Problem 7.5, determine the required footing width to obtain a total settlement of no more than 1.0 in. Select a width that is a multiple of 3 in. Would it be practical to build such a footing?

7.11 Solve Problem 7.6 using the SETTLEMENT.XLS spreadsheet.

7.12 Using the SETTLEMENT.XLS spreadsheet and the data in Problem 7.6, determine the required footing width to obtain a total settlement of no more than 25 mm. Select a width that is a multiple of 100 mm. Would it be practical to build such a footing?

7.13 A proposed building is to be supported on a series of spread footings embedded 36 inches into the ground. The underlying soils consist of silty clays with $C_c/(1+e_0) = 0.12$, $C_r/(1+e_0) = 0.030$, $\sigma_m' = 5000$ lb/ft², and $\gamma = 118$ lb/ft³. This soil strata extends to a great depth and the groundwater table is at a depth of 10 ft below the ground surface. The allowable settlement is 1.0 in. Using the SETTLEMENT.XLS spreadsheet, develop a plot of allowable column load vs. footing width.

7.6 SETTLEMENT ANALYSES BASED ON IN-SITU TESTS

The second category of settlement analysis techniques consists of those based on in-situ tests. Most of these analyses use results from the standard penetration test (SPT) or the cone penetration test (CPT). However, other in-situ tests, especially the dilatometer test (DMT) and the pressuremeter test (PMT) also may be used.

In principle, settlement analyses based on in-situ tests are suitable for all soil types. However, in practice they are most often used on sandy soils, because they are so difficult to sample. Many different methods have been proposed (for example, Meyerhof, 1965, Burland and Burbidge, 1985), but we will consider only Schmertmann's method.

Schmertmann's Method

Schmertmann's method (Schmertmann, 1970, 1978; and Schmertmann, et al., 1978) was developed primarily as a means of computing the settlement of spread footings on sandy soils. It is most often used with cone penetration test (CPT) results, but can be adapted to other in-situ tests. This method was developed from field and laboratory tests, most of which were conducted by the University of Florida. Unlike many of the other methods, which are purely empirical, the Schmertmann method is based on a physical model of settlement, which has been calibrated using empirical data.

Equivalent Modulus of Elasticity

The classical method of computing foundation settlements described the stress–strain properties using the *compression index*, C_c, for normally consolidated soils, or the *recompression index*, C_r, for overconsolidated soils. Both of these parameters are logarithmic, as discussed in Chapter 3.

Schmertmann's method uses the *equivalent modulus of elasticity*, E_s, which is a linear function and thus simplifies the computations. However, soil is not a linear material (i.e., stress and strain are not proportional), so the value of E_s must reflect that of an equivalent unconfined linear material such that the computed settlement will be the same as in the real soil.

The design value of E_s implicitly reflects the lateral strains in the soil. Thus, it is larger than the *modulus of elasticity*, E (also known as *Young's modulus*), but smaller than the *confined modulus*, M.

E_s From Cone Penetration Test (CPT) Results

Schmertmann developed empirical correlations between E_s and the cone resistance, q_c, from a cone penetration test (CPT). This method is especially useful because the CPT

TABLE 7.3 E_S-VALUES FROM CPT RESULTS [Adapted from Schmertmann, et al. (1978), Robertson and Campanella (1989), and other sources.]

Soil Type	USCS Group Symbol	E_s/q_c
Young, normally consolidated clean silica sands (age < 100 years)	SW or SP	2.5–3.5
Aged, normally consolidated clean silica sands (age > 3000 years)	SW or SP	3.5–6.0
Overconsolidated clean silica sands	SW or SP	6.0–10.0
Normally consolidated silty or clayey sands	SM or SC	1.5
Overconsolidated silty or clayey sands	SM or SC	3

provides a continuous plot of q_c vs. depth, so our analysis can model E_s as a function of depth. Table 7.3 presents a range of recommended design values of E_s/q_c. It is usually best to treat all soils as being young and normally consolidated unless there is compelling evidence to the contrary. Such evidence might include:

- Clear indications that the soil is very old. This might be established by certain geologic evidence.
- Clear indications that the soil is overconsolidated. Such evidence would not be based on consolidation tests on the sand (because of soil sampling problems), but might be based on consolidation tests performed on samples from interbedded clay strata. Alternatively, overconsolidation could be deduced from the origin of the soil deposit. For example, lodgement till and compacted fill are clearly overconsolidated.

When interpreting the CPT data for use in Schmertmann's method, do not apply an overburden correction to q_c.

E_s From Standard Penetration Test (SPT) Results

Schmertmann's method also may be used with E_s values based on the standard penetration test. However, these values are not as precise as those obtained from the cone penetration test because:

- The standard penetration test is more prone to error, and is a less precise measurement, as discussed in Chapter 4.
- The standard penetration test provides only a series of isolated data points, whereas the cone penetration test provides a continuous plot.

Nevertheless, SPT data is adequate for many projects, especially those in which the loads are small and the soil conditions are good.

Several direct correlations between E_s and N_{60} have been developed, often producing widely disparate results (Anagnostopoulos, 1990; Kulhawy and Mayne, 1990). This scatter is probably caused in part by the lack of precision in the SPT, and in part to the influence of other factors beside N_{60}. Nevertheless, the following relationship should produce approximate, if somewhat conservative, values of E_s:

$$\boxed{E_s = \beta_0 \sqrt{\mathrm{OCR}} + \beta_1 N_{60}} \tag{7.17}$$

Where:

E_s = equivalent modulus of elasticity
β_0, β_1 = correlation factors from Table 7.4
OCR = overconsolidation ratio
N_{60} = SPT N-value corrected for field procedures

Once again, most analyses should use OCR = 1 unless there is clear evidence of overconsolidation.

E_s From Dilatometer Test (DMT) Results

The dilatometer test (DMT) measures the modulus directly, and this data also could be used as the basis for a Schmertmann analysis. Alternatively, Leonards and Frost (1988) proposed a method of combining CPT and DMT data to assess both compressibility and overconsolidation and use the results in a modified version of Schmertmann's method.

E_s From Pressuremeter Test (PMT) Results

The pressuremeter test also measures the modulus, and also could be used with Schmertmann's method. However, special analysis methods intended specifically for pressuremeter data also are available.

Strain Influence Factor

Schmertmann conducted extensive research on the distribution of vertical strain, ϵ_z, below spread footings. He found the greatest strains do not occur immediately below the footing, as one might expect, but at a depth of 0.5 B to B below the bottom of the footing, where B is the footing width. This distribution is described by the *strain influence factor*, I_ϵ, which is a type of weighting factor. The distribution of I_ϵ with depth has been idealized as two straight lines, as shown in Figure 7.14.

TABLE 7.4 FACTORS FOR EQUATION 7.17.

	β_0		β_1	
Soil Type	(lb/ft^2)	(kPa)	(lb/ft^2)	(kPa)
Clean sands (SW and SP)	100,000	5,000	24,000	1,200
Silty sands and clayey sands (SM and SC)	50,000	2,500	12,000	600

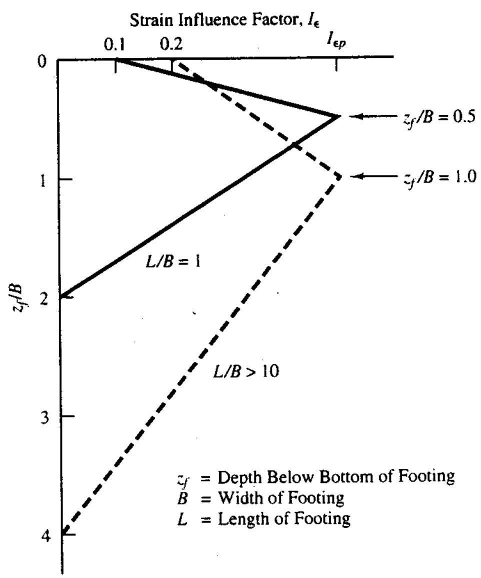


Figure 7.14 Distribution of strain influence factor with depth under square and continuous footings (Adapted from Schmertmann 1978; used with permission of ASCE).

The peak value of the strain influence factor, $I_{\epsilon p}$ is:

$$I_{\epsilon p} = 0.5 + 0.1\sqrt{\frac{q - \sigma'_{zD}}{\sigma'_{zp}}} \tag{7.18}$$

Where:

$I_{\epsilon p}$ = peak strain influence factor
q = bearing pressure
σ'_{zD} = vertical effective stress at a depth D below the ground surface
σ'_{zp} = initial vertical effective stress at depth of peak strain influence factor (for square and circular foundations ($L/B = 1$), compute σ'_{zp} at a depth of $D+B/2$ below the ground surface; for continuous footings ($L/B \geq 10$), compute it at a depth of $D+B$)

The exact value of I_ϵ at any given depth may be computed using the following equations:

Square and circular foundations:

For $z_f = 0$ to $B/2$: $I_\epsilon = 0.1 + (z_f/B)(2I_{\epsilon p} - 0.2)$ (7.19)
For $z_f = B/2$ to $2B$: $I_\epsilon = 0.667 I_{\epsilon p}(2 - z_f/B)$ (7.20)

Continuous foundations ($L/B \geq 10$):

For $z_f = 0$ to B: $I_\epsilon = 0.2 + (z_f/B)(I_{\epsilon p} - 0.2)$ (7.21)
For $z_f = B$ to $4B$: $I_\epsilon = 0.333 I_{\epsilon p}(4 - z_f/B)$ (7.22)

Rectangular foundations ($1 < L/B < 10$):

$$I_\epsilon = I_{\epsilon s} + 0.111(I_{\epsilon c} - I_{\epsilon s})(L/B - 1) \tag{7.23}$$

Where:

z_f = depth from bottom of foundation to midpoint of layer
I_ϵ = strain influence factor
$I_{\epsilon c}$ = I_ϵ for a continuous foundation
$I_{\epsilon p}$ = peak I_ϵ from Equation 7.18
$I_{\epsilon s}$ = I_ϵ for a square foundation ≥ 0

The procedure for computing I_ϵ beneath rectangular foundations requires computation of I_ϵ for each layer using the equations for square foundations (based on the $I_{\epsilon p}$ for square foundations) and the I_ϵ for each layer using the equations for continuous foundations (based on the $I_{\epsilon p}$ for continuous foundations), then combining them using Equation 7.23.

Schmertmann's method also includes empirical corrections for the depth of embedment, secondary creep in the soil, and footing shape. These are implemented through the factors C_1, C_2, and C_3:

$$C_1 = 1 - 0.5\left(\frac{\sigma'_{zD}}{q - \sigma'_{zD}}\right) \tag{7.24}$$

$$C_2 = 1 + 0.2\log\left(\frac{t}{0.1}\right) \tag{7.25}$$

$$C_3 = 1.03 - 0.03\, L/B \geq 0.73 \tag{7.26}$$

Where:

δ = settlement of footing
C_1 = depth factor
C_2 = secondary creep factor (see discussion in Section 7.8)
C_3 = shape factor = 1 for square and circular foundations
q = bearing pressure
σ_{zD}' = effective vertical stress at a depth D below the ground surface
I_ϵ = influence factor at midpoint of soil layer
H = thickness of soil layer
E_s = equivalent modulus of elasticity in soil layer
t = time since application of load (yr) ($t \geq 0.1$ yr)
B = foundation width
L = foundation length

These formulas may be used with any consistent set of units, except that t must be expressed in years. If no time is given, use $t = 50$ yr ($C_2 = 1.54$).

Finally, this information is combined using the following formula to compute the settlement, δ:

$$\delta = C_1 C_2 C_3 (q - \sigma'_{zD}) \Sigma \frac{I_\epsilon H}{E_s} \tag{7.27}$$

Analysis Procedure

The Schmertmann method uses the following procedure:

1. Perform appropriate in-situ tests to define the subsurface conditions.
2. Consider the soil from the base of the foundation to the depth of influence below the base. This depth ranges from $2B$ for square footings or mats to $4B$ for continuous footings. Divide this zone into layers and assign a representative E_s value to each layer. The required number of layers and the thickness of each layer depend on the variations in the E vs. depth profile. Typically 5 to 10 layers are appropriate.
3. Compute the peak strain influence factor, $I_{\epsilon p}$, using Equation 7.18.
4. Compute the strain influence factor, I_ϵ, at the midpoint of each layer. This factor varies with depth as shown in Figure 7.14, but is most easily computed using Equations 7.19 to 7.23.
5. Compute the correction factors, C_1, C_2, and C_3, using Equations 7.24 to 7.26.
6. Compute the settlement using Equation 7.27.

Example 7.6

The results of a CPT sounding performed at McDonald's Farm near Vancouver, British Columbia, are shown in Figure 7.15. The soils at this site consist of young, normally consolidated sands with some interbedded silts. The groundwater table is at a depth of 2.0 m below the ground surface.

A 375 kN/m load is to be supported on a 2.5 m × 30 m footing to be founded at a depth of 2.0 m in this soil. Use Schmertmann's method to compute the settlement of this footing soon after construction and the settlement 50 years after construction.

Solution

Use $E_s = 2.5\, q_c$
1 kPa = 0.01020 kg/cm^2
Depth of influence = $D + 4B = 2.0 + 4(2.5) = 12.0$ m

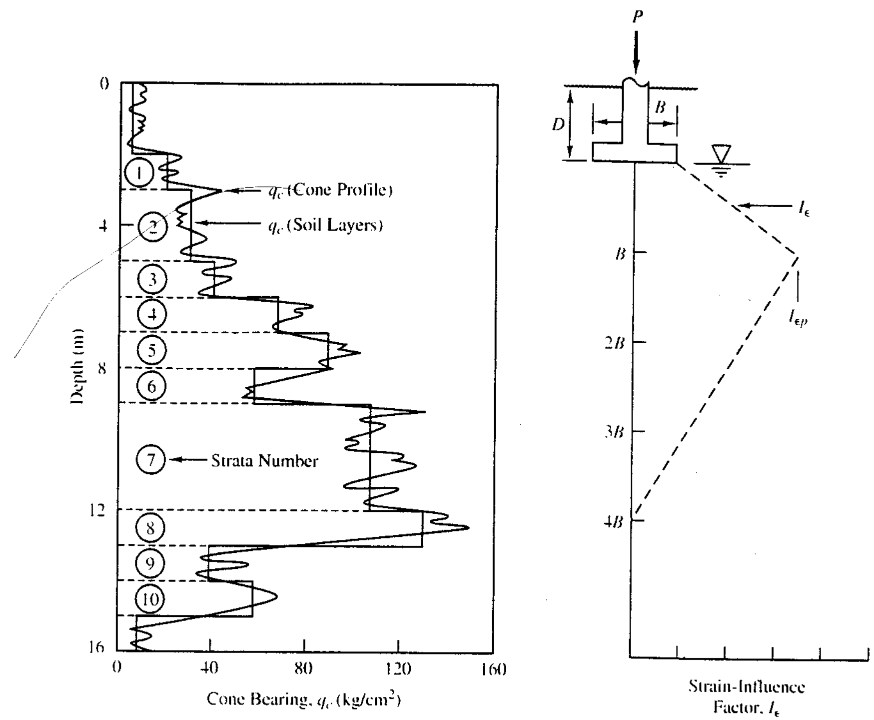


Figure 7.15 CPT results at McDonald's farm (Adapted from Robertson and Campanella, 1988).

Layer No.	Depth (m)	q_c (kg/cm^2)	E_s (kPa)
1	2.0–3.0	20	4,902
2	3.0–5.0	30	7,353
3	5.0–6.0	41	10,049
4	6.0–7.0	68	16,667
5	7.0–8.0	90	22,059
6	8.0–9.0	58	14,216
7	9.0–12.0	108	26,471

$$W_f/b = (2.5 \text{ m})(2.0 \text{ m})(23.6 \text{ kN/m}^3) = 118 \text{ kN/m}$$

$$q = \frac{P/b + W_f/b}{B} - u_D = \frac{375 \text{ kN/m} + 118 \text{ kN/m}}{2.5 \text{ m}} - 0 = 197 \text{ kPa}$$

Use $\gamma = 17 \text{ kN/m}^3$ above groundwater table and 20 kN/m^3 below (from Table 3.2).

$$\sigma'_{zp} \text{ (at } z = D + B) = \Sigma\gamma H - u$$
$$= (17 \text{ kN/m}^3)(2 \text{ m}) + (20 \text{ kN/m}^3)(2.5 \text{ cm}) - (9.8 \text{ kN/m}^3)(2.5 \text{ m})$$
$$= 59 \text{ kPa}$$

$$\sigma'_{zD} = \gamma D = (17)(2) = 34 \text{ kPa}$$

$$I_{\epsilon p} = 0.5 + 0.1\sqrt{\frac{q - \sigma'_{zD}}{\sigma'_{zp}}} = 0.5 + 0.1\sqrt{\frac{197 \text{ kPa} - 34 \text{ kPa}}{59 \text{ kPa}}} = 0.666$$

Layer No.	E_s (kPa)	z_f (m)	I_ϵ	H (m)	$I_\epsilon H/E_s$
1	4,902	0.5	0.293	1.0	5.98×10^{-5}
2	7,353	2.0	0.573	2.0	15.58×10^{-5}
3	10,049	3.5	0.577	1.0	5.74×10^{-5}
4	16,667	4.5	0.488	1.0	2.93×10^{-5}
5	22,059	5.5	0.399	1.0	1.81×10^{-5}
6	14,216	6.5	0.310	1.0	2.18×10^{-5}
7	26,471	8.5	0.133	3.0	1.51×10^{-5}
				$\Sigma =$	35.73×10^{-5}

$$C_1 = 1 - 0.5\left(\frac{\sigma'_{zD}}{q - \sigma'_{zD}}\right) = 1 - 0.5\left(\frac{34 \text{ kPa}}{197 \text{ kPa} - 34 \text{ kPa}}\right) = 0.896$$

$$C_3 = 1.03 - 0.03\, L/B \geq 0.73$$
$$= 1.03 - 0.03\,(30/2.5)$$
$$= 0.67$$
$$\text{Use } C_3 = 0.73$$

At $t = 0.1$ yr:

$$C_2 = 1$$

$$\delta = C_1\, C_2\, C_3\, (q - \sigma'_{zD})\, \Sigma \frac{I_\epsilon H}{E_s}$$
$$= (0.896)(1)(0.73)(197 - 34)(35.73 \times 10^{-5})$$
$$= 0.038 \text{ m}$$
$$= \mathbf{38\ mm} \quad \Leftarrow \textit{Answer}$$

At $t = 50$ yr:

$$C_2 = 1 + 0.2\log\left(\frac{t}{0.1}\right) = 1 + 0.2\log\left(\frac{50}{0.1}\right) = 1.54$$

$$\delta = 38\,(1.54) = \mathbf{59\ mm} \quad \Leftarrow \textit{Answer}$$

Simplified Schmertmann Method

If E_s is constant with depth between the bottom of the foundation and the depth of influence ($2\ z_f/B$ for square and circular foundations to $4\ z_f/B$ for continuous footings), then Equation 7.24 simplifies to the following:

For square and circular foundations ($L/B = 1$):

$$\delta = \frac{C_1\, C_2\, C_3\, (q - \sigma'_{zD})(I_{\epsilon p} + 0.025)B}{E_s} \tag{7.28}$$

For continuous footings ($L/B \geq 10$):

$$\delta = \frac{C_1\, C_2\, C_3\, (q - \sigma'_{zD})(2I_{\epsilon p} + 0.1)B}{E_s} \tag{7.29}$$

Equations 7.28 and 7.29 are especially useful when only minimal subsurface data is available, as is often the case with the SPT, and the soil appears to be fairly homogeneous.

Example 7.7

A 200-k column load is to be supported on a 3-ft deep square footing underlain by a silty sand with an average N_{60} of 28 and $\gamma = 120 \text{ lb/ft}^3$. The groundwater table is at a depth of 50 ft below the ground surface. The allowable total settlement is 0.75 in. Using the simplified Schmertmann method, determine the required footing width.

Solution

$$E_s = \beta_0\sqrt{OCR} + \beta_1 N_{60} = 50{,}000\sqrt{1} + (12{,}000)(28) = 386{,}000 \text{ lb/ft}^2$$
$$\sigma'_{zD} = (120 \text{ lb/ft}^3)(3 \text{ ft}) = 360 \text{ lb/ft}^2$$

Estimate $B = 7$ ft

$$W_f = (7\text{ ft})^2(3\text{ ft})(150\text{ lb/ft}^3) = 22{,}000\text{ lb}$$

$$q = \frac{P + W_f}{B^2} - u = \frac{200{,}000\text{ lb} + 22{,}000\text{ lb}}{(7\text{ ft})^2} - 0 = 4{,}530\text{ lb/ft}^2$$

$$C_1 = 1 - 0.5\left(\frac{\sigma'_{zD}}{q - \sigma'_{zD}}\right) = 1 - 0.5\left(\frac{360\text{ lb/ft}^2}{4530\text{ lb/ft}^2 - 360\text{ lb/ft}^2}\right) = 0.957$$

$$C_2 = 1 + 0.2\log\left(\frac{t}{0.1}\right) = 1 + 0.2\log\left(\frac{50\text{ yr}}{0.1}\right) = 1.54$$

$$C_3 = 1$$

$$\sigma'_{zp}\ @z = 6.5\text{ ft} = (120\text{ lb/ft}^3)(6.5\text{ ft}) = 780\text{ lb/ft}^2$$

$$I_{\epsilon p} = 0.5 + 0.1\sqrt{\frac{1 - \sigma'_D}{\sigma'_{zp}}} = 0.5 + 0.1\sqrt{\frac{4530\text{ lb/ft}^2 - 360\text{ lb/ft}^2}{780\text{ lb/ft}^2}} = 0.731$$

$$\delta = \frac{C_1 C_2 C_3 (q - \sigma'_{zD})(I_{\epsilon p} + 0.025)B}{E_s}$$

$$\frac{0.75\text{ in}}{12\text{ in/ft}} = \frac{(0.957)(1.54)(1)\left(\frac{200{,}000 + 450B^2}{B^2} - 360\text{ lb/ft}^2\right)(0.731 + 0.025)B}{386{,}000\text{ lb/ft}^2}$$

$$B = 9\text{ ft }3\text{ in}$$

Reevaluate with $B = 8$ ft 9 in

$$W_f = (8.75\text{ ft})^2(3\text{ ft})(150\text{ lb/ft}^3) = 34{,}500\text{ lb}$$

$$q = \frac{P + W_f}{B^2} - u = \frac{200{,}000\text{ lb} + 34{,}500\text{ lb}}{(8.75\text{ ft})^2} - 0 = 3062\text{ lb/ft}^2$$

$$C_1 = 1 - 0.5\left(\frac{\sigma'_{zD}}{q - \sigma'_{zD}}\right) = 1 - 0.5\left(\frac{360\text{ lb/ft}^2}{3062\text{ lb/ft}^2 - 360\text{ lb/ft}^2}\right) = 0.933$$

$$\sigma'_{zp}\ @z = 7.37\text{ ft} = (120\text{ lb/ft}^3)(7.37\text{ ft}) = 885\text{ lb/ft}^2$$

$$I_{\epsilon p} = 0.5 + 0.1\sqrt{\frac{q - \sigma'_D}{\sigma'_{zp}}} = 0.5 + 0.1\sqrt{\frac{3062\text{ lb/ft}^2 - 360\text{ lb/ft}^2}{885\text{ lb/ft}^2}} = 0.675$$

$$\delta = \frac{C_1 C_2 C_3 (q - \sigma'_{zD})(I_{\epsilon p} + 0.025)B}{E_s}$$

$$\frac{0.75\text{ in}}{12\text{ in/ft}} = \frac{(0.933)(1.54)(1)\left(\frac{200{,}000 + 450B^2}{B^2} - 360\text{ lb/ft}^2\right)(0.675 + 0.025)B}{386{,}000\text{ lb/ft}^2}$$

$$B = 8.75\text{ ft}$$

Use $B = 8$ **ft 9 in** ⇐ ***Answer** (as a multiple of 3 in)*

Application to Mat Foundations

Schmertmann's method was developed primarily for spread footings, so the various empirical data used to calibrate the method have been developed with this type of foundation in mind. In principle, the method also may be used with mat foundations. However, it tends to overestimate the settlement of mats because their depth of influence is much greater and the equivalent modulus values at these depths is larger than predicted by the methods described earlier in this section.

Therefore, when applying Schmertmann's method to mat foundations, it is best to progressively increase the E_s values with depth, such that E_s at 30 m (100 ft) is about three times that predicted by the methods described earlier in this section.

QUESTIONS AND PRACTICE PROBLEMS

Note: All depths for CPT and SPT data are measured from the ground surface.

7.14 A 250-k column load is to be supported on a 9 ft × 9 ft square footing embedded 2 ft below the ground surface. The underlying soil is a silty sand with an average N_{60} of 32 and a unit weight of 129 lb/ft³. The groundwater table is at a depth of 35 ft. Using the simplified Schmertmann method, compute the settlement of this footing at $t = 50$ yr.

7.15 A 190-k column load is to be supported on a 10-ft square, 3-ft deep spread footing underlain by young, normally consolidated sandy soils. The results of a representative CPT sounding at this site are as follows:

Depth (ft)	q_c (kg/cm²)
0.0–6.0	30
6.0–10.0	51
10.0–18.0	65
18.0–21.0	59
21.0–40.0	110

The groundwater table is at a depth of 15 ft; the unit weight of the soil is 124 lb/ft^3 above the groundwater table and 130 lb/ft^3 below. Using Schmertmann's method with hand computations, compute the total settlement of this footing 30 years after construction.

7.16 A 650-kN column load is supported on a 1.5-m wide by 2.0-m long by 0.5 m deep spread footing. The soil below is a well graded, normally consolidated sand with γ = 17.0 kN/m^3 and the following SPT N_{60} values:

Depth (m)	1.0	2.0	3.0	4.0	5.0
N_{60}	12	13	13	18	22

The groundwater table is at a depth of 25 m. Using Schmertmann's method and hand computations, compute the total settlement at t = 30 yr.

7.7 SCHMERTMANN SPREADSHEET

The SETTLEMENT.XLS spreadsheet described earlier in this chapter can be modified to compute settlements using the Schmertmann method. This has been done, and a spreadsheet called SCHMERTMANN.XLS is available from the Prentice Hall website. Figure 7.16 is a typical screen from this spreadsheet.

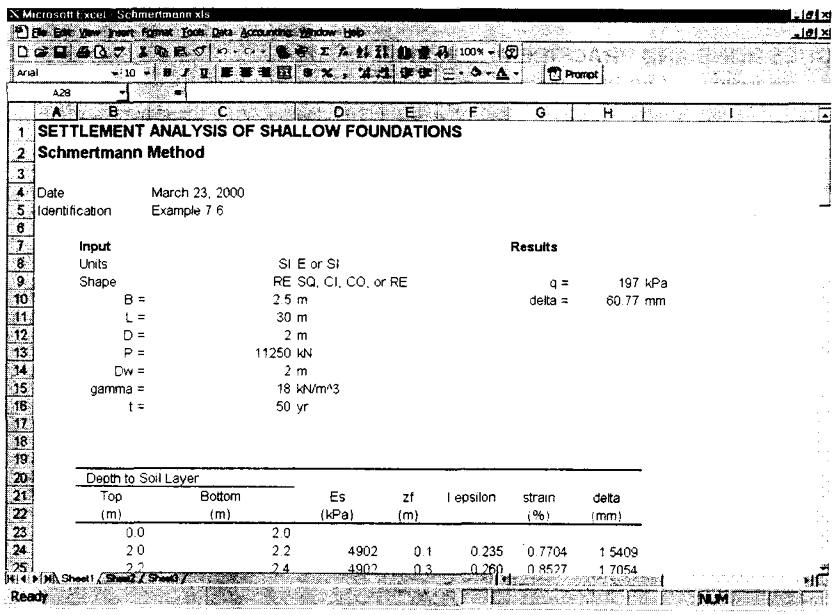


Figure 7.16 Typical screen from SCHMERTMANN.XLS spreadsheet.

QUESTIONS AND PRACTICE PROBLEMS—SPREADSHEET ANALYSES

Note: All depths for CPT and SPT data are measured from the ground surface.

7.17 Solve Problem 7.14 using the SCHMERTMANN.XLS spreadsheet.

7.18 Solve Problem 7.15 using the SCHMERTMANN.XLS spreadsheet.

7.19 Solve Problem 7.16 using the SCHMERTMANN.XLS spreadsheet.

7.20 A 300-k column load is to be supported on a 10-ft square, 4-ft deep spread footing. Cone penetration tests have been conducted at this site, and the results are shown in Figure 7.15. The groundwater table is at a depth of 6 ft, γ = 121 lb/ft^3, and γ_{sat} = 125 lb/ft^3.

a. Compute the settlement of this footing using the SCHMERTMANN.XLS spreadsheet.

b. The design engineer is considering the use of vibroflotation to densify the soils at this site (see discussion in Chapter 19). This process would increase the q_c values by 70 percent, and make the soil slightly overconsolidated. The unit weights would increase by 5 lb/ft^3. Use the spreadsheet to compute the settlement of a footing built and loaded after densification by vibroflotation.

7.21 A proposed building is to be supported on a series of spread footings embedded 36 inches into the ground. The underlying soils consist of silty sands with N_{60} = 30, an estimated overconsolidation ratio of 2, and γ = 118 lb/ft^3. This soil strata extends to a great depth and the groundwater table is at a depth of 10 ft below the ground surface. The allowable settlement is 1.0 in. Using the SCHMERTMANN.XLS spreadsheet, develop a plot of allowable column load vs. footing width.

7.8 SETTLEMENT OF SHALLOW FOUNDATIONS ON STRATIFIED SOILS

When computing the settlement of shallow foundations on soil profiles that are primarily clays or silts, we normally use the methods based on laboratory tests as discussed in Section 7.4. Conversely, when the soil profile consists primarily of sands, we normally use methods based on in-situ tests, as discussed in Section 7.6. However, when the profile is stratified and includes both types of soil, it can be difficult to determine which method to use.

If the soil profile consists predominantly of clays and silts, then it is probably best to use the methods described in Section 7.4. Determine the values of $C_c/(1 + e_0)$ and $C_r/(1 + e_0)$ for the clayey and silty strata using laboratory consolidation tests, and those for the sandy strata using Table 3.7.

Conversely, if the profile is primarily sandy, then it is probably better to use Schmertmann's method. The equivalent modulus, E_s, for normally consolidated clayey layers may be computed using the following equation:

$$E_s = \frac{2.30\,\sigma'_z}{C_c/(1+e_0)} \qquad (7.30)$$

For overconsolidated soils, substitute C_r for C_c in Equation 7.30.

Another option is to conduct two parallel analyses, one for the clayey strata using laboratory test data, and another for the sandy strata using in-situ data, and adding the two computed settlements.

7.9 DIFFERENTIAL SETTLEMENT

Differential settlement, δ_D, is the difference in settlement between two foundations, or the difference in settlement between two points on a single foundation. Excessive differential settlement is troublesome because it distorts the structure and thus introduces serviceability problems, as discussed in Chapter 2.

Normally we design the foundations for a structure such that all of them have the same computed total settlement, δ. Therefore, in theory, there should be no differential settlement. However, in reality differential settlements usually occur anyway. There are many potential sources of these differential settlements, including:

- **Variations in the soil profile.** For example, part of structure might be underlain by stiff natural soils, and part by a loose, uncompacted fill. Such a structure may have excessive differential settlement because of the different compressibility of these soil types, and possibly because of settlement due to the weight of the fill. This source of differential settlements is present to some degree on nearly all sites because of the natural variations in all soils, and is usually the most important source of differential settlement.
- **Variations in the structural loads.** The various foundations in a structure are designed to accommodate different loads according to the portion of the structure they support. Normally each would be designed for the same total settlement under its design load, so in theory the differential settlement should be zero. However, the ratio of actual load to design load may not be same for all of the foundations. Thus, those with a high ratio will settle more than those with a low ratio.
- **Design controlled by bearing capacity.** The design of some of the foundations may have been controlled by bearing capacity, not settlement, so even the design settlement may be less than that of other foundations in the same structure.
- **Construction tolerances.** The as-built dimensions of the foundations will differ from the design dimensions, so their settlement behavior will vary accordingly.

The rigidity of the structure also has an important influence on differential settlements. Some structures, such as the steel frame in Figure 7.17, are very flexible. Each foundation acts nearly independent of the others, so the settlement of one foundation has almost no impact on the other foundations. However, other structures are much stiffer, perhaps because of the presence of shear walls or diagonal bracing. The braced steel frame structure in Figure 7.18 is an example of a more rigid structure. In this case, the

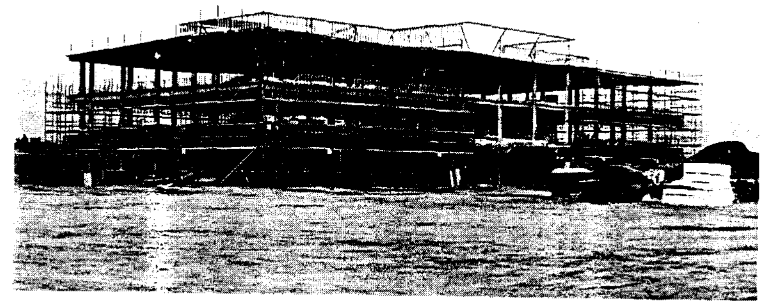


Figure 7.17 This steel-frame structure has no diagonal bracing or shear walls, and thus would be classified as "flexible."

structure tends to smooth over differential settlement problems. For example, if one foundation settles more than the others, a rigid structure will redirect some of its load, as shown in Figure 7.19, thus reducing the differential settlement.

Computing Differential Settlement of Spread Footings

There are at least two methods of predicting differential settlements of spread footings. The first method uses a series of total settlement analyses that consider the expected variations in each of the relevant factors. For example, one analysis might consider the best-

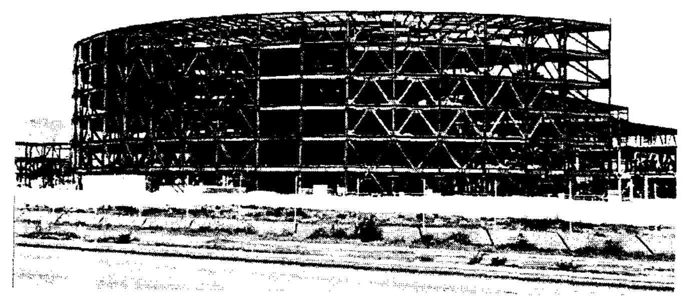


Figure 7.18 The diagonal bracing in this steel-frame structure has been installed to resist seismic loads. However, a side benefit is that this bracing provides more rigidity, which helps even out potential differential settlements. Shear walls have a similar effect. The two bays in the center of the photograph have no diagonal bracing, and thus would be more susceptible to differential settlement problems.

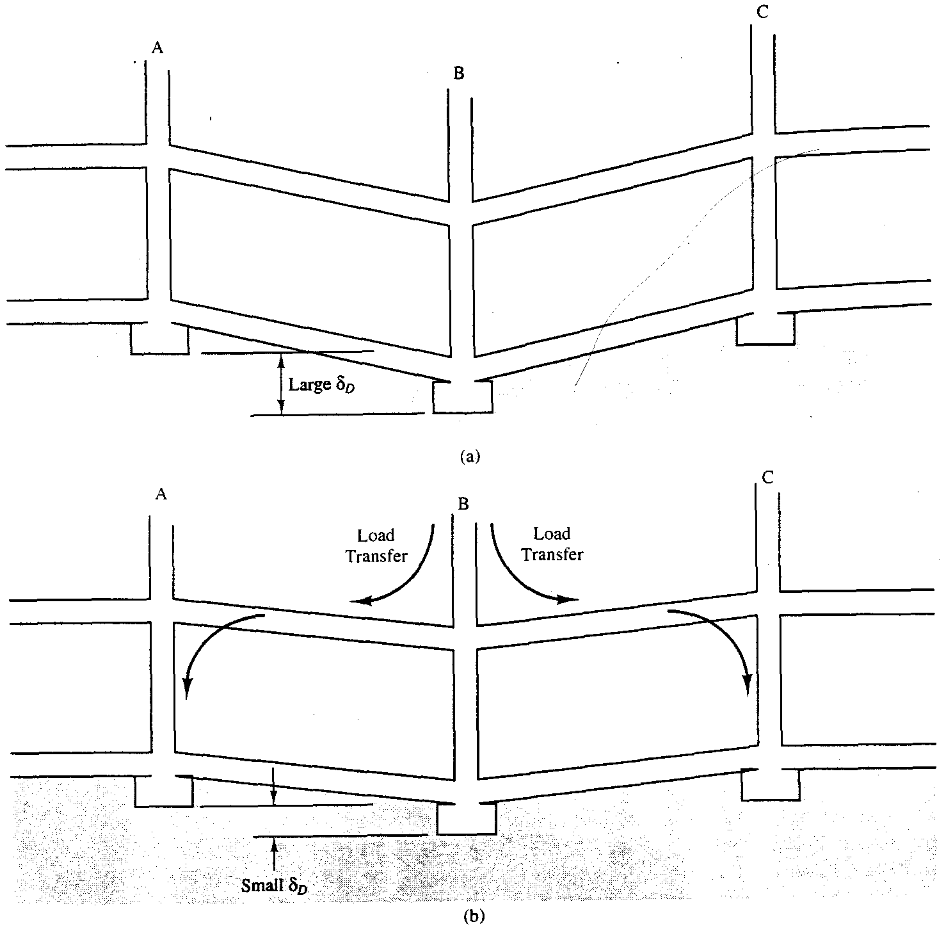


Figure 7.19 Influence of structural rigidity on differential settlements: (a) a very flexible structure has little load transfer, and thus could have larger differential settlements; (b) a more rigid structure has greater capacity for load transfer, and thus provides more resistance to excessive differential settlements.

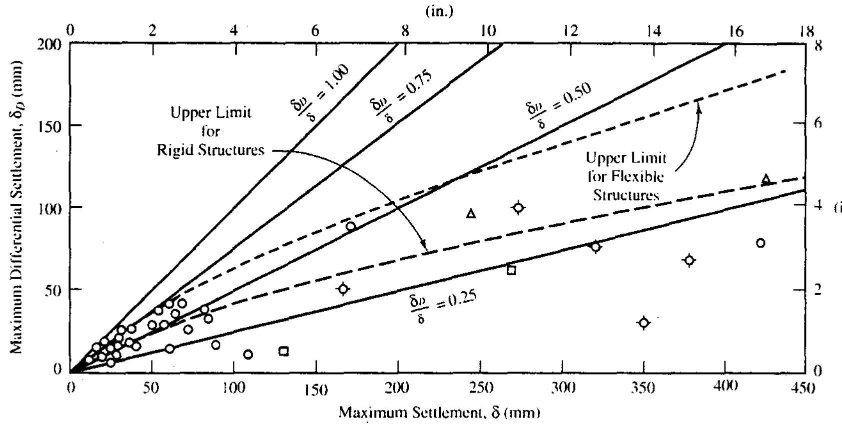


Figure 7.20 Total and differential settlements of spread footings on clays (Adapted from Bjerrum, 1963).

case scenario of soil properties, loading, and so forth, while another would consider the worst-case scenario. The difference between these two total settlements is the differential settlement.

The second method uses δ_D/δ ratios that have been observed in similar structures on similar soil profiles. For example, Bjerrum (1963) compared the total and differential settlements of spread footings on clays and sands, as shown in Figures 7.20 and 7.21. Presumably, this data was obtained primarily from sites in Scandinavia, and thus reflects the very soft soil conditions encountered in that region. This is why much of the data reflects very large settlements.

Sometimes locally-obtained δ_D/δ observations are available. Such data is more useful than generic data, such as Bjerrum's, because it implicitly reflects local soil conditions. This kind of empirical local data is probably the most reliable way to assess δ_D/δ ratios.

In the absence of local data, the generic δ_D/δ ratios in Table 7.5 may be used to predict differential settlements. The values in this table are based on Bjerrum's data and the author's professional judgement, and are probably conservative.

Remedying Differential Settlement Problems

If the computed differential settlements in a structure supported on spread footings are excessive ($\delta_D > \delta_{Da}$), the design must be changed, even if the total settlements are acceptable. Possible remedies include:

- Enlarge all of the footings until the differential settlements are acceptable. This could be done by using the allowable differential settlement, δ_{Da} and the δ_D/δ ratio to compute a new value of δ_a, then sizing the footings accordingly. Example 7.8 illustrates this technique.

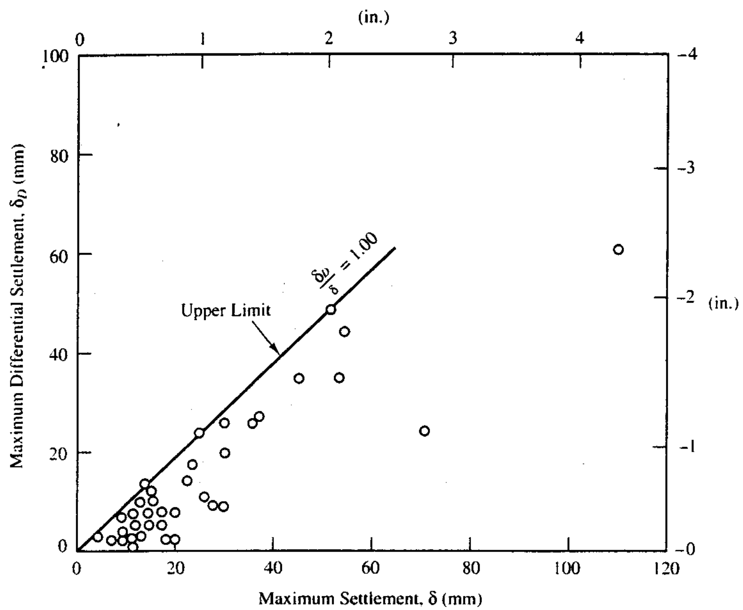


Figure 7.21 Total and differential settlement of spread footings on sands (Adapted from Bjerrum, 1963).

TABLE 7.5 DESIGN VALUES OF δ_D/δ FOR SPREAD FOOTING FOUNDATIONS

	Design Value of δ_D/δ	
Predominant Soil Type Below Footings	Flexible Structures	Rigid Structures
Sandy		
Natural soils	0.9	0.7
Compacted fills of uniform thickness underlain by stiff natural soils	0.5	0.4
Clayey		
Natural soils	0.8	0.5
Compacted fills of uniform thickness underlain by stiff natural soils	0.4	0.3

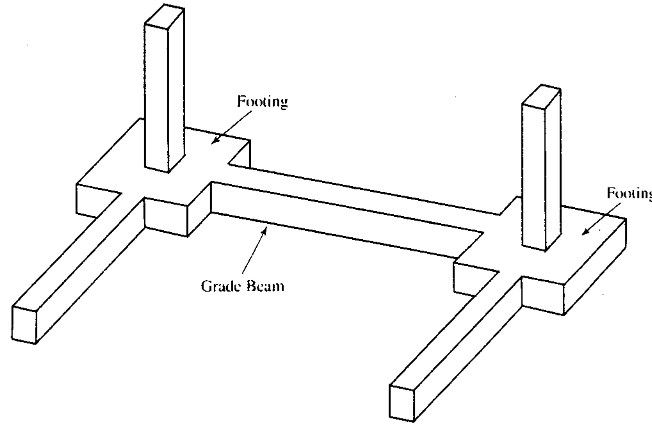


Figure 7.22 Use of grade beams to tie spread footings together.

- Connect the footings with *grade beams*, as shown in Figure 7.22. These beams provide additional rigidity to the foundation system, thus reducing the differential settlements. The effectiveness of this method could be evaluated using a structural analysis.
- Replace the spread footings with a mat foundation. This method provides even more rigidity, and thus further reduces the differential settlements. Chapter 10 discusses the analysis and design of mats.
- Replace the spread footings with a system of deep foundations, as discussed in Chapter 11.
- Redesign the superstructure so that it can accommodate larger differential settlements, so that the structural loads are lower, or both. For example, a masonry structure could be replaced by a wood-frame structure.
- Provide a method of releveling the structure if the differential settlements become excessive. This can be done by temporarily lifting selected columns from the footing and installing shims between the base plate and the footing.
- Accept the large differential settlements and repair any damage as it occurs. For some structures, such as industrial buildings, where minor distress is acceptable, this may be the most cost-effective alternative.

Example 7.8

A "flexible" steel frame building is to be built on a series of spread footing foundations supported on a natural clayey soil. The allowable total and differential settlements are 20 and

12 mm, respectively. The footings have been designed such that their total settlement will not exceed 20 mm, as determined by the analysis techniques described in this chapter.
Will the differential settlements be within tolerable limits?

Solution

According to Table 7.5, the δ_D/δ ratio is about 0.8. Therefore, the differential settlements may be as large as (0.8)(20 mm) = 16 mm. This is greater than the allowable value of 12 mm, and thus is unacceptable. Therefore, it is necessary to design the footings such that their total settlement is no greater than (12 mm)/(0.8) = 15 mm. Thus, in this case the allowable total settlement must be reduced to δ_a = 15 mm.

Mats

Because of their structural continuity, mat foundations generally experience less differential settlement, or at least the differential settlement is spread over a longer distance and thus is less troublesome. In addition, differential settlements in mat foundations are much better suited to rational analysis because they are largely controlled by the structural rigidity of the mat. We will cover these methods in Chapter 10.

QUESTIONS AND PRACTICE PROBLEMS

7.22 A steel frame office building with no diagonal bracing will be supported on spread footings founded in a natural clay. The computed total settlement of these footings is 20 mm. Compute the differential settlement.

7.23 A reinforced concrete building with numerous concrete shear walls will be supported on spread footings founded in a compacted sand. The computed total settlement of these footings is 0.6 in. Compute the differential settlement.

7.10 RATE OF SETTLEMENT

Clays

If the clay is saturated, it is safe to assume the distortion settlement occurs as rapidly as the load is applied. The consolidation settlement will occur over some period, depending on the drainage rate.

Terzaghi's theory of consolidation includes a methodology for computing the rate of consolidation settlement in saturated soils. It is controlled by the rate water is able to squeeze out of the pores and drain away. However, because the soil beneath a footing is able to drain in three dimensions, not one as assumed in Terzaghi's theory, the water will drain away more quickly, so consolidation settlement also will occur more quickly. Davis and Poulos (1968) observed this behavior when they reviewed fourteen case histories. In four of these cases, the rate was very much faster than predicted, and in another four cases, the rate was somewhat faster. In the remaining six cases, the rate was very close to or slightly slower than predicted, but this was attributed to the drainage conditions being close to one-dimensional. They also presented a method of accounting for this effect.

Rate estimates become more complex for some partially saturated soils, as discussed in Chapters 19 and 20.

Sands

The rate of settlement in sands depends on the pattern of loading. If the load is applied only once and then remains constant, then the settlement occurs essentially as fast as the load is applied. The placement of a fill is an example of this kind of loading. The dead load acting on a foundation is another example.

However, if the load varies over time, sands exhibit additional settlement that typically occurs over a period of years or decades. The live loads on a foundation are an example, especially with tanks, warehouses, or other structures in which the live load fluctuates widely and is a large portion of the total load.

A series of long-term measurements on structures in Poland (Bolenski, 1973) has verified this behavior. Bolenski found that footings with fairly constant loads, such as those supporting office buildings, exhibit only a small amount of additional settlement after construction. However, those with varying loads, such as storage tanks, have much more long-term settlement. Burland and Burbidge (1985) indicate the settlement of footings on sand 30 years after construction might be 1.5 to 2.5 times as much as the post-construction settlement. This is the reason for the secondary creep factor, C_2, in Schmertmann's equation.

7.11 ACCURACY OF SETTLEMENT PREDICTIONS

After studying many pages of formulas and procedures, the reader may develop the mistaken impression that settlement analyses are an exact science. This is by no means true. It is good to recall a quote from Terzaghi (1936):

> Whoever expects from soil mechanics a set of simple, hard-and-fast rules for settlement computations will be deeply disappointed. He might as well expect a simple rule for constructing a geologic profile from a single test boring record. The nature of the problem strictly precludes such rules.

Although much progress has been made since 1936, the settlement problem is still a difficult one. The methods described in this chapter should be taken as guides, not dictators, and should be used with engineering judgment. A vital ingredient in this judgement is an understanding of the sources of error in the analysis. These include:

- Uncertainties in defining the soil profile. This is the largest single cause. There have been many cases of unexpectedly large settlements due to undetected compressible layers, such as peat lenses.
- Disturbance of soil samples.
- Errors in in-situ tests (especially the SPT).
- Errors in laboratory tests.

- Uncertainties in defining the service loads, especially when the live load is a large portion of the total load.
- Construction tolerances (i.e., footing not built to the design dimensions).
- Errors in determining the degree of overconsolidation.
- Inaccuracies in the analysis methodologies.
- Neglecting soil-structure interaction effects.

We can reduce some of these errors by employing more extensive and meticulous exploration and testing techniques, but there are economic and technological limits to such efforts.

Because of these errors, the actual settlement of a spread footing may be quite different from the computed settlement. Figure 7.23 shows 90 percent confidence intervals for spread footing settlement computations.

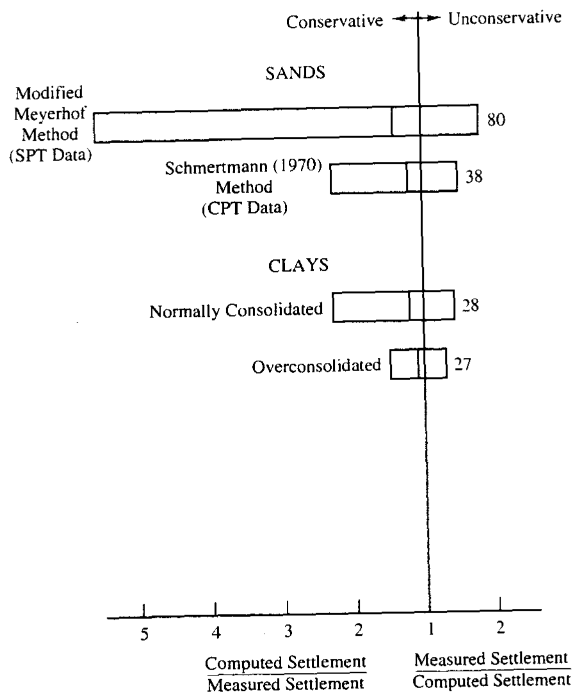


Figure 7.23 Comparison between computed and measured settlements of spread footings. Each bar represents the 90 percent confidence interval (i.e., 90 percent of the settlement predictions will be within this range). The line in the middle of each bar represents the average prediction, and the number to the right indicates the number of data points used to evaluate each method. (Based on data from Burland and Burbridge, 1985; Butler, 1975; Schmertmann, 1970; and Wahls, 1985).

The Leaning Tower of Pisa

During the Middle Ages, Europeans began to build larger and heavier structures, pushing the limits of design well beyond those of the past. In Italy, the various republics erected towers and campaniles to symbolize their power (Kerisel, 1987). Unfortunately, vanity and ignorance often lead to more emphasis on creative architecture than on structural integrity, and many of these structures collapsed. Although some of these failures were caused by poor structural design, many were the result of overloading the soil. Other monuments tilted, but did not collapse. The most famous of these is the campanile in Pisa, more popularly known as the Leaning Tower of Pisa.

Construction of the tower began in the year 1173 under the direction of Bananno Pisano and continued slowly until 1178. This early work included construction of a ring-shaped footing 64.2 ft (19.6 m) in diameter along with three and one-half stories of the tower. By then, the average bearing pressure below the footing was about 6900 lb/ft^2 (330 kPa) and the tower had already begun to tilt. Construction ceased at this level, primarily because of political and economic unrest. We now know that this suspension of work probably saved the tower, because it provided time for the underlying soils to consolidate and gain strength.

Nearly a century later, in the year 1271, construction resumed under the direction of a new architect, Giovanni Di Simone. Although it probably would have been best to tear down the completed portion and start from scratch with a new and larger foundation, Di Simone chose to continue working on the uncompleted tower, attempting to compensate for the tilt by tapering the successive stories and adding extra weight to the high side. He stopped work in 1278 at the seventh cornice. Finally, the tower was completed with the construction of the belfry during a third construction period, sometime between 1360 and 1370. The axis of the belfry is inclined at an angle of 3° from the rest of the tower, which was probably the angle of tilt at that time. Altogether, the project had taken nearly two hundred years to complete.

Both the north and south sides of the tower continued to settle (the tilt has occurred because the south side settled more than the north side), so that by the early nineteenth century, the tower had settled about 2.5 meters into the ground. As a result, the elegant carvings at the base of the columns were no longer visible. To "rectify" this problem, a circular trench was excavated around the perimeter of the tower in 1838 to expose the bottom of the columns. This trench is known as the catino. Unfortunately, construction of the trench disturbed the groundwater table and removed some lateral support from the side of the tower. As a result, the tower suddenly lurched and added about half a meter to the tilt at the top. Amazingly, it did not collapse. Nobody dared do anything else for the next hundred years.

During the 1930s, the Fascist dictator Benito Mussolini decided the leaning tower presented an inappropriate image of the country, and ordered a fix. His workers drilled holes through the floor of the tower and pumped 200 tons of concrete into the underlying soil, but this only aggravated the problem and the tower gained an additional 0.1 degree of tilt.

During most of the twentieth century the tower has been moving at a rate of about 7 seconds of arc per year. By the end of the century the total tilt was about 5.5 degrees to the south which means that the top of the tower structure was 5.2 m (17.0 ft) off of being plumb. The average bearing pressure under the tower is 497 kPa (10,400 lb/ft^2), but the tilting caused its weight to act eccentrically on the foundation, so the bearing pressure is not uniform. By the twentieth century, it ranged from 62 to 930 kPa (1,300–19,600 lb/ft^2), as shown in Figure 7.24. The tower is clearly on the brink of collapse. Even a minor earthquake could cause it to topple, so it became clear that some remedial measure must be taken.

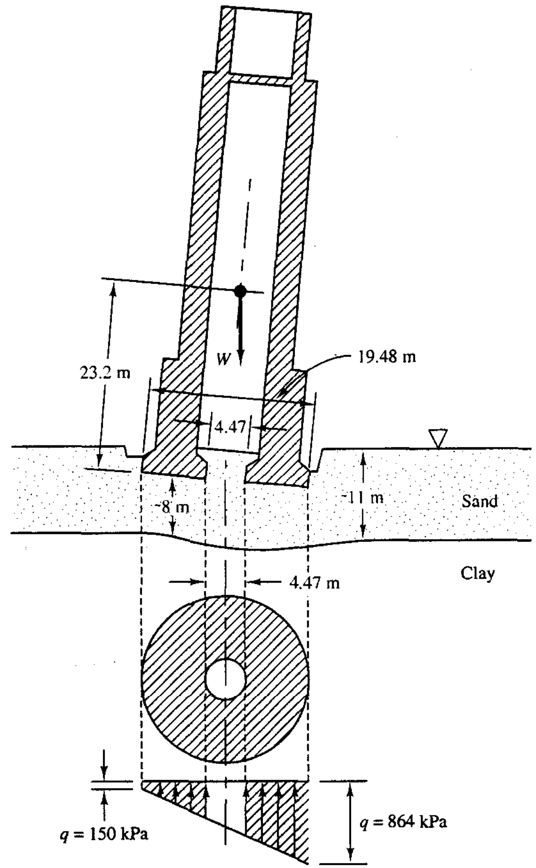


Figure 7.24 Current configuration of the tower (Adapted from Costnazo, Jamiolkowski, Lancellotta and Pepe, 1994 and Terzaghi, 1934a).

The subsurface conditions below the tower have been investigated, including an exhaustive program sponsored by the Italian government that began in 1965. The profile, shown in Figure 7.25, is fairly uniform across the site and consists of sands underlain by fat

This problem has attracted the attention of both amateurs and professionals, and the authorities have received countless "solutions," sometimes at the rate of more than fifty per week. Some are clearly absurd, such as tying helium balloons to the top of the tower, or installing a series of cherub statues with flapping wings. Others, such as large structural supports (perhaps even a large statue leaning against the tower?), may be technically feasible, but aesthetically unacceptable.

In 1990 the interior of the tower was closed to visitors, and in 1993 about 600 tons of lead ingots were placed on the north side of the tower as a temporary stabilization measure.

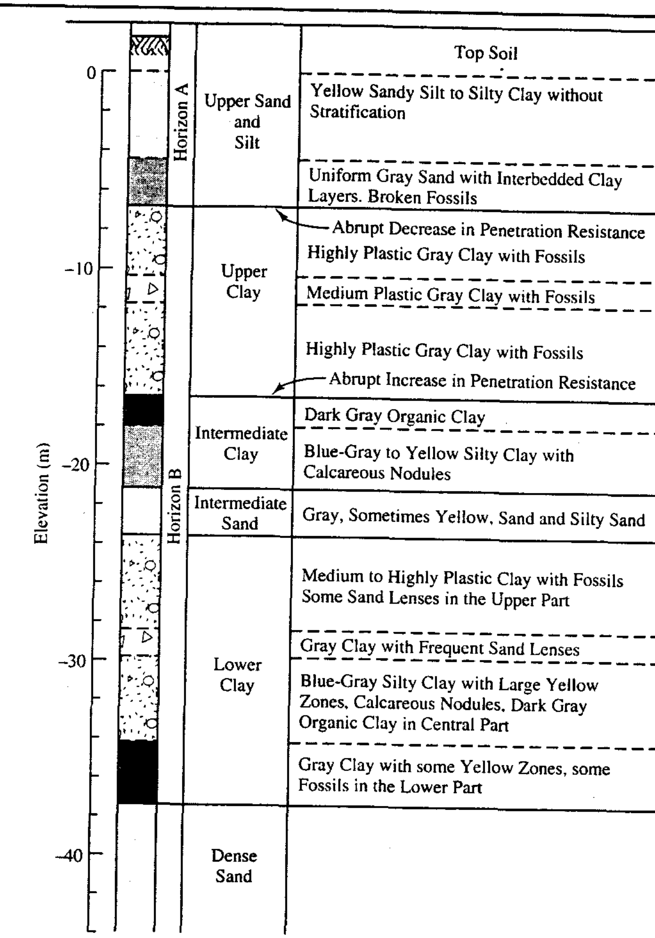


Figure 7.25 Soil profile below tower (Adapted from Mitchell, et al., 1977; Used by permission of ASCE).

Then, in 1995, engineers installed a concrete ring around the foundation and began drilling tiedown anchors through the ring and into the dense sand stratum located at a depth of about 40 ft (see profile in Figure 7.25). The weights caused the underlying soils to compress and slightly reduced the tilt, but construction of the anchors disturbed the soil and produced a sudden increase in the tilt of the tower. In one night the tower moved about 1.5 mm, which is the equivalent to a year's worth of normal movement. As a result, the work was quickly abandoned and more lead ingots were added to the north side.

A period of inactivity followed, but in 1997 an earthquake in nearby Assisi caused a tower in that city to collapse—and that tower was not even leaning! This failure induced a new cycle of activity at Pisa, and the overseeing committee approved a new method of stabilizing the tower: soil extraction.

The method of soil extraction consists of carefully drilling diagonal borings into the ground beneath the north side of the tower and extracting small amounts of soil. The overlying soils then collapse into the newly created void, which should cause the north side of the tower to settle, thus decreasing the tilt. The objective of this effort is to reduce the tilt from 5.5 degrees to 5.0 degrees, which is the equivalent of returning the tower to its position of three hundred years ago. There is no interest in making the tower perfectly plumb.

Soil extraction has been successfully used to stabilize structures in Mexico City, and appears to be the most promising method for Pisa. This process must proceed very slowly, perhaps over a period of months or years, while continuously monitoring the movements of the tower. When this book was published, the soil extraction work had begun and the tilt had been very slightly reduced. If this effort is successful, the temporary lead weights will no longer be necessary, and the life of the tower should be extended for at least three hundred years.

Recommended reference: Costanzo, D.; Jamiolkowski, M., Lancellotta, R., and Pepe, M.C. (1994), *Leaning Tower of Pisa: Description of the Behavior*, Settlement 94 Banquet Lecture, Texas A&M University

We can draw the following conclusions from this data:

- Settlement predictions are conservative more often than they are unconservative (i.e., they tend to overpredict the settlement more often than they underpredict it). However, the range of error is quite wide.
- Settlement predictions made using the Schmertmann method with CPT data are much more precise than those based on the SPT. (Note that these results are based on the 1970 version of Schmertmann's method. Later refinements, as reflected in this chapter, should produce more precise results.)
- Settlement predictions in clays, especially those that are overconsolidated, are usually more precise than those in sands. However, the magnitude of settlement in clays is often greater.

Many of the soil factors that cause the scatter in Figure 7.23 do not change over short distances, so predictions of differential settlements should be more precise than those for total settlements. Therefore, the allowable differential settlement criteria described in Table 2.2 (which include factors of safety of at least 1.5) reflect an appropriate level of conservatism.

SUMMARY

Major Points

1. Foundations must meet two settlement requirements: total settlement and differential settlement.
2. The load on spread footings causes an increase in the vertical stress, $\Delta\sigma_z$, in the soil below. This stress increase causes settlement in the soil beneath the footing.
3. The magnitude of $\Delta\sigma_z$ directly beneath the footing is equal to the bearing pressure, q. It decreases with depth and becomes very small at a depth of about $2B$ below square footings or about $6B$ below continuous footings.
4. The distribution of $\Delta\sigma_z$ below a footing may be calculated using Boussinesq's method, Westergaard's method, or the simplified method.
5. Settlement analyses in clays and silts are usually based on laboratory consolidation tests. The corresponding settlement analysis is an extension of the Terzaghi settlement analyses for fills, as discussed in Chapter 3.
6. Settlement analyses based on laboratory tests may use either the classical method, which assumes one-dimensional consolidation, or the Skempton and Bjerrum method, which accounts for three-dimensional effects.
7. Settlement analyses in sands are usually based on in-situ tests. The Schmertmann method may be used with these test results.
8. Differential settlements may be estimated based on observed ratios of differential to total settlement.
9. Settlement estimates based on laboratory consolidation tests of clays and silts typically range from a 50 percent overestimate (unconservative) to a 100 percent underestimate (conservative).
10. Settlement estimates based on CPT data from sandy soils typically range from a 50 percent overestimate (unconservative) to a 100 percent underestimate (conservative). However, estimates based on the SPT are much less precise.

Vocabulary

Allowable differential settlement
Allowable settlement
Boussinesq's method
Classical method
Differential settlement
Distortion settlement
Induced stress
Plate load test
Rigidity
Schmertmann's method
Settlement
Skempton and Bjerrum method

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

7.24 A 600-mm wide, 500-mm deep continuous footing carries a vertical downward load of 85 kN/m. The soil has γ = 19kN/m^3. Using Boussinesq's method, compute $\Delta\sigma_z$ at a depth of 200 mm below the bottom of the footing at the following locations:

- Beneath the center of the footing
- 150 mm from the center of the footing
- 300 mm from the center of the footing (i.e., beneath the edge)
- 450 mm from the center of the footing

Plot the results in the form of a pressure diagram similar to those in Figure 5.10 in Chapter 5.

Hint: Use the principle of superposition.

7.25 A 3-ft square, 2-ft deep footing carries a column load of 28.2 k. An architect is proposing to build a new 4 ft wide, 2 ft deep continuous footing adjacent to this existing footing. The side of the new footing will be only 6 inches away from the side of the existing footing. The new footing will carry a load of 12.3 k/ft. γ = 119lb/ft^3.

Develop a plot of $\Delta\sigma_z$ due to the new footing vs. depth along a vertical line beneath the center of the existing footing. This plot should extend from the bottom of the existing footing to a depth of 35 ft below the bottom of this footing.

7.26 Using the data from Problem 7.25, $C_r/(1 + e_0)$ =0.08 and γ=119 lb/ft^3, compute the consolidation settlement of the old footing due to the construction and loading of the new footing. The soil is an overconsolidated (case I) silty clay, and the groundwater table is at a depth of 8 ft below the ground surface.

7.27 Using the SCHMERTMANN.XLS spreadsheet and the subsurface data from Example 7.6, develop a plot of footing width, *B*, vs. column load, *P*, for square spread footings embedded 3 ft below the ground surface. Develop a *P* vs. *B* curve for each of the following settlements: 0.5 in, 1.0 in, and 1.5 in, and present all three curves on the same diagram.

8

Spread Footings–Geotechnical Design

Your greatest danger is letting the urgent things crowd out the important.

From *Tyranny of the Urgent* by Charles E. Hummel[1]

This chapter shows how to use the results of bearing capacity and settlement computations, as well as other considerations, to develop spread footing designs that satisfy geotechnical requirements. These are the requirements that relate to the safe transfer of the applied loads from the footing to the ground. Chapter 9 builds on this information, and discusses the structural design aspects, which are those that relate to the structural integrity of the footing and the connection between the footing and the superstructure.

8.1 DESIGN FOR CONCENTRIC DOWNWARD LOADS

The primary load on most spread footings is the downward compressive load, *P*. This load produces a bearing pressure *q* along the bottom of the footing, as described in Section 5.3. Usually we design such footings so that the applied load acts through the centroid (i.e., the column is located in the center of the footing). This way the bearing pressure is uniformly distributed along the base of the footing (or at least it can be assumed to be uniformly distributed) and the footing settles evenly.

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

7.24 A 600-mm wide, 500-mm deep continuous footing carries a vertical downward load of 85 kN/m. The soil has γ = 19kN/m^3. Using Boussinesq's method, compute $\Delta\sigma_z$ at a depth of 200 mm below the bottom of the footing at the following locations:

- Beneath the center of the footing
- 150 mm from the center of the footing
- 300 mm from the center of the footing (i.e., beneath the edge)
- 450 mm from the center of the footing

Plot the results in the form of a pressure diagram similar to those in Figure 5.10 in Chapter 5.

Hint: Use the principle of superposition.

7.25 A 3-ft square, 2-ft deep footing carries a column load of 28.2 k. An architect is proposing to build a new 4 ft wide, 2 ft deep continuous footing adjacent to this existing footing. The side of the new footing will be only 6 inches away from the side of the existing footing. The new footing will carry a load of 12.3 k/ft. γ = 119lb/ft^3.

Develop a plot of $\Delta\sigma_z$ due to the new footing vs. depth along a vertical line beneath the center of the existing footing. This plot should extend from the bottom of the existing footing to a depth of 35 ft below the bottom of this footing.

7.26 Using the data from Problem 7.25, $C_r/(1+e_0)$ =0.08 and γ=119 lb/ft^3, compute the consolidation settlement of the old footing due to the construction and loading of the new footing. The soil is an overconsolidated (case I) silty clay, and the groundwater table is at a depth of 8 ft below the ground surface.

7.27 Using the SCHMERTMANN.XLS spreadsheet and the subsurface data from Example 7.6, develop a plot of footing width, B, vs. column load, P, for square spread footings embedded 3 ft below the ground surface. Develop a P vs. B curve for each of the following settlements: 0.5 in, 1.0 in, and 1.5 in, and present all three curves on the same diagram.

8

Spread Footings–Geotechnical Design

Your greatest danger is letting the urgent things crowd out the important.

From *Tyranny of the Urgent* by Charles E. Hummel[1]

This chapter shows how to use the results of bearing capacity and settlement computations, as well as other considerations, to develop spread footing designs that satisfy geotechnical requirements. These are the requirements that relate to the safe transfer of the applied loads from the footing to the ground. Chapter 9 builds on this information, and discusses the structural design aspects, which are those that relate to the structural integrity of the footing and the connection between the footing and the superstructure.

8.1 DESIGN FOR CONCENTRIC DOWNWARD LOADS

The primary load on most spread footings is the downward compressive load, P. This load produces a bearing pressure q along the bottom of the footing, as described in Section 5.3. Usually we design such footings so that the applied load acts through the centroid (i.e., the column is located in the center of the footing). This way the bearing pressure is uniformly distributed along the base of the footing (or at least it can be assumed to be uniformly distributed) and the footing settles evenly.

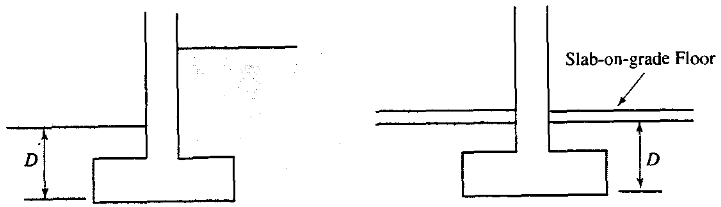


Figure 8.1 Depth of embedment for spread footings.

Footing Depth

The depth of embedment, D, must be at least large enough to accommodate the required footing thickness, T, as shown in Figure 8.1. This depth is measured from the lowest adjacent ground surface to the bottom of the footing. In the case of footings overlain by a slab-on-grade floor, D is measured from the subgrade below the slab.

Tables 8.1 and 8.2 present minimum D values for various applied loads. These are the unfactored loads (i.e., the greatest from Equations 2.1–2.4). These D values are intended to provide enough room for the required footing thickness, T. In some cases, a more detailed analysis may justify shallower depths, but D should never be less than 300 mm (12 in). The required footing thickness, T, is governed by structural concerns, as discussed in Chapter 9.

TABLE 8.1 MINIMUM DEPTH OF EMBEDMENT FOR SQUARE AND RECTANGULAR FOOTINGS

Load P (k)	Minimum D (in)	Load P (kN)	Minimum D (mm)
0–65	12	0–300	300
65–140	18	300–500	400
140–260	24	500–800	500
260–420	30	800–1100	600
420–650	36	1100–1500	700
		1500–2000	800
		2000–2700	900
		2700–3500	1000

TABLE 8.2 MINIMUM DEPTH OF EMBEDMENT FOR CONTINUOUS FOOTINGS

Load P/b (k/ft)	Minimum D (in)	Load P/b (kN/m)	Minimum D (mm)
0–10	12	0–170	300
10–20	18	170–250	400
20–28	24	250–330	500
28–36	30	330–410	600
36–44	36	410–490	700
		490–570	800
		570–650	900
		650–740	1000

Sometimes it is necessary to use embedment depths greater than those listed in Tables 8.1 and 8.2. This situations include the following:

- The upper soils are loose or weak, or perhaps consist of a fill of unknown quality. In such cases, we usually extend the footing through these soils and into the underlying competent soils.
- The soils are prone to frost heave, as discussed later in this section. The customary procedure in such soils is to extend the footings to a depth that exceeds the depth of frost penetration.
- The soils are expansive. One of the methods of dealing with expansive soils is to extend the footings to a greater depth. This gives them additional flexural strength, and places them below the zone of greatest moisture fluctuation. Chapter 19 discusses this technique in more detail.
- The soils are prone to scour, which is erosion caused by flowing water. Footings in such soils must extend below the potential scour depth. This is discussed in more detail later in this chapter.
- The footing is located near the top of a slope in which there is some, even remote, possibility of a shallow landslide. Such footings should be placed deeper than usual in order to provide additional protection against undermining from any such slides.

Sometimes we also may need to specify a maximum depth. It might be governed by such considerations as:

- Potential undermining of existing foundations, structures, streets, utility lines, etc.
- The presence of soft layers beneath harder and stronger near-surface soils, and the desire to support the footings in the upper stratum.

- A desire to avoid working below the groundwater table, and thus avoid construction dewatering expenses.
- A desire to avoid the expense of excavation shoring, which may be needed for footing excavations that are more than 1.5 m (5 ft) deep.

Footing Width

Sometimes bearing capacity and settlement concerns can be addressed by increasing the footing depth. For example, if the near-surface soils are poor, but those at slightly greater depths are substantially better, bearing capacity and settlement problems might be solved by simply deepening the footing until it reaches the higher quality stratum. However, in more uniform soil profiles, we usually satisfy bearing capacity and settlement requirements by adjusting the footing width, B. Increasing B causes the bearing pressure, q, to decrease, which improves the factor of safety against a bearing capacity failure and decreases the settlement.

Most structures require many spread footings, perhaps dozens of them, so it is inconvenient to perform custom bearing capacity and settlement analyses for each one. Instead, geotechnical engineers develop generic design criteria that are applicable to the entire site, then the structural engineer sizes each footing based on its load and these generic criteria. We will discuss two methods of presenting these design criteria: the *allowable bearing pressure method* and the *design chart method.*

Allowable Bearing Pressure Method

The *allowable bearing pressure,* q_A, is the largest bearing pressure that satisfies both bearing capacity and settlement criteria. In other words, it is equal to the allowable bearing capacity, q_a, or the q that produces the greatest acceptable settlement, whichever is less. Normally we develop a single q_A value that applies to the entire site, or at least to all the footings of a particular shape at that site.

Geotechnical engineers develop q_A using the following procedure:

1. Select a depth of embedment, D, as described earlier in this chapter. If different depths of embedment are required for various footings, perform the following computations using the smallest D.
2. Determine the design groundwater depth, D_w. This should be the shallowest groundwater depth expected to occur during the life of the structure.
3. Determine the required factor of safety against a bearing capacity failure (see Figure 6.11).
4. Using the techniques described in Chapter 6, perform a bearing capacity analysis on the footing with the smallest applied normal load. This analysis is most easily performed using the BEARING.XLS spreadsheet. Alternatively, it may be performed as follows:

   a. Using Equation 5.1 or 5.2, write the bearing pressure, q, as a function of B.
   b. Using Equation 6.4, 6.5, 6.6, or 6.13, along with Equation 6.36, write the allowable bearing capacity, q_a, as a function of B.
   c. Set $q = q_a$ and solve for B.
   d. Using Equation 6.4, 6.5, 6.6, or 6.13, along with Equation 6.36 and the B from Step c, determine the allowable bearing capacity, q_a.
5. Using the techniques described in Chapter 2, determine the allowable total and differential settlements, δ_a and δ_{Da}. Normally the structural engineer performs this step and provides these values to the geotechnical engineer.
6. Using local experience or Table 7.5, select an appropriate δ_D/δ ratio.
7. If $\delta_{Da} \geq \delta_a\,(\delta_D/\delta)$, then designing the footings to satisfy the total settlement requirement ($\delta \leq \delta_a$) will implicitly satisfy the differential settlement requirement as well ($\delta_D \leq \delta_{Da}$). Therefore, continue to Step 8 using δ_a. However, if $\delta_{Da} < \delta_a\,(\delta_D/\delta)$, it is necessary to reduce δ_a to keep differential settlement under control (see Example 7.8). In that case, continue to Step 8 using a revised $\delta_a = \delta_{Da} / (\delta_D/\delta)$.
8. Using the δ_a value obtained from Step 7, and the techniques described in Chapter 7, perform a settlement analysis on the footing with the largest applied normal load. This analysis is most easily performed using the SETTLEMENT.XLS or SCHMERTMANN.XLS spreadsheets. Determine the maximum bearing pressure, q, that keeps the total settlement within tolerable limits (i.e., $\delta \leq \delta_a$).
9. Set the allowable bearing pressure, q_A equal to the lower of the q_a from Step 4 or q from Step 8. Express it as a multiple of 500 lb/ft^2 or 25 kPa.

If the structure will include both square and continuous footings, we can develop separate q_A values for each.

We use the most lightly loaded footing for the bearing capacity analysis because it is the one that has the smallest B and therefore the lowest ultimate bearing capacity (per Equations 6.4–6.6). Thus, this footing has the lowest q_a of any on the site, and it is conservative to design the other footings using this value.

However, we use the most heavily loaded footing (i.e., the one with the largest B) for the settlement analysis, because it is the one that requires the lowest value of q to satisfy settlement criteria. To understand why this is so, compare the two footing in Figure 8.2. Both of these footings have the same bearing pressure, q. However, since a greater volume of soil is being stressed by the larger footing, it will settle more than the smaller footing. For footings on clays loaded to the same q, the settlement is approximately proportional to B, while in sands it is approximately proportional to $B^{0.5}$. Therefore, the larger footing is the more critical one for settlement analyses.

The geotechnical engineer presents q_A, along with other design criteria, in a written report. The structural engineer receives this report, and uses the recommended q_A to design the spread footings such that $q \leq q_A$. Thus, for square, rectangular, and circular footings:

$$q = \frac{P + W_f}{A} - u_D \leq q_A \tag{8.1}$$

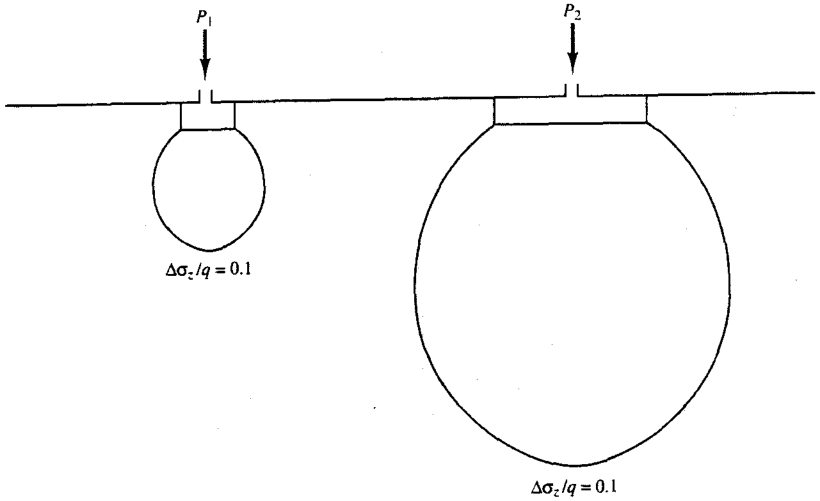


Figure 8.2 These two footings are loaded to the same q, but each has a different width and a correspondingly different P. The larger footing induces stresses to a greater depth in the soil, so it settles more than the smaller footing.

Setting $q = q_A$ and rewriting gives:

$$\boxed{A = \frac{P + W_f}{q_A + u_D}} \tag{8.2}$$

For continuous footings:

$$\boxed{B = \frac{P/b + W_f/b}{q_A + u_D}} \tag{8.3}$$

Where:

A = required base area

for square footings, $A = B^2$

for rectangular footings, $A = BL$

for circular footings, $A = \pi B^2/4$

B = footing width or diameter

L = footing length

P = applied normal load (unfactored)

P/b = applied normal load per unit length (unfactored)

W_f = weight of foundation

W_f/b = weight of foundation per unit length

b = unit length of foundation (normally 1 m or 1 ft)

q_A = allowable bearing pressure

u_D = pore water pressure along base of footing. $u_D = 0$ if the groundwater table is at a depth greater than D. Otherwise, $u_D = \gamma_w (D - D_w)$.

The footing width must be determined using the unfactored design load (i.e., the largest load computed from Equations 2.1–2.4), even if the superstructure has been designed using the factored load.

Example 8.1

As part of an urban redevelopment project, a new parking garage is to be built at a site formerly occupied by two-story commercial buildings. These old buildings have already been demolished and their former basements have been backfilled with well-graded sand, sandy silt, and silty sand. The lower level of the proposed parking garage will be approximately flush with the existing ground surface, and the design column loads range from 250 to 900 k. The allowable total and differential settlements are 1.0 and 0.6 inches, respectively.

A series of five exploratory borings have been drilled at the site to evaluate the subsurface conditions. The soils consist primarily of the basement backfill, underlain by alluvial sands and silts. The groundwater table is at a depth of about 200 ft. Figure 8.3 shows a design soil profile compiled from these borings, along with all of the standard penetration test N_{60} values.

The basement backfills were not properly compacted and only encompass portions of the site. Therefore, in the interest of providing more uniform support for the proposed spread footing foundations, the upper ten feet of soil across the entire site will be excavated and recompacted to form a stratum of properly compacted fill. This fill will have an estimated overconsolidation ratio of 3 and an estimated N_{60} of 60. A laboratory direct shear test on a compacted sample of this soil produced $c' = 0$ and $\phi' = 35°$.

Determine the allowable bearing pressures, q_A, for square and continuous footings at this site, then use this q_A to determine the required dimensions for a square footing that will support a 300 k column load.

Solution

Step 1 — Use an estimated D of 3 ft

Step 2 — The groundwater table is very deep, and is not a concern at this site

Step 3 — Use $F = 2.5$

Step 4 — Using the BEARING.XLS spreadsheet with P = 250 k, the computed allowable bearing pressure, q_a is 10,500 lb/ft^2

Step 5 — Per the problem statement, δ_a = 1.0 in and δ_{Da} = 0.6 in

Step 6 — Using Table 7.5 and assuming the parking garage is a "flexible" structure, the design value of δ_D/δ is 0.5

Step 7 — $\delta_{Da} > \delta_a (\delta_D/\delta)$, so the total settlement requirement controls the settlement analysis

Step 8 — Using Table 7.3 and Equation 7.17, the equivalent modulus values for each SPT data point are as follows:

Boring No.	Depth (ft)	Soil Type	N_{60}	β_0	β_1	E_s (lb/ft^2)
1	14	SW	20	100,000	24,000	580,000
1	25	SP	104	100,000	24,000	2,596,000
1	35	SP	88	100,000	24,000	2,212,000
1	45	SW	96	100,000	24,000	2,404,000
2	14	SW	44	100,000	24,000	1,156,000
2	25	SP	122	100,000	24,000	3,028,000
3	14	SP & SW	72	100,000	24,000	1,828,000
3	25	SW	90	100,000	24,000	2,260,000
4	15	SW	46	100,000	24,000	1,204,000
4	25	SW	102	100,000	24,000	2,548,000
5	19	SW	92	100,000	24,000	2,308,000
5	29	SP	68	100,000	24,000	1,732,000
5	40	SW	74	100,000	24,000	1,876,000
5	45	SW	60	100,000	24,000	1,540,000
5	49	SW	56	100,000	24,000	1,444,000

The equivalent modulus of the proposed compacted fill is:

$$E_s = B_0\sqrt{\text{OCR}} + \beta_1 N_{60}$$
$$= 7000\sqrt{3} + (16{,}000)(60)$$
$$\doteq 1{,}100{,}000 \text{ lb/ft}^2$$

Based on this data, we can perform the settlement analysis using the following equivalent modulus values:

Depth (ft)	E_s (lb/ft^2)
0 – 10	1,100,000
10 – 20	1,000,000
> 20	1,700,000

Using the SCHMERTMANN.XLS spreadsheet with P = 900 k and δ_a = 1.0 in produces q = 6,700 lb/ft^2

Step 9 — 6,700 < 10,500, so settlement controls the design. Rounding to a multiple of 500 lb/ft^2 gives:

$$q_A = 6500 \text{ lb/ft}^2 \quad \Leftarrow \textit{Answer}$$

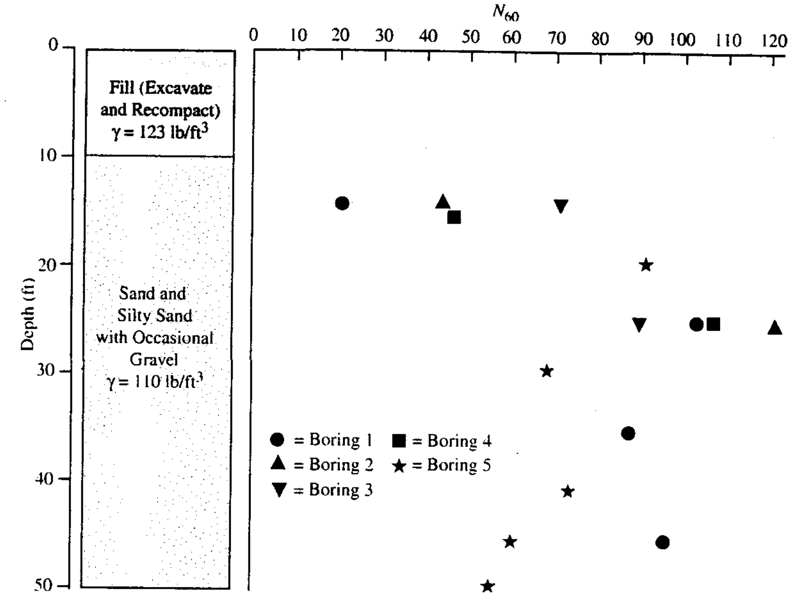


Figure 8.3 Design soil profile and SPT results for proposed parking garage site.

For a 300 k column load, $W_f = (3 \text{ ft})(150 \text{ lb/ft}^3)\, B^2 = 450\, B^2$

$$B = \sqrt{A} = \sqrt{\frac{P + W_f}{q_A + u_D}}$$
$$= \sqrt{\frac{300{,}000 + 450\,B^2}{6500 + 0}}$$
$$= \textbf{7 ft 0 in} \quad \Leftarrow \textit{Answer}$$

Design Chart Method

The allowable bearing pressure method is sufficient for most small to medium-size structures. However, larger structures, especially those with a wide range of column loads, warrant a more precise method: the design chart. This added precision helps us reduce both differential settlements and construction costs.

Instead of using a single allowable bearing pressure for all footings, it is better to use a higher pressure for small ones and a lower pressure for large ones. This method reduces the differential settlements and avoid the material waste generated by the allowable bearing pressure method. This concept is implicit in a design chart such as the one in Figure 8.4.

Use the following procedure to develop design charts:

1. Determine the footing shape (i.e., square, continuous, etc.) for this design chart. If different shapes are to be used, each must have its own design chart.
2. Select the depth of embedment, D, using the guidelines described earlier in this chapter. If different D values are required for different footings, perform these computations using the smallest D.
3. Determine the design groundwater depth, D_w. This should be the shallowest groundwater depth expected to occur during the life of the structure.
4. Select the design factor of safety against a bearing capacity failure (see Figure 6.11).

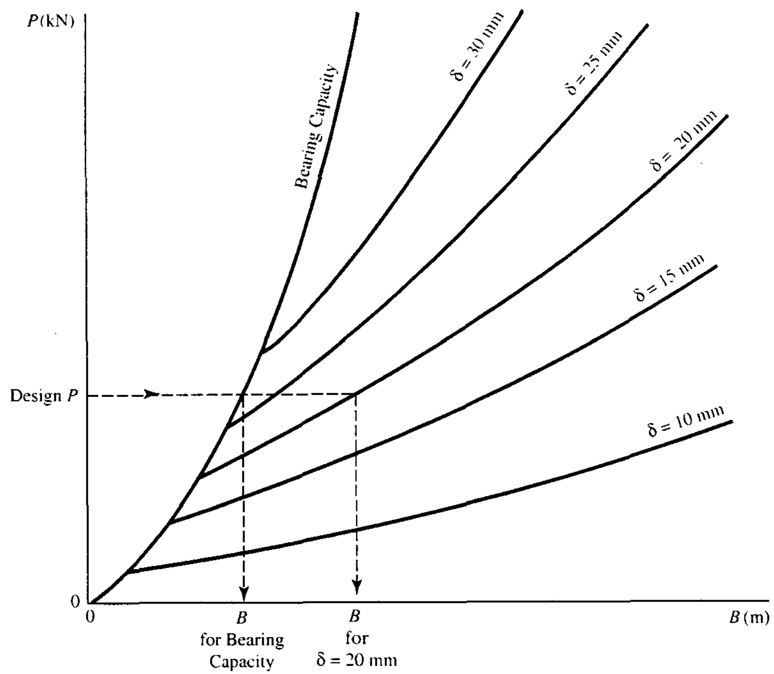


Figure 8.4 A typical design chart for spread footings.

5. Set the footing width B equal to 300 mm or 1 ft, then conduct a bearing capacity analysis and compute the column load that corresponds to the desired factor of safety. Plot this (B, P) data point on the design chart. Then select a series of new B values, compute the corresponding P, and plot the data points. Continue this process until the computed P is slightly larger than the maximum design column load. Finally, connect these data points with a curve labeled "bearing capacity." The spreadsheet developed in Chapter 6 makes this task much easier.
6. Develop the first settlement curve as follows:
   a. Select a settlement value for the first curve (e.g., 0.25 in).
   b. Select a footing width, B, that is within the range of interest and arbitrarily select a corresponding column load, P. Then, compute the settlement of this footing using the spreadsheets developed in Chapters 6 and 7, or some other suitable method.
   c. By trial-and-error, adjust the column load until the computed settlement matches the value assigned in step a. Then, plot the point B, P_a on the design chart.
   d. Repeat steps b and c with new values of B until a satisfactory settlement curve has been produced.
7. Repeat step 6 for other settlement values, thus producing a family of settlement curves on the design chart. These curves should encompass a range of column loads and footing widths appropriate for the proposed structure.
8. Using the factors in Table 7.5, develop a note for the design chart indicating the design differential settlements are ___% of the total settlements.

Once the design chart has been obtained, the geotechnical engineer gives it to the structural engineer who sizes each footing using the following procedure:

1. Compute the design load, P, which is the largest load computed from Equations 2.1, 2.2, 2.3a, or 2.4a. Note that this is the unfactored load, even if the superstructure has been designed using the factored load.
2. Using the bearing capacity curve on the design chart, determine the minimum required footing width, B, to support the load P while satisfying bearing capacity requirements.
3. Using the settlement curve that corresponds to the allowable total settlement, δ_a, determine the footing width, B, that corresponds to the design load, P. This is the minimum width required to satisfy total settlement requirements.
4. Using the δ_D/δ ratio stated on the design chart, compute the differential settlement, δ_D, and compare it to the allowable differential settlement, δ_{Da}.
5. If the differential settlement is excessive ($\delta_D > \delta_{Da}$), then use the following procedure:
   a. Use the allowable differential settlement, δ_{Da}, and the δ_D/δ ratio to compute a new value for allowable total settlement, δ_a. This value implicitly satisfies both total and differential settlement requirements.

b. Using the settlement curve on the design chart that corresponds to this revised δ_a, determine the required footing width, B. This footing width is smaller than that computed in step 3, and satisfies both total and differential settlement criteria.

6. Select the larger of the B values obtained from the bearing capacity analysis (step 2) and the settlement analysis (step 3 or 5b). This is the design footing width.
7. Repeat steps 1 to 6 for the remaining columns.

These charts clearly demonstrate how the bearing capacity governs the design of narrow footings, whereas settlement governs the design of wide ones.

The advantages of this method over the allowable bearing pressure method include:

- The differential settlements are reduced because the bearing pressure varies with the footing width.
- The selection of design values for total and differential settlement becomes the direct responsibility of the structural engineer, as it should be. (With the allowable bearing pressure method, the structural engineer must give allowable settlement data to the geotechnical engineer who incorporates it into q_A.)
- The plot shows the load-settlement behavior, which we could use in a soil-structure interaction analysis.

Example 8.2

Develop a design chart for the proposed arena described in Example 8.1, then use this chart to determine the required width for a footing that is to support a 300-k column load.

Solution

Bearing capacity analyses (based on BEARING.XLS spreadsheet)

B (ft)	P (k)
2	29
3	74
4	146
5	251
6	395
7	582
8	818
9	1109

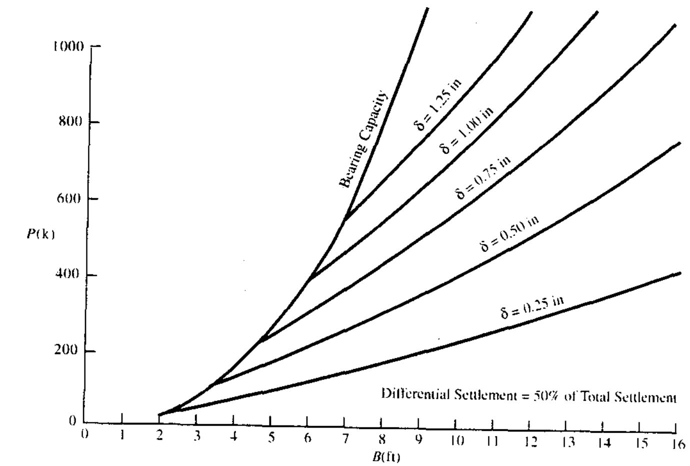


Figure 8.5 Design chart for Example 8.2.

Settlement analyses (based on SCHMERTMANN.XLS spreadsheet)
Column loads to obtain a specified total settlement

	B = 2 ft	B = 7 ft	B = 12 ft	B = 17 ft
δ = 0.25 in	32 k	145 k	280 k	450 k
δ = 0.50 in	56 k	260 k	510 k	830 k
δ = 0.75 in	77 k	365 k	720 k	1190 k
δ = 1.00 in	97 k	465 k	925 k	
δ = 1.25 in	115 k	555 k	1120 k	

The result of these analyses are plotted in Figure 8.5.

According to this design chart, a 300-k column load may be supported on a 5 ft, 6 in wide footing. This is much smaller than the 7 ft, 0 in wide footing in Example 8.1.

Use **B = 5 ft 6 in** ⇐ ***Answer***

QUESTIONS AND PRACTICE PROBLEMS

8.1 Which method of expressing footing width criteria (allowable bearing pressure or design chart) would be most appropriate for each of the following structures?

a. A ten-story reinforced concrete building
b. A one-story wood frame house
c. A nuclear power plant
d. A highway bridge

8.2 Explain why an 8-ft wide footing with q = 3000 lb/ft^2 will settle more than a 3-ft wide one with the same q.

8.3 Under what circumstances would bearing capacity most likely control the design of spread footings? Under what circumstances would settlement usually control?

8.4 A proposed building will have column loads ranging from 40 to 300 k. All of these columns will be supported on square spread footings. When computing the allowable bearing pressure, q_A, which load should be used to perform the bearing capacity analyses? Which should be used to perform the settlement analyses?

8.5 A proposed building will have column loads ranging from 50 to 250 k. These columns are to be supported on spread footings which will be founded in a silty sand with the following engineering properties: γ = 119 lb/ft^3 above the groundwater table and 122 lb/ft^3 below, c' = 0, ϕ' = 32°, N_{60} = 30. The groundwater table is 15 ft below the ground surface. The required factor of safety against a bearing capacity failure must be at least 2.5 and the allowable settlement, δ_a, is 0.75 in.

Compute the allowable bearing pressure for square spread footings founded 2 ft below the ground surface at this site. You may use the spreadsheets described in Chapters 6 and 7 to perform the computations, or you may do so by hand. Then, comment on the feasibility of using spread footings at this site.

8.6 A proposed office building will have column loads between 200 and 1000 kN. These columns are to be supported on spread footings which will be founded in a silty clay with the following engineering properties: γ = 15.1 kN/m^3 above the groundwater table and 16.5 kN/m^3 below, s_u = 200 kPa, $C_r/(1+e_0)$ = 0.020, σ_m' = 400 kPa. The groundwater table is 5 m below the ground surface. The required factor of safety against a bearing capacity failure must be at least 3 and the allowable settlement, δ_a, is 20 mm.

Compute the allowable bearing pressure for square spread footings founded 0.5 m below the ground surface at this site. You may use the spreadsheets described in Chapters 6 and 7 to perform the computations, or you may do so by hand. Then, comment on the feasibility of using spread footings at this site.

8.7 A series of columns carrying vertical loads of 20 to 90 k are to be supported on 3-ft deep square footings. The soil below is a clay with the following engineering properties: γ = 105 lb/ft^3 above the groundwater table and 110 lb/ft^3 below, s_u = 3000 lb/ft^2, $C_r/(1 + e_0)$ = 0.03 in the upper 10 ft and 0.05 below. Both soil strata are overconsolidated Case I. The groundwater table is 5 ft below the ground surface. The factor of safety against a bearing capacity failure must be at least 3. Use the spreadsheets described in Chapters 6 and 7 to compute the allowable bearing pressure, q_A. The allowable settlement is 1.4 in.

8.8 Using the information in Problem 8.7, develop a design chart. Consider footing widths of up to 12 ft.

8.9 Several cone penetration tests have been conducted in a young, normally consolidated silica sand. Based on these tests, an engineer has developed the following design soil profile:

Depth (m)	q_c (kg/cm^2)
0–2.0	40
2.0–3.5	78
3.5–4.0	125
4.0–6.5	100

This soil has an average unit weight of 18.1 kN/m^3 above the groundwater table and 20.8 kN/m^3 below. The groundwater table is at a depth of 3.1 m.

Using this data with the spreadsheets described in Chapters 6 and 7, create a design chart for 1.0-m deep square footings. Consider footing widths of up to 4 m and column loads up to 1500 kN, a factor of safety of 2.5, and a design life of 50 years.

Hint: In a homogeneous soil, the critical shear surface for a bearing capacity failure extends to a depth of approximately B below the bottom of the footing. See Chapter 4 for a correlation between q_c in this zone and ϕ'.

8.2 DESIGN FOR ECCENTRIC OR MOMENT LOADS

Sometimes it becomes necessary to build a footing in which the downward load, P, does not act through the centroid, as shown in Figure 8.6a. One example is in an exterior footing in a structure located close to the property line, as shown in Figure 5.2. The bearing pressure beneath such footings is skewed, as discussed in Section 5.3.

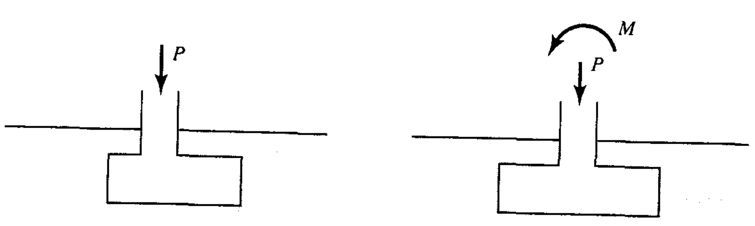


Figure 8.6 (a) Spread footing subjected to an eccentric downward load; (b) Spread footing subjected to a moment load.

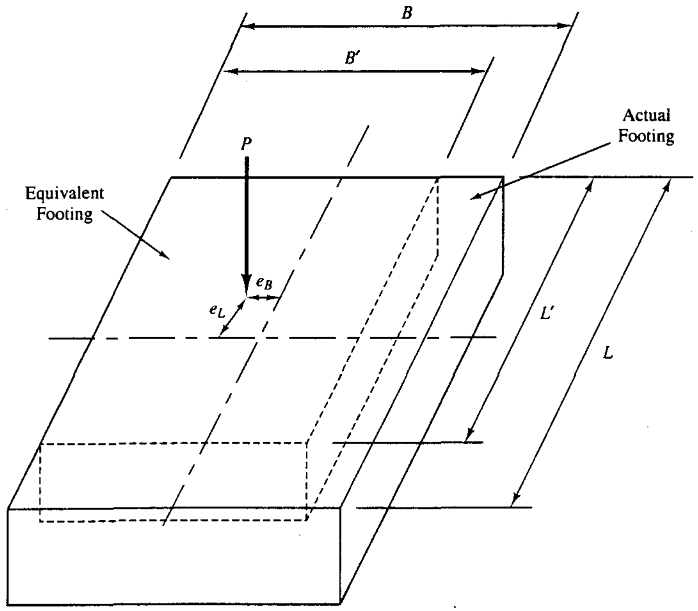


Figure 8.7 Equivalent footing for evaluating the bearing capacity of footings with eccentric or applied moment loads. Note that the equivalent footing has no eccentricity.

Another, more common possibility is a footing that is subjected to an applied moment load, M, as shown in Figure 8.6b. This moment may be permanent, but more often it is a temporary load due to wind or seismic forces acting on the structure. These moment loads also produce a skewed bearing pressure.

Use the following process to design for footings with eccentric or moment loads:

1. Develop preliminary values for the plan dimensions B and L. If the footing is square, then $B = L$. These values might be based on a concentric downward load analysis, as discussed in Section 8.1, or on some other method.
2. Determine if the resultant of the bearing pressure acts within the middle third of the footing (for one-way loading) or within the kern (for two-way loading). The tests for these conditions are described in Equations 5.9 and 5.10, and illustrated in Examples 5.4 and 5.5. If these criteria are not satisfied, then some of the footing will tend to lift off the soil, which is unacceptable. Therefore, any such footings need to be modified by increasing the width or length, as illustrated in Example 5.5.
3. Using the following procedure, determine the effective footing dimensions, B' and L', as shown in Figure 8.7 (Meyerhof, 1963; Brinch Hansen, 1970):
   a. Using Equations 5.3 to 5.6, compute the bearing pressure eccentricity in the B and/or L directions (e_B, e_L).

   b. Compute the effective footing dimensions:

$$\boxed{B' = B - 2e_B} \quad (8.4)$$

$$\boxed{L' = L - 2e_L} \quad (8.5)$$

   This produces an equivalent footing with an area $A' = B' \times L'$ as shown in Figure 8.7.
4. Compute the equivalent bearing pressure using:

$$\boxed{q_{equiv} = \frac{P + W_f}{B'L'} - u_D} \quad (8.6)$$

5. Compare q_{equiv} with the allowable bearing pressure, q_a. If $q_{equiv} \le q_a$ then the design is satisfactory. If not, then increase the footing size as necessary to satisfy this criterion.

Example 8.3

A 5-ft square, 2-ft deep footing supports a vertical load of 80 k and a moment load of 60 ft-k. The underlying soil has an allowable bearing pressure, q_A, of 3500 lb/ft^2 and the groundwater table is at a great depth. Is this design satisfactory?

Solution

$$W_f = (5\text{ ft})(5\text{ ft})(2\text{ ft})(150\text{ lb/ft}^3) = 7500\text{ lb}$$

Using Equation 5.5:

$$e = \frac{M}{P + W_f} = \frac{60\text{ ft-k}}{80\text{ k} + 7.5\text{ k}} = 0.686\text{ ft}$$

$$\frac{B}{6} = \frac{5\text{ ft}}{6} = 0.833\text{ ft}$$

$$e \le \frac{B}{6} \quad \therefore \text{OK for eccentric loading}$$

$$B' = B - 2e_B = 5 - (2)(0.686) = 3.63\text{ ft}$$

$$q_{equiv} = \frac{P + W_f}{A} - u_D = \frac{800{,}000\text{ lb} + 7{,}500\text{ lb}}{(3.63\text{ ft})(5\text{ ft})} - 0 = 4821\text{ lb/ft}^2$$

Since $q_{equiv} > q_A$ (4821 > 3500), this design is not satisfactory. This is true even though eccentric loading requirement ($e \le B/6$) has been met. Therefore, a larger B is required. ⇐ ***Answer***

Further trials will demonstrate that $B = 6$ ft 0 in satisfies all of the design criteria.

8.3 DESIGN FOR SHEAR LOADS

Some footings are also subjected to applied shear loads, as shown in Figure 8.8. These loads may be permanent, as those from retaining walls, or temporary, as with wind or seismic loads on buildings.

Shear loads are resisted by passive pressure acting on the side of the footing, and by sliding friction along the bottom. The allowable shear capacity, V_{fa}, for footings located above the groundwater table at a site with a level ground surface is:

$$V_{fa} = \frac{(P + W_f)\mu + P_p - P_a}{F} \tag{8.7}$$

Passive and active forces are discussed in Chapter 23. However, rather than individually computing them for each footing, it is usually easier to compute λ, which is the net result of the active and passive pressures expressed in terms of an equivalent fluid density. In other words, we evaluate the problem as if the soil along one side of the footing is replaced with a fluid that has a unit weight of λ, then using the principles of fluid statics to compute the equivalent of $P_p - P_a$. Thus, for square footings, Equation 8.7 may be rewritten as:

$$\boxed{V_{fa} = (P + W_f)\mu_a + 0.5\lambda_a BD^2} \tag{8.8}$$

$$\boxed{\mu_a = \frac{\mu}{F}} \tag{8.9}$$

$$\boxed{\lambda_a = \frac{\gamma[\tan^2(45° + \phi/2) - \tan^2(45° - \phi/2)]}{F}} \tag{8.10}$$

Equation 8.10 considers only the frictional strength of the soil. In some cases, it may be appropriate to also consider the cohesive strength using the techniques described in Chapter 23.

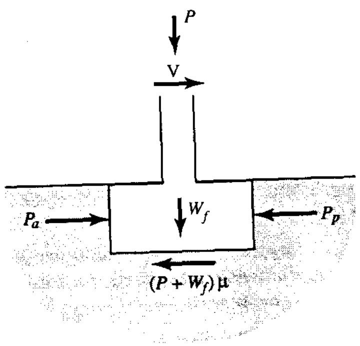


Figure 8.8 Shear load acting on a spread footing.

The footing must then be designed so that:

$$\boxed{V \leq V_a} \tag{8.11}$$

Where:

V = applied shear load
V_{fa} = allowable footing shear load capacity
P = downward load acting on the footing
W_f = weight of footing
B = footing width
D = depth of embedment
μ = coefficient of friction (from Table 8.3 or Equation 8.12)
μ_a = allowable coefficient of friction
λ = equivalent passive fluid density
λ_a = allowable equivalent passive fluid density
ϕ = friction angle of soil (use ϕ' for drained loading conditions or ϕ_T for undrained loading conditions)
F = factor of safety (typically 1.5 to 2.0 for μ and 2 to 3 for λ)

$$\boxed{\mu = \tan(0.7\phi')} \tag{8.12}$$

When quoting or using μ and λ, it is important to clearly indicate whether they are ultimate and allowable values. This is often a source of confusion, which can result in compounding factors of safety, or designing without a factor of safety. Normally, geotechnical engineering reports quote allowable values of these parameters.

The engineer also must be careful to use the proper value of P in Equation 8.8. Typically, multiple load combinations must be considered, and the shear capacity must be satisfactory for each combination. Thus, the P for a particular analysis must be the minimum

TABLE 8.3 DESIGN VALUES OF μ FOR CAST-IN-PLACE CONCRETE (U.S. Navy, 1982b)

Soil or Rock Classification	μ
Clean sound rock	0.70
Clean gravel, gravel-sand mixtures, coarse sand	0.55–0.60
Clean fine-to-medium sand, silty medium-to-coarse sand, silty or clayey gravel	0.45–0.55
Clean fine sand, silty or clayey fine to medium sand	0.35–0.45
Fine sandy silt, nonplastic silt	0.30–0.35
Very stiff and hard residual or overconsolidated clay	0.40–0.50
Medium stiff and stiff clay and silty clay	0.30–0.35

normal load that would be present when the design shear load acts on the footing. For example, if V is due to wind loads on a building, P should be based on dead load only because the live load might not be present when the wind loads occur. If the wind load also causes an upward normal load on the footing, then P would be equal to the dead load minus the upward wind load.

Example 8.4

A 6 ft × 6 ft × 2.5 ft deep footing supports a column with the following design loads: $P = 112$ k, $V = 20$ k. The soil is a silty fine-to-medium sand with $\phi' = 29°$, and the groundwater table is well below the bottom of the footing. Check the shear capacity of this footing and determine if the design will safely withstand the design shear load.

Solution

Per Table 8.3: $\mu = 0.35–0.45$
Per Equation 8.11: $\mu = \tan[0.7(29)] = 0.37$
$\therefore$ Use $\mu = 0.38$

$$\mu_a = \frac{\mu}{F} = \frac{0.38}{1.5} = 0.25$$

$$\lambda_a = \frac{120[\tan^2(45 + 29/2) - \tan^2(45 - 29/2)]}{2} = 152\ \text{lb/ft}^3$$

$$W_f = (6)(6)(2.5)(150) = 13{,}500\ \text{lb}$$

$$V_{fa} = (112 + 12.5)(0.25) + (0.5)\left(\frac{152}{1000}\right)(6)(2.5^2) = 34\ \text{k} \quad \Leftarrow \textit{Answer}$$

$V \le V_{fa}(20 \le 34)$ so the footing has sufficient lateral load capacity ⇐ *Answer*

Footings subjected to applied shear loads also have a smaller ultimate bearing capacity, which may be assessed using the i factors in Vesic's method, as described in Chapter 6. This reduction in bearing capacity is often ignored when the shear load is small (i.e., less than about 0.20 P), but it can become significant with larger shear loads.

QUESTIONS AND PRACTICE PROBLEMS

8.10 A square spread footing with $B = 1000$ mm and $D = 500$ mm supports a column with the following design loads: $P = 150$ kN, $M = 22$ kN-m. The underlying soil has an allowable bearing pressure of 200 kPa. Is this design acceptable? If not, compute the minimum required footing width and express it as a multiple of 100 mm.

8.11 A 3 ft × 7 ft rectangular footing is to be embedded 2 ft into the ground and will support a single centrally-located column with the following design loads: $P = 50$ k, $M = 80$ ft-k (acts in long direction only). The underlying soil is a silty sand with $c' = 0$, $\phi' = 31°$, $\gamma = 123$ lb/ft^3, and a very deep groundwater table. Using a factor of safety of 2.5, determine if this design is acceptable for bearing capacity.

8.12 A 4-ft square spread footing embedded 1.5 ft into the ground is subjected to the following design loads: $P = 25$ k, $V = 6$ k. The underlying soil is a well-graded sand with $c' = 0$, $\phi' = 36°$, $\gamma = 126$ lb/ft^3, and a very deep groundwater table. Using a factor of safety of 2.5 on bearing capacity, 2 on passive pressure, and 1.5 on sliding friction, determine if this design is acceptable for bearing capacity and for lateral load capacity.

8.4 DESIGN FOR WIND OR SEISMIC LOADS

Many spread footings are subjected to wind or seismic loads in addition to the static loads. For purposes of foundation design, these loads are nearly always expressed in terms of equivalent static loads, as discussed in Section 2.1. In some cases, engineers might use dynamic analyses to evaluate the seismic loads acting on foundations, but such methods are beyond the scope of this book.

Wind and seismic loads are primarily horizontal, so they produce shear loads on the foundations and thus require a shear load capacity evaluation as discussed in Section 8.3. In addition, wind and seismic loads on the superstructure can produce additional normal loads (either downward or upward) on the foundations. These loads are superimposed on the static normal loads. In some cases, such as single pole transmission towers, wind and seismic loads impart moments onto the foundation.

When these loads are expressed as equivalent static loads, the methods of evaluating the load capacity of foundations is essentially the same as for static loads. However, geotechnical engineers usually permit a 33 percent greater load-bearing capacity for load comminations that include wind or seismic components. This increase is based on the following considerations:

- The shear strength of soils subjected to rapid loading, such as from wind or seismic loads, is greater than that during sustained loading, especially in sands. Therefore the ultimate bearing capacity and ultimate lateral capacity is correspondingly larger.
- We are willing to accept a lower factor of safety against a bearing capacity failure or a lateral sliding failure under the transitory loads because these loads are less likely to occur.
- Settlement in soils subjected to transitory loading is generally less than that under an equal sustained load, because the soil has less time to respond.
- We can tolerate larger settlements under transitory loading conditions. In other words, most people would accept some cracking and other minor distress in a structure following a design windstorm or a design earthquake, both of which would be rare events.

For example, if the allowable bearing pressure, q_A, for static loads computed using the technique described in Section 8.1 is 150 kPa, the allowable bearing pressure for load combinations that include wind or seismic components would be (1.33)(150 kPa) = 200 kPa.

Most building codes allow this one-third increase for short term loads [ICBO 1612.3, 1809.2, and Table 18-I-A], [BOCA 1805.2], [ICC 1605.3.2 and Table 1804.2]. In addition, most building codes permit the geotechnical engineer to specify allowable bearing pressures based on a geotechnical investigation, and implicitly allow the flexibility to express separate allowable bearing pressures for short- and long-term loading conditions.

This one-third increase is appropriate for most soil conditions. However, it probably should not be used for foundations supported on soft clayey soils, because they may have lower strength when subjected to strong wind or seismic loading (Krinitzky, et al., 1993). In these soils, the foundations should be sized using a design load equal to the greatest of Equations 2.1 to 2.4 and the q_A value from Chapter 8.

There are two ways to implement this one-third increase in the design process for downward loads:

Method 1:

1. Compute the long duration load as the greater of that produced by Equations 2.1 and 2.2.
2. Size the foundation using the load from Step 1, the q_A from Chapter 8, and Equation 8.2 or 8.3.
3. Compute the short duration load as the greater of that produced by Equations 2.3 and 2.4.
4. Size the foundation using the load from Step 3, 1.33 times the q_A from Chapter 8, and Equation 8.2 or 8.3.
5. Use the larger of the footing sizes from Steps 2 and 4 (i.e., the final design may be controlled by either the long term loading condition or the short term loading condition).

This method is a straightforward application of the principle described above, but can be tedious to implement. The second method is an attempt to simplify the analysis while producing the same design:

Method 2:

1. Compute the design load as the greatest of that produced by Equations 2.1, 2.2, 2.3a, and 2.4a.
2. Size the foundation using the load from Step 1, the q_A from Chapter 8, and Equation 8.2 or 8.3.

Therefore, the author recommends using Method 2.

The design process for shear loads also may use either of these two methods. Once again, it is often easier to use Method 2.

Special Seismic Considerations

Loose sandy soils pose special problems when subjected to seismic loads, especially if these soils are saturated. The most dramatic problem is *soil liquefaction*, which is the sudden loss of shear strength due to the build-up of excess pore water pressures (see Coduto,

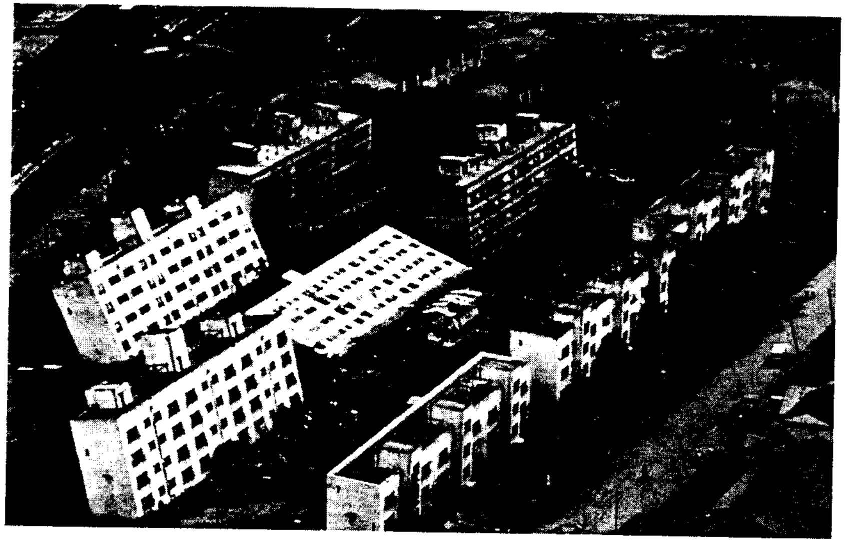


Figure 8.9 The soils beneath these apartment buildings in Niigata, Japan liquified during the 1964 earthquake, which produced bearing capacity failures. These failures reportedly occurred very slowly, and the buildings were very strong and rigid, so they remained virtually intact as they tilted. Afterwards, the occupants of the center building were able to evacuate by walking down the exterior wall (Earthquake Engineering Research Center Library, University of California, Berkeley, Steinbrugge Collection).

1999). This loss in strength can produce a bearing capacity failure, as shown in Figure 8.9. Another problem with loose sands, even if they are not saturated and not prone to liquefaction, is earthquake-induced settlement. In some cases, such settlements can be very large.

Earthquakes also can induce landslides, which can undermine foundations built near the top of a slope. This type of failure occurred in Anchorage, Alaska, during the 1964 earthquake, as well as elsewhere. The evaluation of such problems is a slope stability concern, and thus is beyond the scope of this book.

8.5 LIGHTLY-LOADED FOOTINGS

The principles of bearing capacity and settlement apply to all sizes of spread footings. However, the design process can be simplified for lightly-loaded footings. For purposes of geotechnical foundation design, we will define *lightly-loaded footings* as those subjected to vertical loads less than 200 kN (45 k) or 60 kN/m (4 k/ft). These include typical one- and two-story wood-frame buildings, and other similar structures. The foundations for such structures are small, and do not impose large loads onto the ground, so extensive subsurface investigation and soil testing programs are generally not cost-effective. Normally it is less expensive to use conservative designs than it is to conduct extensive investigations and analyses.

Presumptive Allowable Bearing Pressures

Spread footings for lightweight structures are often designed using *presumptive allowable bearing pressures* (also known as *prescriptive bearing pressures*) which are allowable bearing pressures obtained directly from the soil classification. These presumptive bearing pressures appear in building codes, as shown in Table 8.4. They are easy to implement, and do not require borings, laboratory testing, or extensive analyses. The engineer simply obtains the q_A value from the table and uses it with Equation 8.2 or 8.3 to design the footings.

TABLE 8.4 PRESUMPTIVE ALLOWABLE BEARING PRESSURES FROM VARIOUS BUILDING CODES[a,c]

	Allowable Bearing Pressure, q_A lb/ft^2 (kPa)		
Soil or Rock Classification	Uniform Building Code[b] (ICBO, 1997)	National Building Code (BOCA, 1996) and International Building Code (ICC, 2000)	Canadian Code (NRCC, 1990)
Massive crystalline bedrock	4,000–12,000 (200–600)	12,000 (600)	40,000–200,000 (2,000–10,000)
Sedimentary rock	2,000–6,000 (100–300)	6,000 (300)	10,000–80,000 (500–4000)
Sandy gravel or gravel	2,000–6,000 (100–300)	5,000 (250)	4,000–12,000 (200–600)
Sand, silty sand, clayey sand, silty gravel, or clayey gravel	1,500–4,500 (75–225)	3,000 (150)	2,000–8,000 (100–400)
Clay, sandy clay, silty clay, or clayey silt	1,000–3,000 (50–150)	2,000 (100)	1,000–12,000 (50–600)

[a]The values in this table are for illustrative purposes only and are not a complete description of the code provisions. Portions of the table include the author's interpretations to classify the presumptive bearing values into uniform soil groups. Refer to the individual codes for more details.

[b]The Uniform Building Code values in soil are intended to provide a factor of safety of at least 3 against a bearing capacity failure, and a total settlement of no more than 0.5 in (12 mm) (ICBO, 1997). The lower value for each soil is intended for footings with B = 12 in (300 mm) and D = 12 in (300 mm) and may be increased by 20 percent for each additional foot of width and depth to the maximum value shown. Exception: No increase for additional width is allowed for clay, sandy clay, silty clay, or clayey silt.

[c]The Standard Building Code (SBCCI, 1997) does not include any presumptive allowable bearing pressures.

Presumptive allowable bearing pressures have been used since the late nineteenth century, and thus predate bearing capacity and settlement analyses. Today they are used primarily for lightweight structures on sites known to be underlain by good soils. Although presumptive bearing pressures are usually conservative (i.e., they produce larger footings), the additional construction costs are small compared to the savings in soil exploration and testing costs.

However, it is inappropriate to use presumptive bearing pressures for larger structures founded on soil because they are not sufficiently reliable. Such structures warrant more extensive engineering and design, including soil exploration and testing. They also should not be used on sites underlain by poor soils.

Minimum Dimensions

If the applied loads are small, such as with most one- or two-story wood-frame structures, bearing capacity and settlement analyses may suggest that extremely small footings would be sufficient. However, from a practical perspective, very small footings are not acceptable for the following reasons:

- Construction of the footing and the portions of the structure that connect to it would be difficult.
- Excavation of soil to build a footing is by no means a precise operation. If the footing dimensions were very small, the ratio of the construction tolerances to the footing dimensions would be large, which would create other construction problems.
- A certain amount of flexural strength is necessary to accommodate nonuniformities in the loads and local inconsistencies in the soil, but an undersized footing would have little flexural strength.

Therefore, all spread footings should be built with certain minimum dimensions. Figure 8.10 shows typical minimums. In addition, building codes sometimes dictate other minimum dimensions. For example, the *Uniform Building Code* and the International Building Code stipulate certain minimum dimensions for footings that support wood-frame structures. The minimum dimensions for continuous footings are presented in Table 8.5, and those for square footings are presented in Note 3 of the table. This code also allows the geotechnical engineer to supercede these minimum dimensions [UBC 1806.1, IBC 1805.21].

Potential Problems

Although the design of spread footings for lightweight structures can be a simple process, as just described, be aware that such structures are not immune to foundation problems. Simply following these presumptive bearing pressures and code minimums does not necessarily produce a good design. Engineers need to know when these simple design guidelines are sufficient, and when additional considerations need to be included.

Most problems with foundations of lightweight structures are caused by the soils below the foundations, rather than high loads from the structure. For example, founda-

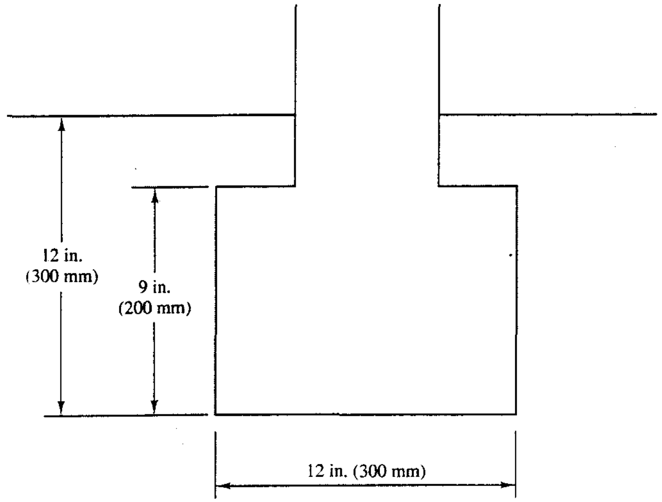


Figure 8.10 Minimum dimensions for spread footings. If the footing is reinforced, the thickness should be at least 12 in (3000 mm).

TABLE 8.5 MINIMUM DIMENSIONS FOR CONTINUOUS FOOTINGS THAT SUPPORT WOOD-FRAME BEARING WALLS PER UBC AND IBC (ICBO, 1997 and ICC, 2000)

Number of floors supported by the foundation	Thickness of Foundation Wall		Footing Width, *B*		Footing Thickness, *T*		Footing Depth Below Undisturbed Ground Surface, *D*	
	(mm)	(in)	(mm)	(in)	(mm)	(in)	(mm)	(in)
1	150	6	300	12	150	6	300	12
2	200	8	375	15	175	7	450	18
3	250	10	450	18	200	8	600	24

1. Where unusual conditions or frost conditions are found, footings and foundations shall be as required by UBC Section 1806.1 or IBC Section 1805.2.1.
2. The ground under the floor may be excavated to the elevation of the top of the footing.
3. Interior stud bearing walls may be supported by isolated footings. The footing width and length shall be twice the width shown in this table and the footings shall be spaced not more than 6 ft (1829 mm) on center.
4. In Seismic Zone 4, continuous footings shall be provided with a minimum of one No. 4 bar top and bottom.
5. Foundations may support a roof in addition to the stipulated number of floors. Foundations supporting roofs only shall be as required for supporting one floor.

tions placed in loose fill may settle because of the weight of the fill or because of infiltration of water into the fill. Expansive soils, collapsible soils, landslides, and other problems also can affect foundations of lightweight structures. These problems may justify more extensive investigation and design effort.

QUESTIONS AND PRACTICE PROBLEMS

8.13 A certain square spread footing for an office building is to support the following downward design loads: dead load = 800 kN, live load = 500 kN, seismic load = 400 kN. The 33 percent increase for seismic load capacity is applicable to this site.

a. Compute the design load.

b. Using the design chart from Example 8.2, determine the required width of this footing such that the total settlement is no more than 20 mm.

8.14 A three-story wood-frame building is to be built on a site underlain by sandy clay. This building will have wall loads of 1900 lb/ft on a certain exterior wall. Using the minimum dimensions presented in Table 8.4 and presumptive bearing pressures from the International Building Code as presented in Table 8.5, compute the required width and depth of this footing. Show your final design in a sketch.

8.6 FOOTINGS ON OR NEAR SLOPES

Vesic's bearing capacity formulas in Chapter 6 are able to consider footings near sloping ground, and we also could compute the settlement of such footings. However, it is best to avoid this condition whenever possible. Special concerns for such situations include:

- The reduction in lateral support makes bearing capacity failures more likely.
- The foundation might be undermined if a shallow (or deep!) landslide were to occur.
- The near-surface soils may be slowly creeping downhill, and this creep may cause the footing to move slowly downslope. This is especially likely in clays.

However, there are circumstances where footings must be built on or near a slope. Examples include abutments of bridges supported on approach embankments, foundations for electrical transmission towers, and some buildings.

Shields, Chandler, and Garnier (1990) produced another solution for the bearing capacity of footings located on sandy slopes. This method, based on centrifuge tests, relates the bearing capacity of footings at various locations with that of a comparable footing with $D = 0$ located on level ground. Figures 8.11 to 8.13 give this ratio for 1.5:1 and 2:1 slopes.

The Uniform Building Code and the International Building Code require setbacks as shown in Figure 8.14. We can meet these criteria either by moving the footing away from the slope or by making it deeper.

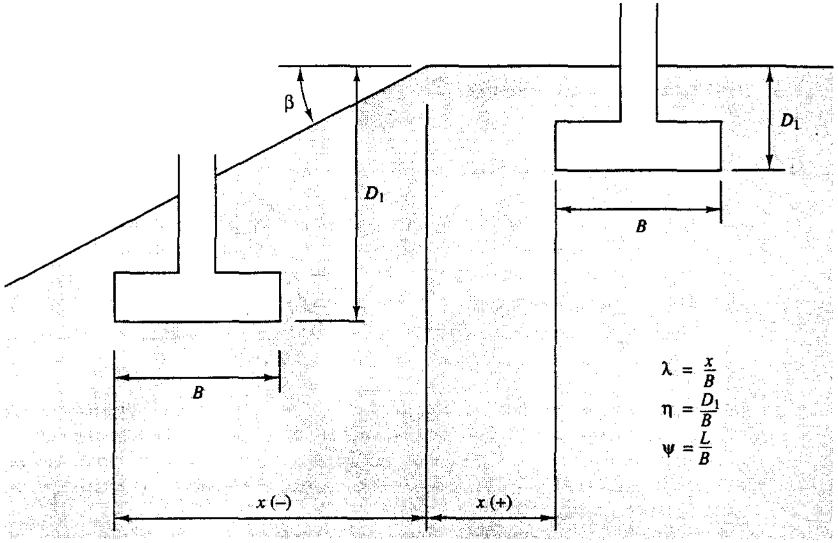


Figure 8.11 Definition of terms for computing bearing capacity of footings near or on sandy slopes (Adapted from Shields, Chandler and Garnier, 1990; Used by permission of ASCE).

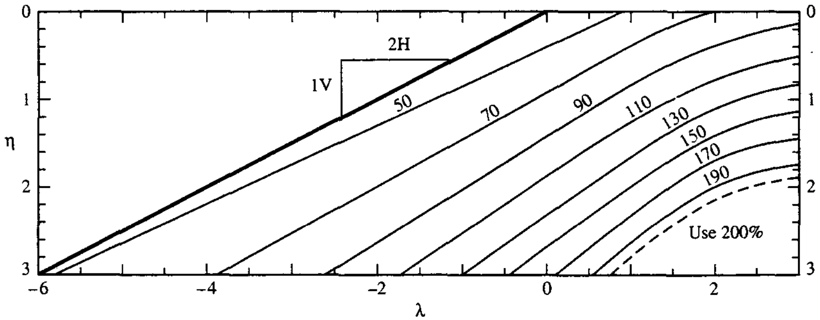


Figure 8.12 Bearing capacity of footings near or on a 2H:1V sandy slopes. The contours are the bearing capacity divided by the bearing capacity of a comparable footing located at the surface of level ground, expressed as a percentage. (Adapted from Shields, Chandler and Garnier, 1990; Used by permission of ASCE).

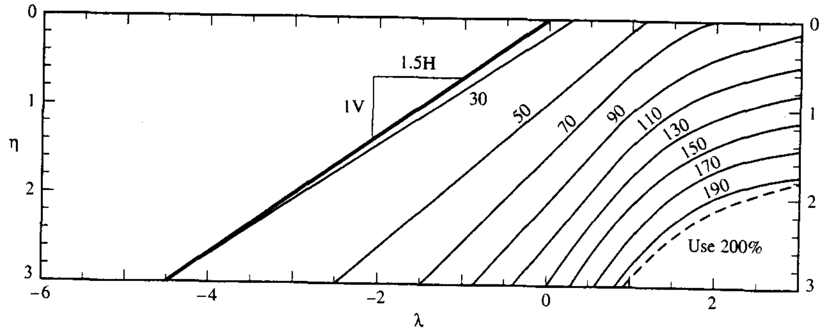


Figure 8.13 Bearing capacity of footings near or on a 1.5H:1V sandy slopes. The contours are the bearing capacity divided by the bearing capacity of a comparable footing located at the surface of level ground, expressed as a percentage. (Adapted from Shields, Chandler and Garnier, 1990; Used by permission of ASCE).

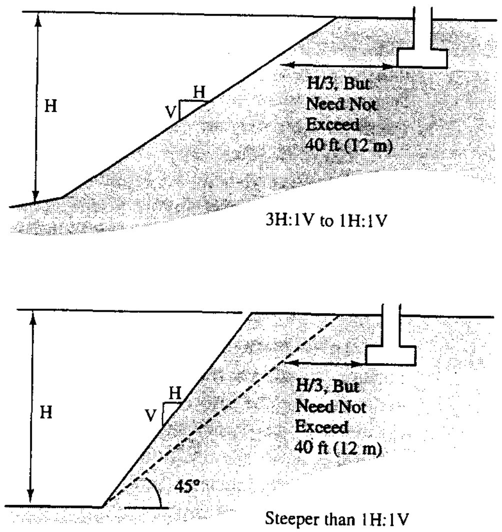


Figure 8.14 Footing setback as required by the Uniform Building Code [1806.5] and the International Building Code [1805.3] for slopes steeper than 3 horizontal to 1 vertical. The horizontal distance from the footing to the face of the slope should be at least $H/3$, but need not exceed 40 ft (12 m). For slopes that are steeper than 1 horizontal to 1 vertical, this setback distance should be measured from a line that extends from the toe of the slope at an angle of 45°. (Adapted from the 1997 edition of the *Uniform Building Code*, © 1997, with the permission of the publisher, the International Conference of Building Officials and the 2000 edition of the International Building Code).

However, this setback criteria does not justify building foundations above unstable slopes. Therefore, we also should perform appropriate slope stability analyses to verify the overall stability.

8.7 FOOTINGS ON FROZEN SOILS

In many parts of the world, the air temperature in the winter often falls below the freezing point of water (0° C) and remains there for extended periods. When this happens, the ground becomes frozen. In the summer, the soils become warmer and return to their unfrozen state. Much of the northern United States, Canada, central Europe, and other places with similar climates experience this annual phenomenon.

The greatest depth to which the ground might become frozen at a given locality is known as the *depth of frost penetration*. This distance is part of an interesting thermodynamics problem and is a function of the air temperature and its variation with time, the initial soil temperature, the thermal properties of the soil, and other factors. The deepest penetrations are obtained when very cold air temperatures are maintained for a long duration. Typical depths of frost penetration in the United States are shown in Figure 8.15.

These annual freeze–thaw cycles create special problems that need to be considered in foundation design.

Frost Heave

The most common foundation problem with frozen soils is *frost heave*, which is a rising of the ground when it freezes.

When water freezes, it expands about 9 percent in volume. If the soil is saturated and has a typical porosity (say, 40 percent), it will expand about 9% × 40% ≈ 4% in volume when it freezes. In climates comparable to those in the northern United States, this could correspond to surface heaves of as much as 25 to 50 mm (1–2 in). Although such heaves are significant, they are usually fairly uniform, and thus cause relatively little damage.

However, there is a second, more insidious source of frost heave. If the groundwater table is relatively shallow, capillary action can draw water up to the frozen zone and form ice lenses, as shown in Figure 8.16. In some soils, this mechanism can move large quantities of water, so it is not unusual for these lenses to produce ground surface heaves of 300 mm (1 ft) or more. Such heaves are likely to be very irregular and create a hummocky ground surface that could extensively damage structures, pavements, and other civil engineering works.

In the spring, the warmer weather permits the soil to thaw, beginning at the ground surface. As the ice melts, it leaves a soil with much more water than was originally present. Because the lower soils will still be frozen for a time, this water temporarily cannot drain away, and the result is a supersaturated soil that is very weak. This condition is often the cause of ruts and potholes in highways and can also effect the performance of shallow foundations and floor slabs. Once all the soil has thawed, the excess water drains down and the soil regains its strength. This annual cycle is shown in Figure 8.17.

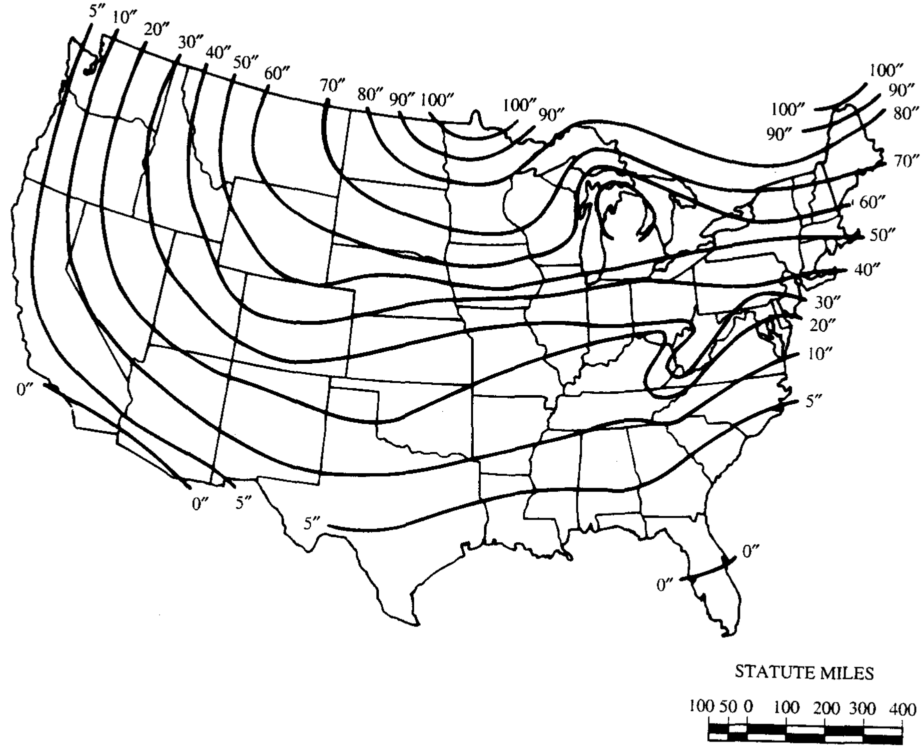


Figure 8.15 Approximate depth of frost penetration in the United States (U.S. Navy, 1982a).

To design foundations in soils that are prone to frost heave, we need to know the depth of frost penetration. This depth could be estimated using Figure 8.15, but as a practical matter it is normally dictated by local building codes. For example, the Chicago Building Code specifies a design frost penetration depth of 1.1 m (42 in). Rarely, if ever, would a rigorous thermodynamic analysis be performed in practice.

Next, the engineer will consider whether ice lenses are likely to form within the frozen zone, thus causing frost heave. This will occur only if both of the following conditions are met:

1. There is a nearby source of water; and
2. The soil is *frost-susceptible*.

To be considered frost-susceptible, a soil must be capable of drawing significant quantities of water up from the groundwater table into the frozen zone. Clean sands and

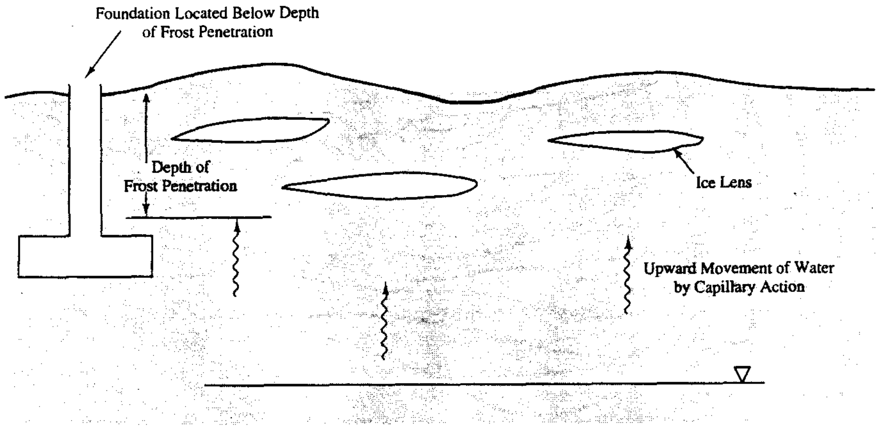


Figure 8.16 Formation of ice lenses. Water is drawn up by capillary action and freezes when it nears the surface. The frozen water forms ice lenses that cause heaving at the ground surface. Foundations placed below the depth of frost penetration are not subject to heaving.

gravels are not frost-susceptible because they are not capable of significant capillary rise. Conversely, clays are capable of raising water through capillary rise, but they have a low permeability and are therefore unable to deliver large quantities of water. Therefore,.clays are capable of only limited frost heave. However, intermediate soils, such as silts and fine sands, have both characteristics: They are capable of substantial capillary rise and have a high permeability. Large ice lenses are able to form in these soils, so they are considered to be very frost-susceptible.

The U.S. Army Corps of Engineers has classified frost-susceptible soils into four groups, as shown in Table 8.6. Higher group numbers correspond to greater frost susceptibility and more potential for formation of ice lenses. Clean sands and gravels (i.e., < 3% finer than 0.02 mm) may be considered non frost-susceptible and are not included in this table.

The most common method of protecting foundations from the effects of frost heave is to build them at a depth below the depth of frost penetration. This is usually wise in all soils, whether or not they are frost-susceptible and whether or not the groundwater table is nearby. Even "frost-free" clean sands and gravels often have silt lenses that are prone to heave, and groundwater conditions can change unexpectedly, thus introducing new sources of water. The small cost of building deeper foundations is a wise investment in such cases. However, foundations supported on bedrock or interior foundations in heated buildings normally do not need to be extended below the depth of frost penetration.

Builders in Canada and Scandinavia often protect buildings with slab-on-grade floors using thermal insulation, as shown in Figure 8.18. This method traps heat stored in the ground during the summer and thus protects against frost heave, even though the

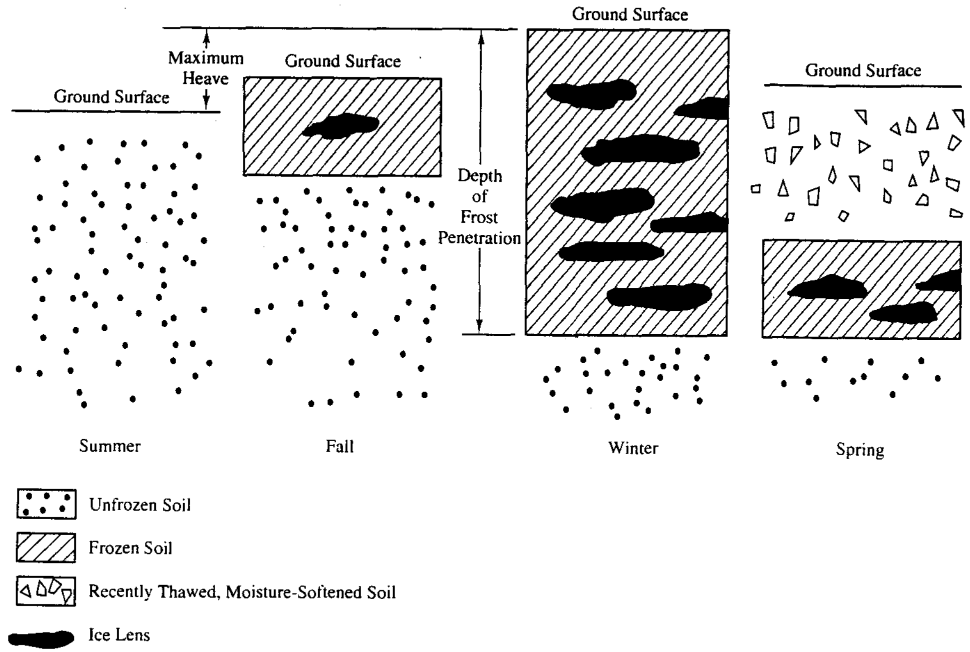


Figure 8.17 Idealized freeze-thaw cycle in temperate climates. During the summer, none of the ground is frozen. During the fall and winter, it progressively freezes from the ground surface down. Then, in the spring, it progressively thaws from the ground surface down.

foundations are shallower than the normal frost depth. Both heated and nonheated buildings can use this technique (NAHB, 1988 and 1990).

Alternatively, the natural soils may be excavated to the frost penetration depth and replaced with soils that are known to be frost-free. This may be an attractive alternative for unheated buildings with slab floors to protect both the floor and the foundation from frost heave.

Although frost heave problems are usually due to freezing temperatures from natural causes, it is also possible to freeze the soil artificially. For example, refrigerated buildings such as cold-storage warehouses or indoor ice skating rinks can freeze the soils below and be damaged by frost heave, even in areas where natural frost heave is not a concern (Thorson and Braun, 1975). Placing insulation or air passages between the building and the soil and/or using non–frost-susceptible soils usually prevents these problems.

A peculiar hazard to keep in mind when foundations or walls extend through frost-susceptible soils is *adfreezing* (CGS, 1992). This is the bonding of soil to a wall or foundation as it freezes. If heaving occurs after the adfreezing, the rising soil will impose a large

TABLE 8.6 FROST SUSCEPTIBILITY OF VARIOUS SOILS ACCORDING TO THE U.S. ARMY CORPS OF ENGINEERS (Adapted from Johnston, 1981).

Group	Soil Types[a]	USCS Group Symbols[b]
F1 (least susceptible)	Gravels with 3–10% finer than 0.02 mm	GW, GP, GW-GM, GP-GM
F2	a. Gravels with 10–20% finer than 0.02 mm	GM, GW-GM, GP-GM
	b. Sands with 3–15% finer than 0.02 mm	SW, SP, SM, SW-SM, SP-SM
F3	a. Gravels with more than 20% finer than 0.02 mm	GM, GC
	b. Sands, except very fine silty sands, with more than 15% finer than 0.02 mm	SM, SC
	c. Clays with $I_P > 12$, except varved clays	CL, CH
F4 (most susceptible)	a. Silts and sandy silts	ML, MH
	b. Fine silty sands with more than 15% finer than 0.02 mm	SM
	c. Lean clays with $I_P < 12$	CL, CL-ML
	d. Varved clays and other fine-grained, banded sediments	

[a] I_P = Plasticity Index (explained in Chapter 3).
[b] See Chapter 3 for an explanation of USCS group symbols.

upward load on the structure, possibly separating structural members. Placing a 10-mm (0.5 in) thick sheet of rigid polystyrene between the foundation and the frozen soil reduces the adfreezing potential.

Permafrost

In areas where the mean annual temperature is less than 0°C, the penetration of freezing in the winter may exceed the penetration of thawing in the summer and the ground can become frozen to a great depth. This creates a zone of permanently frozen soil known as *permafrost*. In the harshest of cold climates, such as Greenland, the frozen ground is continuous, whereas in slightly "milder" climates, such as central Alaska, central Canada, and much of Siberia, the permafrost is discontinuous. Areas of seasonal and continuous permafrost in Canada are shown in Figure 8.19.

In areas where the summer thaws occur, the upper soils can be very wet and weak and probably not capable of supporting any significant loads, while the deeper soils remain permanently frozen. Foundations must penetrate through this seasonal zone and well into the permanently frozen ground below. It is very important that these foundations be designed so that they do not transmit heat to the permafrost, so buildings are typically built with raised floors and a ducting system to maintain subfreezing air temperatures between the floor and the ground surface.

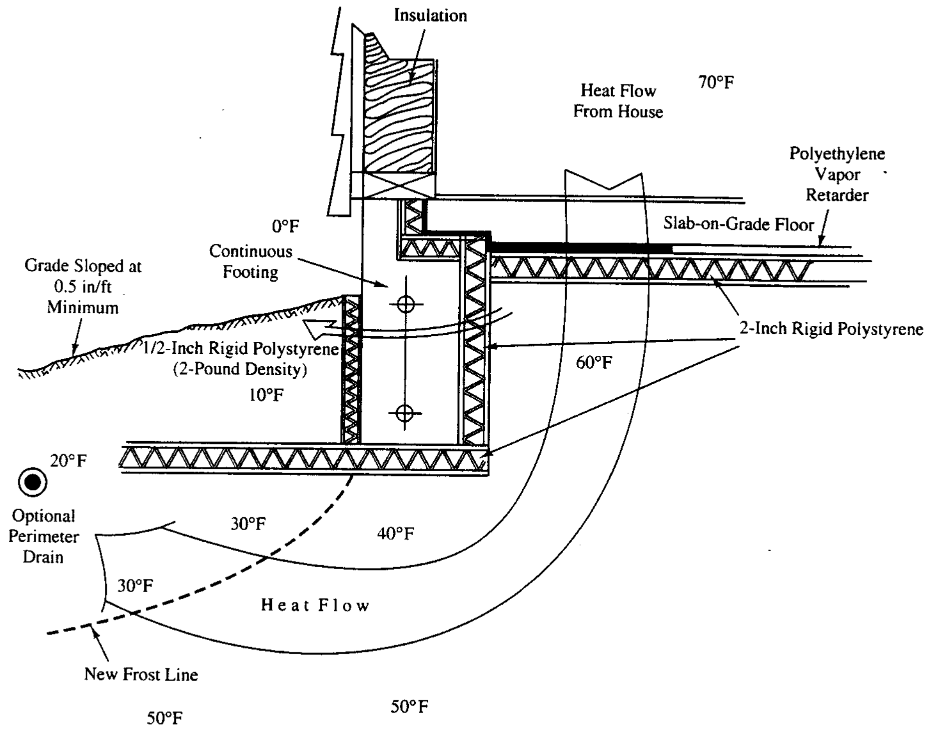


Figure 8.18 Thermal insulation traps heat in the soil, thus protecting a foundation from frost heave (NAHB, 1988, 1990).

The Alaska Pipeline project is an excellent example of a major engineering work partially supported on permafrost (Luscher et. al, 1975).

8.8 FOOTINGS ON SOILS PRONE TO SCOUR

Scour is the loss of soil because of erosion in river bottoms or in waterfront areas. This is an important consideration for design of foundations for bridges, piers, docks, and other structures, because the soils around and beneath the foundation could be washed away.

Scour around the foundations is the most common cause of bridge failure. For example, during spring 1987, there were seventeen bridge failures caused by scour in the northeastern United States alone (Huber, 1991). The most notable of these was the collapse of the Interstate Route 90 bridge over Schoharie Creek in New York (Murillo,

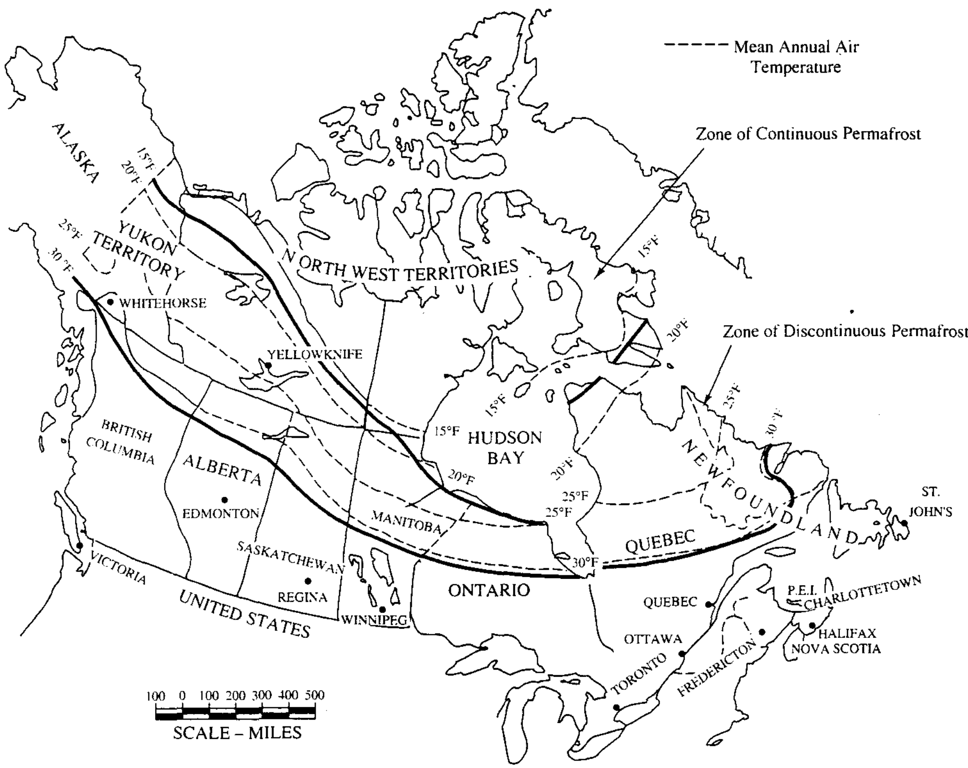


Figure 8.19 Zones of continous and discontinuous permafrost in Canada (Adapted from Crawford and Johnson, 1971).

1987), a failure that killed ten people. Figures 8.20 and 8.21 show another bridge that collapsed as a result of scour.

Scour is part of the natural process that moves river-bottom sediments downstream. It can create large changes in the elevation of the river bottom. For example, Murphy (1908) describes a site on the Colorado River near Yuma, Arizona, where the river bed consists of highly erodible fine silty sands and silts. While passing a flood, the water level at this point rose 4.3 m (14 ft) and the bottom soils scoured to depths of up to 11 m (36 ft)! If a bridge foundation located 10.7 m (35 ft) below the river bottom had been built at this location, it would have been completely undermined by the scour and the bridge would have collapsed.

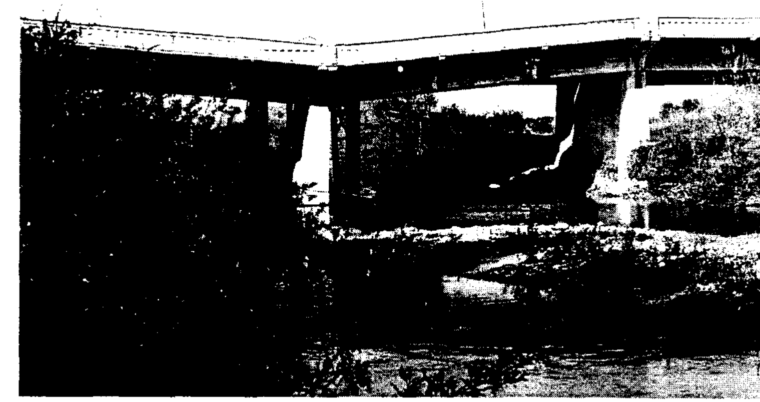


Figure 8.20 One of the mid-channel piers supporting this bridge sank about 1.5 m when it was undermined by scour in the river channel.

Figure 8.21 Deck view of the bridge shown in Figure 8.20. The lanes on the right side of the fence are supported by a separate pier that was not undermined by the scour.

Scour is often greatest at places where the river is narrowest and constrained by levees or other means. Unfortunately, these are the locations most often selected for bridges. The presence of a bridge pier also creates water flow patterns that intensify the scour. However, methods are available to predict scour depths (Richardson et al., 1991) and engineers can use preventive measures, such as armoring, to prevent scour problems (TRB, 1984).

8.9 FOOTINGS ON ROCK

In comparison to foundations on soil, those on bedrock usually present few difficulties for the designer (Peck, 1976). The greatest problems often involve difficulties in construction, such as excavation problems and proper removal of weathered or disturbed material to provide good contact between the footing and the bedrock.

The allowable bearing pressure on rock may be determined in at least four ways (Kulhawy and Goodman, 1980):

- Presumptive allowable bearing pressures
- Empirical rules
- Rational methods based on bearing capacity and settlement analyses
- Full-scale load tests

When supported on good quality rock, spread footings are normally able to support moderately large loads with very little settlement. Engineers usually design them using presumptive bearing pressures, preferably those developed for the local geologic conditions. Typical values are listed in Table 8.7.

If the rock is very strong, the strength of the concrete may govern the bearing capacity of spread footings. Therefore, do not use an allowable bearing value, q_a, greater than one-third of the compressive strength of the concrete ($0.33 f_c'$).

When working with bedrock, be aware of certain special problems. For example, soluble rocks, including limestone, may have underground cavities that might collapse, causing *sinkholes* to form at the ground surface. These have caused extensive damage to buildings in Florida and elsewhere.

Soft rocks, such as siltstone, claystone, and mudstone, are very similar to hard soil, and often can be sampled, tested, and evaluated using methods developed for soils.

TABLE 8.7 TYPICAL PRESUMPTIVE ALLOWABLE BEARING PRESSURES FOR FOUNDATIONS ON BEDROCK (Adapted from US Navy, 1982b)

Rock Type	Rock Consistency	Allowable Bearing Pressure, q_A (lb/ft^2)	(kPa)
Massive crystalline igneous and metamorphic rock: Granite, diorite, basalt, gneiss, thoroughly cemented conglomerate	Hard and sound (minor cracks OK)	120,000–200,000	6000–10,000
Foliated metamorphic rock: Slate, schist	Medium hard, sound (minor cracks OK)	60,000–80,000	3000–4000
Sedimentary rock: Hard-cemented shales, siltstone, sandstone, limestone without cavities	Medium hard, sound	30,000–50,000	1500–2500
Weathered or broken bedrock of any kind; compaction shale or other argillaceous rock in sound condition	Soft	16,000–24,000	800–1200

QUESTIONS AND PRACTICE PROBLEMS

8.15 A 4 ft square, 2 ft deep spread footing carries a compressive column load of 50 k. The edge of this footing is 1 ft behind the top of a 40 ft tall, 2H:1V descending slope. The soil has the following properties: $c = 200$ lb/ft^2, $\phi = 31°$, $\gamma = 121$ lb/ft^3, and the groundwater table is at a great depth. Compute the factor of safety against a bearing capacity failure and comment on this design.

8.16 Classify the frost susceptibility of the following soils:

a. Sandy gravel (GW) with 3% finer than 0.02 mm.
b. Well graded sand (SW) with 4% finer than 0.02 mm.
c. Silty sand (SM) with 20% finer than 0.02 mm.
d. Fine silty sand (SM) with 35% finer than 0.02 mm.
e. Sandy silt (ML) with 70% finer than 0.02 mm.
f. Clay (CH) with plasticity index = 60.

8.17 A compacted fill is to be placed at a site in North Dakota. The following soils are available for import: Soil 1 - silty sand; Soil 2 - lean clay; Soil 3 - Gravelly coarse sand. Which of these soils would be least likely to have frost heave problems?

8.18 Would it be wise to use slab-on-grade floors for houses built on permafrost? Explain.

8.19 What is the most common cause of failure in bridges?

8.20 A single-story building is to be built on a sandy silt in Detroit. How deep must the exterior footings be below the ground surface to avoid problems with frost heave?

SUMMARY

Major Points

1. The depth of embedment, *D*, must be great enough to accommodate the required footing thickness, *T*. In addition, certain geotechnical concerns, such as loose soils or frost heave, may dictate an even greater depth.
2. The required footing width, *B*, is a geotechnical problem, and is governed by bearing capacity and settlement criteria. It is inconvenient to satisfy these criteria by performing custom bearing capacity and settlement computations for each footing, so we present the results of generic computations in a way that is applicable to the entire site. There are two methods of doing so: the allowable bearing pressure method and the design chart method.
3. Footings subjected to eccentric or moment loads need to be evaluated using the "equivalent footing."
4. Footings can resist applied shear loads through passive pressure and sliding friction.
5. Wind and seismic loads are normally treated as equivalent static loads. For most soils, load combinations that include wind or seismic components may be evaluated using a 33 percent greater allowable bearing pressure.
6. The design of lightly-loaded footings is often governed by minimum practical dimensions.
7. Lightly-loaded footings are often designed using an presumptive allowable bearing pressure, which is typically obtained from the applicable building code.
8. The design of footings on or near slopes needs to consider the sloping ground.
9. Footings on frozen soils need special considerations. The most common problem is frost heave, and the normal solution is to place the footing below the depth of frost penetration.
10. Footings in or near riverbeds are often prone to scour, and must be designed accordingly.
11. Rock usually provides excellent support for spread footings. Such footings are typically designed using a presumptive allowable bearing pressure.

Vocabulary

Allowable bearing pressure	Equivalent footing	Permafrost
Concentric downward load	Frost heave	Presumptive allowable bearing pressure
Depth of frost penetration	Frost-susceptible soil	Scour
Design chart	Lightly-loaded footing	Shear load
Eccentric load	Moment load	

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

8.21 A 2.0-m square, 0.5-m deep spread footing carries a concentrically applied downward load of 1200 kN and a moment load of 300 m-kN. The underlying soil has an undrained shear strength of 200 kPa. The design must satisfy the eccentric load requirements described in Chapter 5, and it must have a factor of safety of at least 3 against a bearing capacity failure. Determine if these requirements are being met. If not, adjust the footing dimensions until both requirements have been satisfied.

8.22 The soil at a certain site has the following geotechnical design parameters: $q_A = 4000$ lb/ft^2, $\mu_a = 0.28$, and $\lambda_a = 200$ lb/ft^3. The groundwater table is at a depth of 20 ft. A column that is to

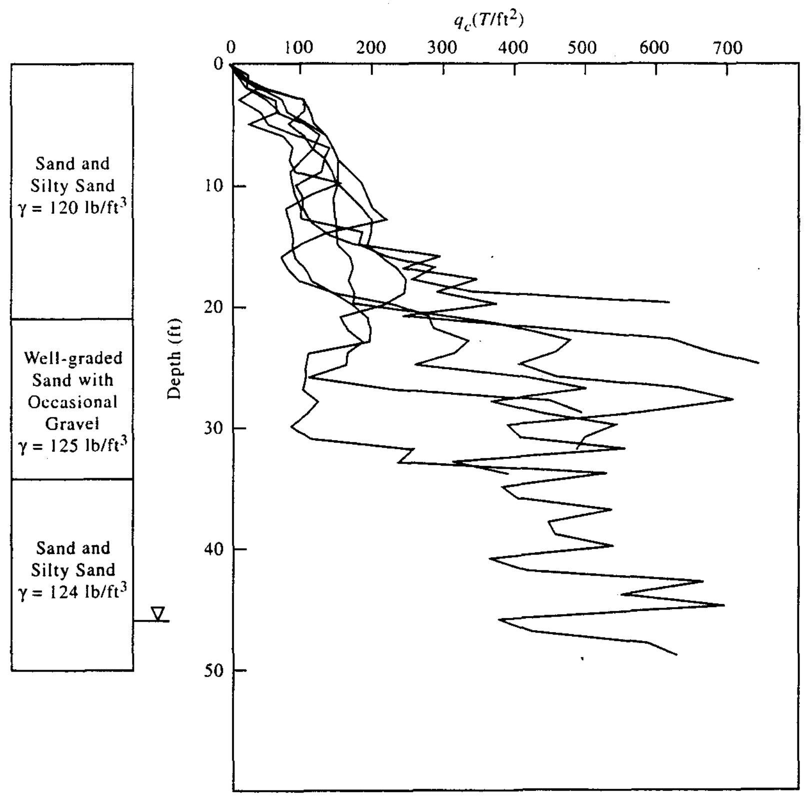


Figure 8.22 CPT data and synthesis of boring for Problems 8.23 and 8.24.

be supported on a square spread footing on this soil will impart the following load combinations onto the footing: $P = 200$ k, $V = 18$ k.

Determine the required footing width and depth of embedment.

8.23 Six cone penetration tests and four exploratory borings have been performed at the site of a proposed warehouse building. The underlying soils are natural sands and silty sands with occasional gravel. The CPT results and a synthesis of the borings are shown in Figure 8.22. The warehouse will be supported on 3 ft deep square footings that will have design downward loads of 100 to 600 k. The allowable total settlement is 1.0 in and the allowable differential settlement is 0.5 in. Using this data with reasonable factors of safety, develop values of q_A, μ_a, and λ_a. Use Figure 4.16 to estimate the friction angle.

8.24 Using the design values in Problem 8.23, determine the required width of a footing that must support the following load combinations:

Load combination 1: $P_D = 200$ k, $P_L = 0$, $V = 0$

Load combination 2: $P_D = 200$ k, $P_L = 0$, $V_E = 21$ k

Load combination 3: $P_D = 200$ k, $P_L = 240$, $V_E = 40$ k

Load combination 4: $P_D = 200$ k, $P_L = 240$, $P_E = -20$ k, $V_E = 40$ k

9

Spread Footings—Structural Design

Foundations ought to be twice as thick as the wall to be built on them; and regard in this should be had to the quality of the ground, and the largeness of the edifice; making them greater in soft soils, and very solid where they are to sustain a considerable weight.

The bottom of the trench must be level, that the weight may press equally, and not sink more on one side than on the other, by which the walls would open. It was for this reason the ancients paved the said bottom with tivertino, and we usually put beams or planks, and build on them.

The foundations must be made sloping, that is, diminished in proportion as they rise; but in such a manner, that there may be just as much set off one side as on the other, that the middle of the wall above may fall plumb upon the middle of that below: Which also must be observed in the setting off of the wall above ground; because the building is by this method made much stronger than if the diminutions were done any other way.

Sometimes (especially in fenny places, and where the columns intervene) to lessen the expence, the foundations are not made continued, but with arches, over which the building is to be.

It is very commendable in great fabricks, to make some cavities in the thickness of the wall from the foundation to the roof, because they give vent to the winds and vapours, and cause them to do less damage to the building. They save expence, and are of no little use if there are to be circular stairs from the foundation to the top of the edifice.

The First Book of Andrea Palladio's Architecture (1570),
as translated by Isaac Ware (1738)

The plan dimensions and minimum embedment depth of spread footings are primarily geotechnical concerns, as discussed in Chapters 6 to 8. Once these dimensions have been set, the next step is to develop a structural design that gives the foundation enough integrity to safely transmit the design loads from the structure to the ground. The structural design process for reinforced concrete foundations includes:

- Selecting a concrete with an appropriate strength
- Selecting an appropriate grade of reinforcing steel
- Determining the required foundation thickness, T, as shown in Figure 9.1
- Determining the size, number, and spacing of the reinforcing bars
- Designing the connection between the superstructure and the foundation.

The structural design aspects of foundation engineering are far more codified than are the geotechnical aspects. These codes are based on the results of research, the performance of existing structures, and the professional judgment of experts. Engineers in North America use the *Building Code Requirements for Structural Concrete* (ACI 318-99 and ACI 318M-99) for most projects. This code is published by the American Concrete Institute (ACI, 1999). The most notable alternative to ACI is the *Standard Specifications for Highway Bridges* published by the American Association of State Highway and Transportation Officials (AASHTO, 1996). The model building codes (ICBO, 1997; BOCA, 1996; SBCCI, 1997; ICC, 2000) contain additional design requirements.

This chapter covers the major principles of structural design of spread footings, and often refers to specific code requirements, with references shown in brackets [ ]. However, it is not a comprehensive discussion of every code provision, and thus is not a substitute for the code books.

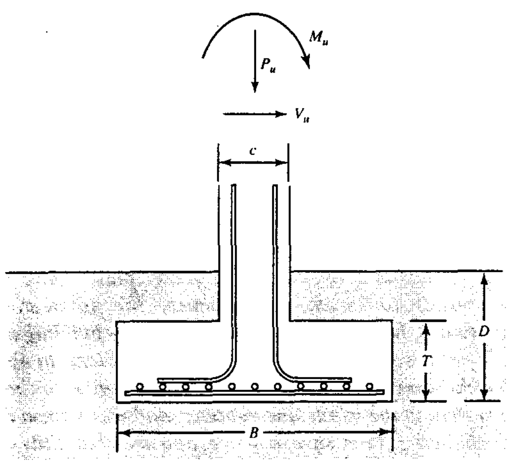


Figure 9.1 Cross section of a spread footing showing applied loads, reinforcing steel, and relevant dimensions.

9.1 SELECTION OF MATERIALS

Unlike geotechnical engineers, who usually have little or no control over the engineering properties of the soil, structural engineers can, within limits, select the engineering properties of the structural materials. In the context of spread footing design, we must select an appropriate concrete strength, f_c', and reinforcing steel strength, f_y.

When designing a concrete superstructure, engineers typically use concrete that has $f_c' = 20$–35 MPa (3000–5000 lb/in^2). In very tall structures, f_c' might be as large as 70 MPa (10,000 lb/in^2). The primary motive for using high-strength concrete in the superstructure is that it reduces the section sizes, which allows more space for occupancy and reduces the weight of the structure. These reduced member weights also reduce the dead loads on the underlying members.

However, the plan dimensions of spread footings are governed by bearing capacity and settlement concerns and will not be affected by changes in the strength of the concrete; only the thickness, T, will change. Even then, the required excavation depth, D, may or may not change because it might be governed by other factors. In addition, saving weight in a footing is of little concern because it is the lowest structural member and does not affect the dead load on any other members. In fact, additional weight may actually be a benefit in that it increases the uplift capacity.

Because of these considerations, and because of the additional materials and inspection costs of high strength concrete, spread footings are usually designed using an f_c' of only 15–20 MPa (2000–3000 lb/in^2). For footings that carry relatively large loads, perhaps greater than about 2000 kN (500 k), higher strength concrete might be justified to keep the footing thickness within reasonable limits, perhaps using an f_c' as high as 35 MPa (5000 lb/in^2).

Since the flexural stresses in footings are small, grade 40 steel (metric grade 300) is usually adequate. However, this grade is readily available only in sizes up through #6 (metric #22), and grade 60 steel (metric grade 420) may be required on the remainder of the project. Therefore, engineers often use grade 60 (metric grade 420) steel in the footings for reinforced concrete buildings so only one grade of steel is used on the project. This makes it less likely that leftover grade 40 (metric grade 300) bars would accidently be placed in the superstructure.

9.2 BASIS FOR DESIGN METHODS

Before the twentieth century, the design of spread footings was based primarily on precedent. Engineers knew very little about how footings behaved, so they followed designs that had worked in the past.

The first major advance in our understanding of the structural behavior of reinforced concrete footings came as a result of full-scale load tests conducted at the University of Illinois by Talbot (1913). He tested 197 footings in the laboratory and studied the mechanisms of failure. These tests highlighted the importance of shear in footings.

During the next five decades, other individuals in the United States, Germany, and elsewhere conducted additional tests. These tests produced important experimental information on the flexural and shear resistance of spread footings and slabs as well as the response of new and improved materials. Richart's (1948) tests were among the most significant of these. He tested 156 footings of various shapes and construction details by placing them on a bed of automotive coil springs that simulated the support from the soil and loaded them using a large testing machine until they failed. Whitney (1957) and Moe (1961) also made important contributions.

A committee of engineers (ACI-ASCE, 1962) synthesized this data and developed the analysis and design methodology that engineers now use. Because of the experimental nature of its development, this method uses simplified, and sometimes arbitrary, models of the behavior of footings. It also is conservative.

As often happens, theoretical studies have come after the experimental studies and after the establishment of design procedures (Jiang, 1983; Rao and Singh, 1987). Although work of this type has had some impact on engineering practice, it is not likely that the basic approach will change soon. Engineers are satisfied with the current procedures for the following reasons:

- Spread footings are inexpensive, and the additional costs of a conservative design are small.
- The additional weight that results from a conservative design does not increase the dead load on any other member.
- The construction tolerances for spread footings are wider than those for the superstructure, so additional precision in the design probably would be lost during construction.
- Although perhaps crude when compared to some methods available to analyze superstructures, the current methods are probably more precise than the geotechnical analyses of spread footings and therefore are not the weak link in the design process.
- Spread footings have performed well from a structural point-of-view. Failures and other difficulties have usually been due to geotechnical or construction problems, not bad structural design.
- The additional weight of conservatively designed spread footings provides more resistance to uplift loads.

Standard design methods emphasize two modes of failure: shear and flexure. A shear failure, shown in Figure 9.2, occurs when part of the footing comes out of the bottom. This type of failure is actually a combination of tension and shear on inclined failure surfaces. We resist this mode of failure by providing an adequate footing thickness, T.

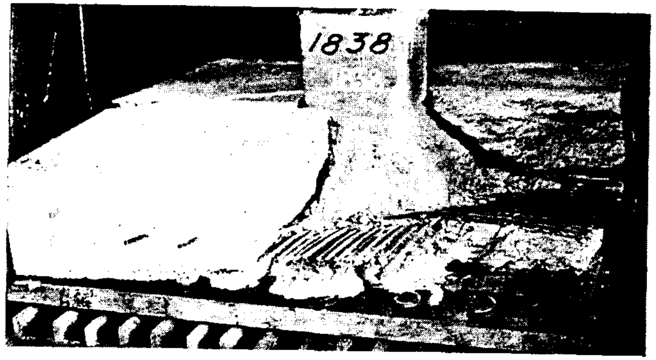


Figure 9.2 "Shear" failure in a spread footing loaded in a laboratory (Talbot, 1913). Observe how this failure actually is a combination of tension and shear.

A flexural failure is shown in Figure 9.3. We analyze this mode of failure by treating the footing as an inverted cantilever beam and resisting the flexural stresses by placing tensile steel reinforcement near the bottom of the footing.

9.3 DESIGN LOADS

The structural design of spread footings is based on LRFD methods (ACI calls it *ultimate strength design* or *USD*), and thus uses the factored loads as defined in Equations 2.7 to 2.15. Virtually all footings support a compressive load, P_u, and it should be computed

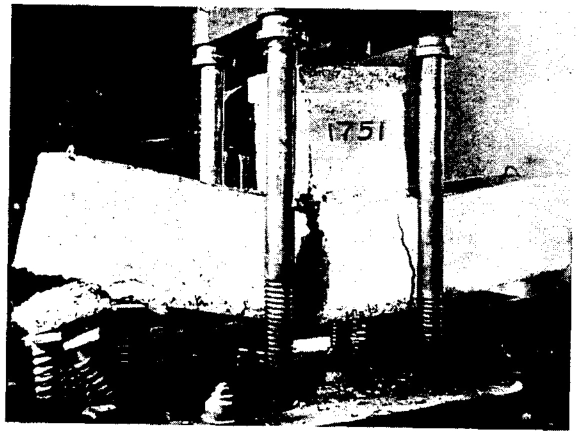


Figure 9.3 Flexural failure in a spread footing loaded in a laboratory (Talbot, 1913).

without including the weight of the footing because this weight is evenly distributed and thus does not produce shear or moment in the footing. Some footings also support shear (V_u) and/or moment (M_u) loads, as shown in Figure 9.1, both of which must be expressed as the factored load. This is often a point of confusion, because the geotechnical design of the same footing is normally based on ASD methods, and thus use the unfactored load, as defined in Equations 2.1 to 2.4. In addition, the geotechnical design must include the weight of the footing.

Therefore, when designing spread footings, be especially careful when computing the load. The footing width, B, is based on geotechnical requirements and is thus based on the unfactored load, as discussed in Chapter 8, whereas the thickness, T, and the reinforcement are structural concerns, and thus are based on the factored load. Examples 9.1 and 9.2 illustrate the application of these principles.

9.4 MINIMUM COVER REQUIREMENTS AND STANDARD DIMENSIONS

The ACI code specifies the minimum amount of concrete cover that must be present around all steel reinforcing bars [7.7]. For concrete in contact with the ground, such as spread footings, at least 70 mm (3 in) of concrete cover is required, as shown in Figure 9.4. This cover distance is measured from the edge of the bars, not the centerlines. It provides proper anchorage of the bars and corrosion protection. It also allows for irregularities in the excavation and accommodates possible contamination of the lower portion of the concrete.

Sometimes it is appropriate to specify additional cover between the rebar and the soil. For example, it is very difficult to maintain smooth footing excavation at sites with loose sands or soft clays, so more cover may be appropriate. Sometimes contractors place a thin layer of lean concrete, called a *mud slab* or a *leveling slab*, in the bottom of the footing excavation at such sites before placing the steel, thus providing a smooth working surface.

For design purposes, we ignore any strength in the concrete below the reinforcing steel. Only the concrete between the top of the footing and the rebars is considered in our analyses. This depth is the *effective depth*, d, as shown in Figure 9.4.

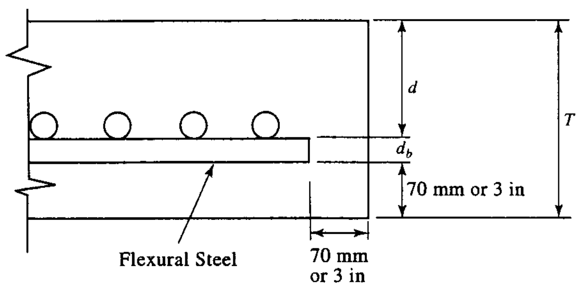


Figure 9.4 In square spread footings, the effective depth is the distance from the top of the concrete to the contact point of the flexural steel.

Footings are typically excavated using backhoes, and thus do not have precise as-built dimensions. Therefore, there is no need to be overly precise when specifying the footing thickness T. Round it to a multiple of 3 in or 100 mm (i.e., 12, 15, 18, 21, etc. inches or 300, 400, 500, etc. mm). The corresponding values of d are:

$$\boxed{d = T - 3\text{ in} - d_b} \quad (9.1\text{ English})$$

$$\boxed{d = T - 70\text{ mm} - d_b} \quad (9.1\text{ SI})$$

Where d_b is the nominal diameter of the steel reinforcing bars (see Table 9.1).

ACI [15.7] requires d be at least 6 in (150 mm), so the minimum acceptable T for reinforced footings is 12 in or 300 mm.

9.5 SQUARE FOOTINGS

This section considers the design of square footings supporting a single centrally-located column. Other types of footings are covered in subsequent sections.

In most reinforced concrete design problems, the flexural analysis is customarily done first. However, with spread footings, it is most expedient to do the shear analysis first. This is because it is not cost-effective to use shear reinforcement (stirrups) in most

TABLE 9.1 DESIGN DATA FOR STEEL REINFORCING BARS

Bar Size Designation		Available Grades		Nominal Dimensions: Diameter, d_b		Cross-Sectional Area, A_s	
English	SI	English	SI	(in)	(mm)	(in^2)	(mm^2)
#3	#10	40, 60	300, 420	0.375	9.5	0.11	71
#4	#13	40, 60	300, 420	0.500	12.7	0.20	129
#5	#16	40, 60	300, 420	0.625	15.9	0.31	199
#6	#19	40, 60	300, 420	0.750	19.1	0.44	284
#7	#22	60	420	0.875	22.2	0.60	387
#8	#25	60	420	1.000	25.4	0.79	510
#9	#29	60	420	1.128	28.7	1.00	645
#10	#32	60	420	1.270	32.3	1.27	819
#11	#36	60	420	1.410	35.8	1.56	1006
#14	#43	60	420	1.693	43.0	2.25	1452
#18	#57	60	420	2.257	57.3	4.00	2581

spread footings, and because we neglect the shear strength of the flexural steel. The only source of shear resistance is the concrete above the flexural reinforcement, so the effective depth, d, as shown in Figure 9.4, must be large enough to provide sufficient shear capacity. We then perform the flexural analysis using this value of d.

Designing for Shear

ACI defines two modes of shear failure, *one-way shear* (also known as *beam shear* or *wide-beam shear*) and *two-way shear* (also known as *diagonal tension shear*). In the context of spread footings, these two modes correspond to the failures shown in Figure 9.5. Although the failure surfaces are actually inclined, as shown in Figure 9.2, engineers use these idealized vertical surfaces to simplify the computations.

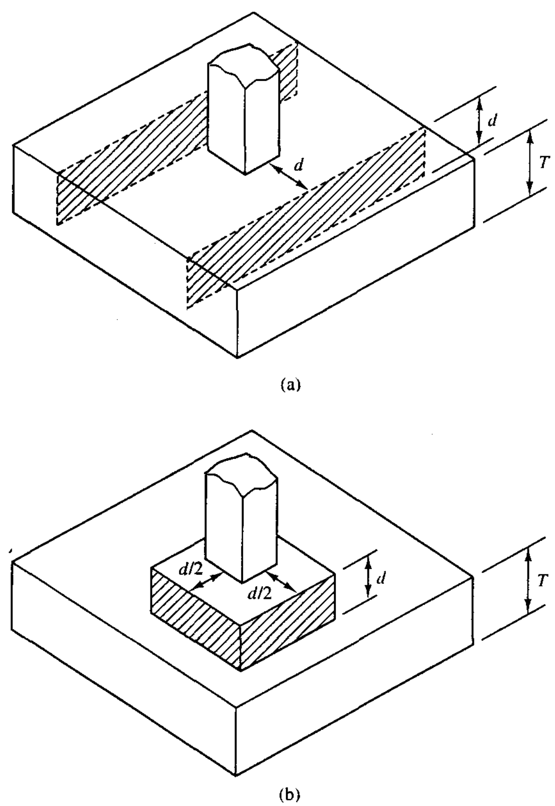


Figure 9.5 The two modes of shear failure: (a) one-way shear, and (b) two-way shear.

Various investigators have suggested different locations for the idealized critical shear surfaces shown in Figure 9.5. The ACI code [11.12.1] specifies that they be located a distance d from the face of the column for one-way shear and a distance $d/2$ for two-way shear.

The footing design is satisfactory for shear when it satisfies the following condition on all critical shear surfaces:

$$\boxed{V_{uc} \leq \phi V_{nc}} \tag{9.2}$$

Where:

V_{uc} = factored shear force on critical surface

ϕ = resistance factor for shear = 0.85

V_{nc} = nominal shear capacity on the critical surface

The nominal shear load capacity, V_{nc}, on the critical shear surface is [11.1]:

$$\boxed{V_{nc} = V_c + V_s} \tag{9.3}$$

Where:

V_c = nominal shear load capacity of concrete

V_s = nominal shear load capacity of reinforcing steel

For spread footings, we neglect V_s and rely only on the concrete for shear resistance.

Two-Way Shear

The footing may be subjected to applied normal, moment, and shear loads, P_u, M_u, and V_u, all of which produce shear forces on the critical shear surfaces.

To visualize the shear force on the critical surface, V_{uc}, caused by the applied normal load, P_u, we divide the footing into two blocks, one inside the shear surface and the other outside, as shown in Figure 9.6. The factored normal load, P_u, is applied to the top of the inner block and is transferred to a uniform pressure acting on the base of both blocks. Some of this load is transferred to the soil beneath the inner block, while the remainder must pass through the critical shear surface and enters the soil beneath the lower block. Only the later portion produces a shear force on the critical shear surface. In other words, the percentage of P_u that produces shear along the critical surfaces is the ratio of the base area of the outer block to the total base area.

If an applied moment load, M_u, is present, it produces an additional shear force on two opposing faces of the inner block, as shown in Figure 9.7. The shear force on one of the faces acts in the same direction as the shear force induced by the normal load, while that on the other face acts in the opposite direction. Therefore, the face with both forces

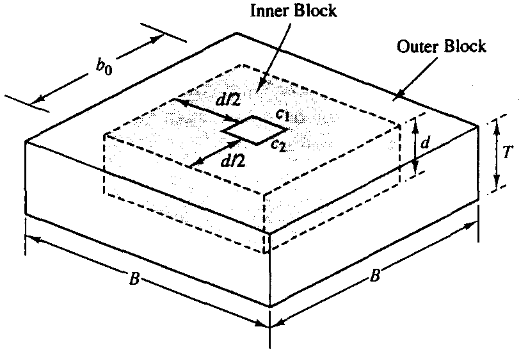


Figure 9.6 The inner block is the portion of the footing inside the critical section for two-way shear. The factored shear force acting along the perimeter of this block, V_{uc}, must not exceed ϕV_n. The factored shear force, V_{uc}, is the portion of the factored column load, P_u, that must pass through the outside surfaces of the inner block before reaching the ground.

acting in the same direction has the greatest shear force, and thus controls the design. The force on this face is:

$$V_{uc} = \left(\frac{P_u}{4} + \frac{M_u}{c+d}\right)\left(\frac{\text{base area of outer block}}{\text{total base area}}\right) = \left(\frac{P_u}{4} + \frac{M_u}{c+d}\right)\left(\frac{B^2 - (c+d)^2}{B^2}\right) \quad (9.4)$$

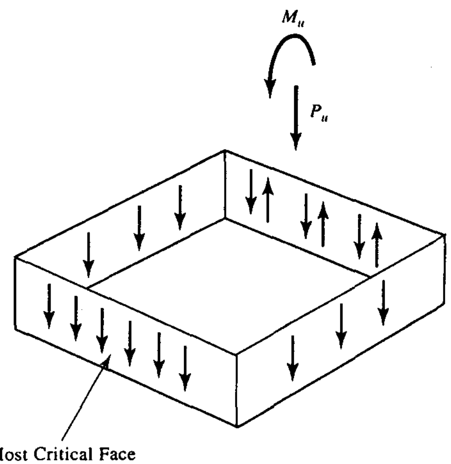


Figure 9.7 Distribution of shear forces on the critical shear surfaces for two-way shear when the footing is subjected to both normal and moment loads.

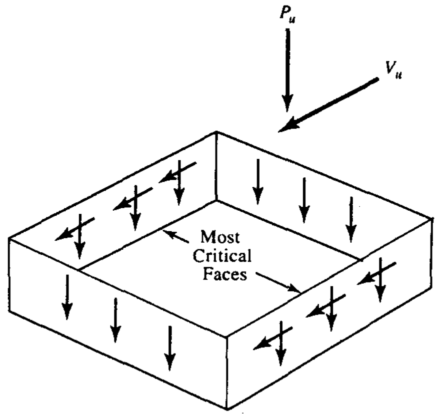


Figure 9.8 Distribution of shear forces on the critical shear surfaces for two-way shear when the footing is subjected to both normal and shear loads.

If an applied shear load, V_u, is present and it acts in the same direction as the moment load (which is the usual case), it produces a shear force on the other two faces, as shown in Figure 9.8. If we assume the applied shear force is evenly divided between these two faces, then the shear force on each face is:

$$V_{uc} = \left(\frac{\text{base area of outer block}}{\text{total base area}}\right)\sqrt{\left(\frac{P_u}{4}\right)^2 + \left(\frac{V_u}{2}\right)^2} = \left(\frac{B^2 - (c+d)^2}{B^2}\right)\sqrt{\left(\frac{P_u}{4}\right)^2 + \left(\frac{V_u}{2}\right)^2} \quad (9.5)$$

Where:

V_{uc} = factored shear force on the most critical face
P_u = applied normal load
M_u = applied moment load
V_u = applied shear load
c = column width or diameter (for concrete columns) or base plate width (for steel columns)
d = effective depth
B = footing width

The design should be based on the larger of the V_{uc} values obtained from Equations 9.4 and 9.5, thus accounting for applied normal, shear, and/or moment loads.

For square footings supporting square or circular columns located in the interior of the footing (i.e., not on the edge or corner), the nominal two-way shear capacity is [ACI 11.12.2.1]:

$$V_{nc} = V_c = 4\, b_0\, d\sqrt{f'_c} \quad \text{(9.6 English)}$$

$$V_{nc} = V_c = \frac{1}{3}\, b_0\, d\, \sqrt{f'_c} \quad \text{(9.6 SI)}$$

$$b_0 = c + d \quad (9.7)$$

Where:

V_{nc} = nominal two-way shear capacity on the critical section (lb, N)
V_c = nominal two-way shear capacity of concrete (lb, N)
b_0 = length of critical shear surface = length of one face of inner block
c = column width (in, mm)
d = effective depth (in, mm)
f'_c = 28-day compressive strength of concrete (lb/in^2, MPa)

Other criteria apply if the column has another shape, or if it is located along edge or corner of the footing [ACI 11.12.2.1]. Special criteria also apply if the footing is made of prestressed concrete [ACI 11.12.2.2], but spread footings are rarely, if ever, prestressed.

The objective of this analysis is to find the effective depth, d, that satisfies Equation 9.2. Both V_{uc} and V_{nc} depend on the effective depth, d, but there is no direct solution. Therefore, it is necessary to use the following procedure:

1. Assume a trial value for d. Usually a value approximately equal to the column width is a good first trial. When selecting trial values of d, remember T must be a multiple of 3 in or 100 mm, as discussed in Section 9.4, so the corresponding values of d are the only ones worth considering. Assuming $d_b \approx 1$ in (25 mm), the potential values of d are 8, 11, 14, 17, etc. inches or 200, 300, 400, etc. mm.
2. Compute V_{uc} and V_{nc}, and check if Equation 9.2 has been satisfied.
3. Repeat Steps 1 and 2 as necessary until finding the smallest d that satisfies Equation 9.2.
4. Using Equation 9.1 with $d_b = 1$ in or 25 mm, compute the footing thickness, T. Express it as a multiple of 3 in or 100 mm. T must be at least 12 in or 300 mm.

The final value of d_b will be determined as a part of the flexural analysis, and may be different from the 1 in or 25 mm assumed here. However, this difference is small compared to the construction tolerances, so there is no need to repeat the shear analysis with the revised d_b.

One-Way Shear

Two-way shear always governs the design of square footings subjected only to vertical loads. There is no need to check one-way shear in such footings. However, if applied shear and/or moment loads are present, both kinds of shear need to be checked.

To analyze this situation, we will make the following assumptions:

- The shear stress caused by the applied vertical load, P_u, is uniformly distributed across the two critical vertical planes shown in Figure 9.5a.
- The shear stress on the vertical planes caused by the applied moment load, M_u, is expressed by the flexure formula, $\sigma = Mc/I$, and thus is greatest at the left and right edges of these planes.
- The shear stress caused by the applied shear load is uniformly distributed across the planes.
- The applied normal, moment, and shear loads must be multiplied by $(B - c - 2d)/B$ before applying them to the critical vertical planes. This factor is the ratio of the footing base area outside the critical planes to the total area, and thus reflects the percentage of the applied loads that must be transmitted through the critical vertical planes.
- The maximum shear stress on the critical vertical surfaces is the vector sum of those due to the applied normal, moment, and shear loads.
- The factored shear stress on the critical vertical surfaces is the greatest shear stress multiplied by the area of the shear surfaces. This may be greater than the integral of the shear stress across the shear surfaces, but is useful because it produces a design that keeps the maximum shear stress within acceptable limits.

Based on these assumptions, we compute the factored shear force on the critical vertical surfaces, V_{uc}, as follows:

$$V_{uc} = \left(\frac{B - c - 2d}{B}\right)\sqrt{\left(P_u + \frac{6\,M_u}{B}\right)^2 + V_u^2} \quad (9.8)$$

Where:

V_{uc} = shear force on critical shear surfaces
B = footing width
c = column width
d = effective depth
P_u = applied normal load
M_u = applied moment load
V_u = applied shear load

The nominal one-way shear load capacity on the critical section [11.3.1.1] is:

$$V_{nc} = V_c = 2\, b_w d \sqrt{f_c'} \qquad (9.9\ \text{English})$$

$$V_{nc} = V_c = \frac{1}{6} b_w d \sqrt{f_c'} \qquad (9.9\ \text{SI})$$

Where:

V_{nc} = nominal one-way shear capacity on the critical section (lb, N)
V_c = nominal one-way shear capacity of concrete (lb, N)
b_w = length of critical shear surface = $2B$ (in, mm)
d = effective depth (in, mm)
f_c' = 28-day compressive strength of concrete (lb/in^2, MPa)

Once again, the design is satisfactory when Equation 9.2 has been satisfied.

Both V_{uc} and V_{nc} depend on the effective depth, d, which must be determined using Equations 9.8 and 9.9 with the procedure described under two-way shear. The final design value of d is the larger of that obtained from the one-way and two-way shear analyses.

The final value of d_b will be determined as a part of the flexural analysis, and may be different from the 1 in or 25 mm assumed here. However, this difference is small compared to the construction tolerances, so there is no need to repeat the shear analysis with the revised d_b.

Example 9.1— Part A

A 21-inch square reinforced concrete column carries a vertical dead load of 380 k and a vertical live load of 270 k. It is to be supported on a square spread footing that will be founded on a soil with an allowable bearing pressure of 6500 lb/ft^2. The groundwater table is well below the bottom of the footing. Determine the required width, B, thickness, T, and effective depth, d.

Solution

Unfactored load—Equation 2.2 governs

$$P = P_D + P_L + \ldots = 380\text{ k} + 270\text{ k} + 0 = 650\text{ k}$$

Per Table 8.1, use $D = 36$ in

$$W_f = B^2 D \gamma_c = B^2 (3\text{ ft})(150\text{ lb/ft}^3) = 450\, B^2$$

$$B = \sqrt{\frac{P + W_f}{q_A + u_D}} = \sqrt{\frac{650{,}000 + 450\, B^2}{6500 + 0}}$$

$$B = 10.36\text{ ft} \rightarrow \text{use } B = 10\text{ ft } 6\text{ in } (126\text{ in})$$

Factored load (Equation 2.7 governs)

$$P_u = 1.4\, P_D + 1.7\, P_L = (1.4)(380) + (1.7)(270) = 991\text{ k}$$

Because of the large applied load and because this is a large spread footing, we will use $f_c' = 4000$ lb/in^2 and $f_y = 60$ k/in^2.

Since there are no applied moment or shear loads, there is no need to check one-way shear. Determine required thickness based on a two-way shear analysis.

Try $T = 24$ in:

$$d = T - 1\text{ bar diameter} - 3\text{ in} = 24 - 1 - 3 = 20\text{ in}$$

$$V_{uc} = \left(\frac{P_u}{4} + \frac{M_u}{c + d}\right)\left(\frac{B^2 - (c + d^2)}{B^2}\right)$$
$$= \left(\frac{991{,}000\text{ lb}}{4} + 0\right)\left(\frac{(126\text{ in})^2 - (21\text{ in} + 20\text{ in})^2}{(126\text{ in})^2}\right)$$
$$= 221{,}500\text{ lb}$$

$$b_0 = c + d = 21 + 20 = 41\text{ in}$$

$$V_{nc} = 4\, b_0 d \sqrt{f_c'}$$
$$= 4(41\text{ in})(20\text{ in})\sqrt{4000\text{ lb/in}^2}$$
$$= 207{,}400\text{ lb}$$

$$\phi\, V_{nc} = (0.85)(207{,}400\text{ lb}) = 176{,}300\text{ lb}$$

$$V_{uc} > \phi\, V_{nc} \quad \therefore \text{Not acceptable}$$

Try $T = 27$ in:

$$d = T - 1\text{ bar diameter} - 3\text{ in} = 27 - 1 - 3 = 23\text{ in}$$

$$V_{uc} = \left(\frac{P_u}{4} + \frac{M_u}{c + d}\right)\left(\frac{B^2 - (c + d)^2}{B^2}\right)$$
$$= \left(\frac{991{,}000\text{ lb}}{4} + 0\right)\left(\frac{(126\text{ in})^2 - (21\text{ in} + 23\text{ in})^2}{(126\text{ in})^2}\right)$$
$$= 217{,}500\text{ lb}$$

$$b_0 = c + d = 21 + 23 = 44\text{ in}$$

$$V_{nc} = 4\, b_0\, d\sqrt{f'_c}$$
$$= 4(41 \text{ in})(23 \text{ in})\sqrt{4000 \text{ lb/in}^2}$$
$$= 256{,}000 \text{ lb}$$

$$\phi\, V_n = (0.85)(256{,}000 \text{ lb}) = 217{,}600 \text{ lb}$$

$$V_{uc} \leq \phi\, V_{nc} \quad \textbf{OK}$$

∴ Use **B = 10 ft 6 in; T = 27 in; d = 23 in** ⇐ ***Answer***

Note 1: In this case, V_{uc} is almost exactly equal to ϕV_{nc}. However, since we are considering only certain values of d, it is unusual for them to match so closely.

Note 2: The depth of embedment of 3 ft, as obtained from Table 8.1, is not needed here (unless frost depth or other concerns dictate it). We could use D = 30 in and still have plenty of room for a 27 inch-thick footing.

Designing for Flexure

Once we have completed the shear analysis, the design process can move to the flexural analysis.

ACI Flexural Design Standards

Reinforcing Steel

Concrete is strong in compression, but weak in tension. Therefore, engineers add reinforcing steel, which is strong in tension, to form *reinforced concrete*. This reinforcement is necessary in members subjected to pure tension, and those that must resist *flexure* (bending). Reinforcing steel may consist of either *deformed bars* (more commonly known as *reinforcing bars*, or *rebars*) or *welded wire fabric*. However, wire fabric is rarely used in foundations.

Manufacturers produce reinforcing bars in various standard diameters, typically ranging between 9.5 mm (3/8 in) and 57.3 mm (2 1/4 in). In the United States, the English and metric bars are the same size (i.e., we have used a soft conversion), and are identified by the *bar size* designations in Table 9.1.

Rebars are available in various strengths, depending on the steel alloys used to manufacture them. The two most common bar strengths used in the United States are:

- Grade 40 bars (also known as metric grade 300), which have a yield strength, f_y, of 40 k/in^2 (300 MPa)
- Grade 60 bars (also known as metric grade 420), which have a yield strength, f_y, of 60 k/in^2 (420 MPa)

Flexural Design Principles

The primary design problem for flexural members is as follows: Given a factored moment on the critical surface, M_{uc}, determine the necessary dimensions of the member and the necessary size and location of reinforcing bars. Fortunately, flexural design in foundations is simpler than that for some other structural members because geotechnical concerns dictate some of the dimensions.

The amount of steel required to resist flexure depends on the *effective depth, d*, which is the distance from the extreme compression fiber to the centroid of the tension reinforcement, as shown in Figure 9.9.

The nominal moment capacity of a flexural member made of reinforced concrete with $f'_c \leq$ 30 MPa (4000 lb/in^2) as shown in Figure 9.9 is:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \tag{9.10}$$

$$a = \frac{\rho\, d\, f_y}{0.85\, f'_c} \tag{9.11}$$

$$\boxed{\rho = \frac{A_s}{b\, d}} \tag{9.12}$$

Setting $M_u = \phi\, M_n$, where M_u is the factored moment at the section being analyzed, and solving for A_s gives:

$$\boxed{A_s = \left(\frac{f'_c\, b}{1.176 f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353\, M_{uc}}{\phi f'_c\, b}}\right)} \tag{9.13}$$

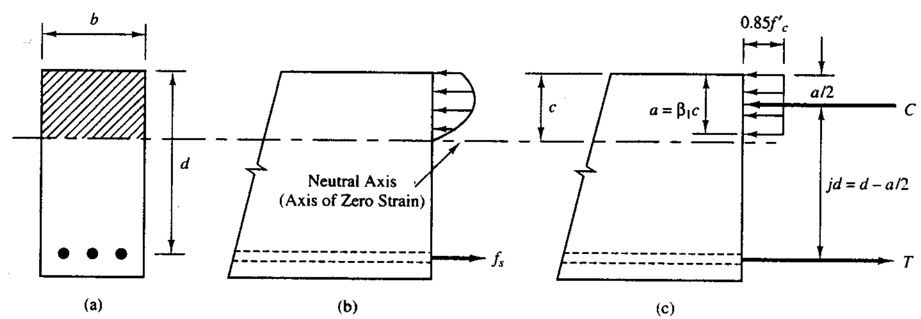


Figure 9.9 The reinforcing bars are placed in the portion of the member that is subjected to tension. (a) Cross section, (b) actual stress distribution, and (c) equivalent rectangular stress distribution. The effective depth, d, is the distance from the extreme compression fiber to the centroid of the reinforcement. β_1 is an empirical factor that ranges between 0.65 and 0.85. (Adapted from MacGregor, 1996).

Where:

A_s = cross-sectional area of reinforcing steel
f_c' = 28-day compressive strength of concrete
f_y = yield strength of reinforcing steel
ρ = steel ratio
b = width of flexural member
d = effective depth
ϕ = 0.9 for flexure in reinforced concrete
M_{uc} = factored moment at the section being analyzed

Two additional considerations also enter the design process: minimum steel and maximum steel. The minimum steel in footings is governed by ACI 10.5.4 and 7.12.2, because footings are treated as "structural slabs of uniform thickness" (MacGregor, 1996). These requirements are as follows:

For grade 40 (metric grade 300) steel $A_s \geq 0.0020\, A_g$
For grade 60 (metric grade 420) steel $A_s \geq 0.0018\, A_g$

Where:

A_g = gross cross-sectional area

The ρ_{min} criteria in ACI 10.5.1 do not apply to footings.

The maximum steel requirement [10.3] is intended to maintain sufficient ductility. It never governs the design of simple footings, but it may be of concern in combined footings or mats.

We can supply the required area of steel, computed using Equation 9.13 by any of several combinations of bar size and number of bars. This selection must satisfy the following minimum and maximum spacing requirements:

- The clear space between bars must be at least equal to d_b, 25 mm (1 in), or 4/3 times the nominal maximum aggregate size [3.3.2 and 7.6.1], whichever is greatest.
- The center-to-center spacing of the reinforcement must not exceed $3T$ or 500 mm (18 in), whichever is less [10.5.4].

Notice how one of these criteria is based on the "clear space" which is the distance between the edges of two adjacent bars, while the other is based on the center-to-center spacing, which is the distance between their centerlines.

Development Length

The rebars must extend a sufficient distance into the concrete to develop proper anchorage [ACI 15.6]. This distance is called the *development length.* Provides the clear spacing between the bars is at least $2d_b$, and the concrete cover is at least d_b, the ratio of the minimum required development length, l_d, to the bar diameter, d_b is [ACI 12.2.3]:

$$\frac{l_d}{d_b} = \frac{3}{40} \frac{f_y}{\sqrt{f_c'}} \frac{\alpha \beta \gamma \lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \quad \text{(9.14 English)}$$

$$\frac{l_d}{d_b} = \frac{9}{10} \frac{f_y}{\sqrt{f_c'}} \frac{\alpha \beta \gamma \lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \quad \text{(9.14 SI)}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500\, s\, n} \quad \text{(9.15 English)}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{10\, s\, n} \quad \text{(9.15 SI)}$$

For spread footings, use $K_{tr} = 0$, which is conservative.

Where:

l_d = minimum required development length (in, mm)
d_b = nominal bar diameter (in, mm)
f_y = yield strength of reinforcing steel (lb/in^2, MPa)
f_{yt} = yield strength of transverse reinforcing steel (lb/in^2, MPa)
f_c' = 28-day compressive strength of concrete (lb/in^2, MPa)
α = reinforcement location factor
  α = 1.3 for horizontal reinforcement with more than 300 mm (12 in) of fresh concrete below the bar
  α = 1.0 for all other cases
β = coating factor
  β = 1.5 for epoxy coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$
  β = 1.2 for other epoxy coated bars or wires
  β = 1.0 for uncoated bars or wires
γ = reinforcement factor
  γ = 0.8 for #6 (metric #19) and smaller bars
  γ = 1.0 for #7 (metric #22) and larger bars
λ = lightweight concrete factor = 1.0 for normal concrete (lightweight concrete is not used in foundations)
c = spacing or cover dimension (in, mm) = the smaller of the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bars
A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and which crosses the potential plane of splitting through the re-

inforcement being developed (in^2, mm^2)—may conservatively be taken to be zero

s = maximum center-to-center spacing of transverse reinforcement within l_d (in, mm)

The term $(c + K_{tr})/d_b$ must be no greater than 2.5, and the product $\alpha\beta$ need not exceed 1.7. In addition, the development length must always be at least 300 mm (12 in).

Application to Spread Footings

Principles

A square footing bends in two perpendicular directions as shown in Figure 9.10a, and therefore might be designed as a *two-way slab* using methods similar to those that might be applied to a floor slab that is supported on all four sides. However, for practical purposes, it is customary to design footings as if they were a *one-way slab* as shown in Figure 9.10b. This conservative simplification is justified because of the following:

- The full-scale load tests on which this analysis method is based were interpreted this way.
- It is appropriate to design foundations more conservatively than the superstructure.
- The flexural stresses are low, so the amount of steel required is nominal and often governed by ρ_{min}.
- The additional construction cost due to this simplified approach is nominal.

Once we know the amount of steel needed to carry the applied load in one-way bending, we place the same steel area in the perpendicular direction. In essence the footing is reinforced twice, which provides more reinforcement than required by a more rigorous two-way analysis.

Steel Area

The usual procedure for designing flexural members is to prepare a moment diagram and select an appropriate amount of steel for each portion of the member. However, for simple spread footings, we again simplify the problem and design all the steel for the moment that occurs at the *critical section for bending*. The location of this section for various types of columns is shown in Figure 9.11.

We can simplify the computations by defining a distance l, measured from the critical section to the outside edge of the footing. In other words, l is the cantilever distance. It is computed using the formulas in Table 9.2.

The factored bending moment at the critical section, M_{uc}, is:

$$\boxed{M_{uc} = \frac{P_u l^2}{2B} + \frac{2 M_u l}{B}} \qquad (9.16)$$

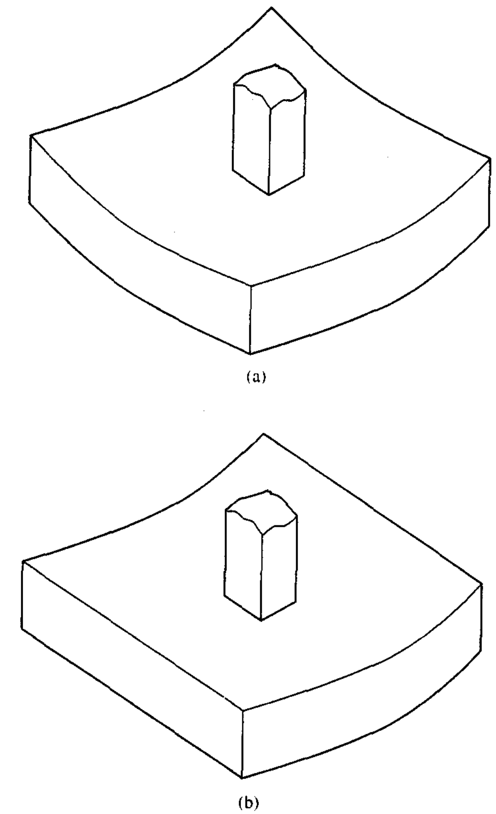


Figure 9.10 (a) A spread footing is actually a two-way slab, bending in both the "north-south" and "east-west" directions; (b) For purposes of analysis, engineers assume that the footing is a one-way slab that bends in one axis only.

Where:

M_{uc} = factored moment at critical section for bending
P_u = factored compressive load from column
M_u = factored moment load from column
l = cantilever distance (from Table 9.2)
B = footing width

The first term in Equation 9.16 is based on the assumption that P_u acts through the centroid of the footing. The second term is based on a soil bearing pressure with an assumed eccentricity of $B/3$, which is conservative (see Figure 5.15).

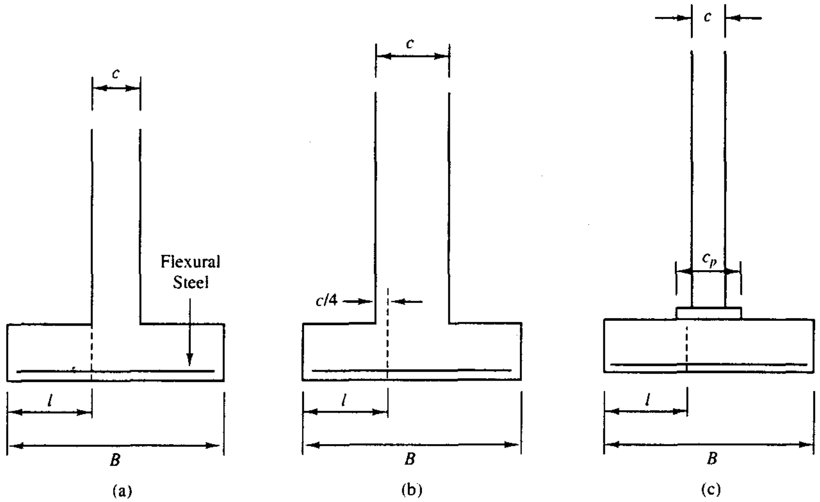


Figure 9.11 Location of critical section for bending: (a) with a concrete column; (b) with a masonry column; and (c) with a steel column.

After computing M_{uc}, find the steel area, A_s, and reinforcement ratio, ρ, using Equations 9.12 and 9.15. Check if the computed ρ is less than ρ_{min}. If so, then use ρ_{min}. Rarely will ρ be larger than 0.0040. This light reinforcement requirement develops because we made the effective depth d relatively large to avoid the need for stirrups.

TABLE 9.2 DESIGN CANTILEVER DISTANCE FOR USE IN DESIGNING REINFORCEMENT IN SPREAD FOOTINGS [15.4.2].

Type of Column	l
Concrete	$(B - c)/2$
Masonry	$(B - c/2)/2$
Steel	$(2B - (c + c_p))/4$

1. ACI does not specify the location of the critical section for timber columns, but in this context, it seems reasonable to treat them in the same way as concrete columns.
2. If the column has a circular, octagonal, or other similar shape, use a square with an equivalent cross-sectional area.
3. B = footing width; c = column width; c_p = base plate width. If column has a circular or regular polygon cross section, base the analysis on an equivalent square.

The required area of steel for each direction is:

$$\boxed{A_s = \rho B d} \tag{9.17}$$

Carry the flexural steel out to a point 70 mm (3 in) from the edge of the footing as shown in Figure 9.4.

Development Length

ACI [15.6] requires the flexural steel in spread footings meet standard development length requirements. This development length is measured from the critical section for bending to the end of the bars as defined in Figure 9.11 to the end of the bars, which is 70 mm (3 in) from the edge of the footing. Thus, the supplied development length, $(l_d)_{supplied}$ is:

$$\boxed{(l_d)_{supplied} = l - 70 \text{ mm (3 in)}} \tag{9.18}$$

Where:

$(l_d)_{supplied}$ = supplied development length

l = cantilever distance (per Table 9.2)

This supplied development length must be at least equal to the required development length, as computed using Equation 9.14 or 9.15. If this criteria is not satisfied, we do not enlarge the footing width, B. Instead, it is better to use smaller diameter bars, which have a correspondingly shorter required development length.

If the supplied development length is greater than the required development length, we still extend the bars to 70 mm (3 in) from the edge of the footing. Do not cut them off at a different location.

Example 9.1—Part B

Using the results from Part A, design the required flexural steel.

Solution

Find the required steel area

$$l = \frac{B - c}{2} = \frac{126 - 21}{2} = 52.5 \text{ in}$$

$$M_{uc} = \frac{P_u l^2}{2B} + 0 = \frac{(991{,}000)(52.5)^2}{(2)(126)} = 10{,}800{,}000 \text{ in-lb}$$

$$A_s = \left(\frac{f'_c b}{1.176 f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353\, M_u}{\phi f'_c b}}\right)$$

$$= \left(\frac{(4000\ \text{lb/in}^2)(126\ \text{in})}{(1.176)(60{,}000\ \text{lb/in}^2)}\right)\left(23\ \text{in} - \sqrt{(23\ \text{in})^2 - \frac{2.353\,(10{,}800{,}000\ \text{in-lb})}{(0.9)(4000\ \text{lb/in}^2)(126\ \text{in})}}\right)$$

$$= 8.94\ \text{in}^2$$

Check minimum steel

$$A_s \geq 0.018\,(27)\,(126)$$

$$\geq 6.12\ \text{in}^2$$

$$8.94 \geq 6.12 \quad \text{ok}$$

Use 12 #8 bars each way (A_s = 9.42 in^2) ⇐ ***Answer***

Clear space between bars = 126/13-1 = 8.7 in OK

Check development length

$$(l_d)_{supplied} = l - 3 = 52.5 - 3 = 49.5\ \text{in}$$

$$\frac{c + K_{tr}}{d_b} = \frac{3.5 + 0}{1} = 3.5 > 2.5 \quad \text{use } 2.5$$

$$\frac{l_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} = \frac{3}{40}\frac{60{,}000}{\sqrt{4000}}\frac{(1)(1)(1)(1)}{2.5} = 28$$

$$l_d = 28\, d_b = (28)(1) = 28\ \text{in}$$

$l_d < (l_d)_{supplied}$, so development length is OK.

The final design is shown in Figure 9.12.

QUESTIONS AND PRACTICE PROBLEMS

The ASD load for determining the required footing width should be computed using the largest of Equations 2.1, 2.2, 2.3a, or 2.4a, unless otherwise stated. The factored loads should be computed using the ACI load factors (Equations 2.7–2.17).

9.1 A column carries the following vertical downward loads: dead load = 400 kN, live load = 300 kN, wind load = 150 kN, and earthquake load = 250 kN. Compute the unfactored load, P, and the factored load, P_u, to be used in the design of a concrete footing.

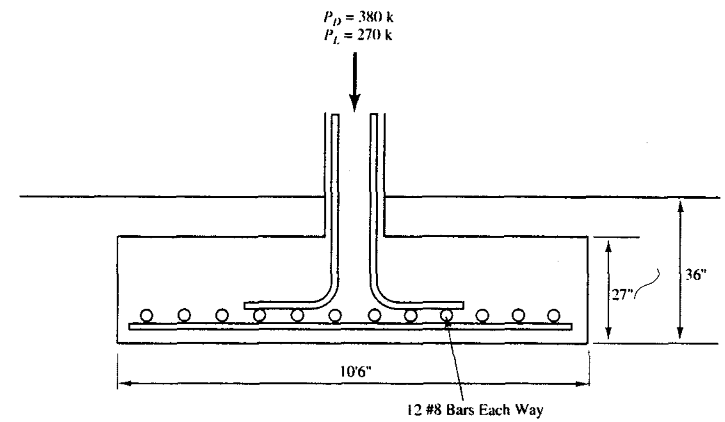


Figure 9.12 Footing design for Example 9.1.

9.2 A flexural member has a dead load moment of 200 ft-k and a live load moment of 150 ft-k. The computed nominal moment capacity, M_n, is 600 ft-k. Is the design of this member satisfactory? Use the ACI ultimate strength criterion.

9.3 Why are spread footings usually made of low-strength concrete?

9.4 Explain the difference between the shape of the actual shear failure surfaces in footings with those used for analysis and design.

9.5 A 400-mm square concrete column that carries a factored vertical downward load of 450 kN and a factored moment load of 100 kN-m is supported on a 1.5-m square footing. The effective depth of the concrete in this footing is 500 mm. Compute the ultimate shear force that acts on the most critical section for two-way shear failure in the footing.

9.6 A 16-in square concrete column carries vertical dead and live loads of 150 and 100 k, respectively. It is to be supported on a square footing with f'_c = 3000 lb/in^2 and f_y = 60 k/in^2. The soil has an allowable bearing pressure of 4500 lb/ft^2 and the groundwater table is at a great depth. Because of frost heave considerations, the bottom of this footing must be at least 30 inches below the ground surface. Determine the required footing thickness, size the flexural reinforcement, and show your design in a sketch.

9.7 A W16×50 steel column with a 22-inch square base plate is to be supported on a square spread footing. This column has a design dead load of 200 k and a design live load of 120 k. The footing will be made of concrete with f'_c = 2500 lb/in^2 and reinforcing steel with f_y = 60 k/in^2. The soil has an allowable bearing pressure of 3000 lb/ft^2 and the groundwater table is at a great depth. Determine the required footing thickness, size the flexural reinforcement, and show your design in a sketch.

9.8 A 500-mm square concrete column carries vertical dead and live loads of 500 and 280 kN, respectively. It is to be supported on a square footing with $f_c' = 17$ MPa and $f_y = 420$ MPa. The soil has an allowable bearing pressure of 200 kPa and the groundwater table is at a great depth. Determine the required footing thickness, size the flexural reinforcement, and show your design in a sketch.

9.6 CONTINUOUS FOOTINGS

The structural design of continuous footings is very similar to that for square footings. The differences, described below, are primarily the result of the differences in geometry.

Designing for Shear

As with square footings, the depth of continuous footings is governed by shear criteria. However, we only need to check one-way shear because it is the only type that has any physical significance. The critical surfaces for evaluating one-way shear are located a distance d from the face of the wall as shown in Figure 9.13.

The factored shear force acting on a unit length of the critical shear surface is:

$$V_{uc}/b = (P_u/b)\left(\frac{B - c - 2d}{B}\right) \qquad (9.19)$$

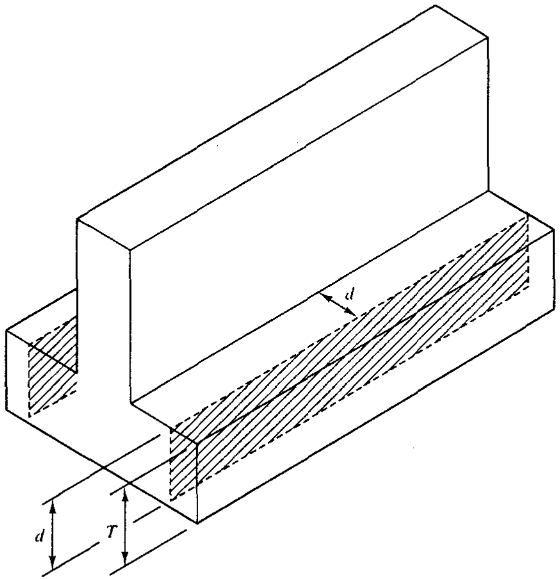


Figure 9.13 Location of idealized critical shear surface for one-way shear in a continuous footing.

Where:

V_{uc}/b = factored shear force on critical shear surface per unit length of footing

P_u/b = factored applied compressive load per unit length of footing

c = width of wall

b = unit length of footing (usually 1 ft or 1 m)

Setting $V_{uc} = \phi V_{nc}$, equating Equations 9.9 and 9.19, and solving for d gives:

$$d = \frac{(P_u/b)(B - c)}{48\,\phi\,B\sqrt{f_c'} + 2\,P_u/b} \qquad (9.20 \text{ English})$$

$$d = \frac{1500\,(P_u/b)(B - c)}{500\,\phi\,B\sqrt{f_c'} + 3\,P_u/b} \qquad (9.20 \text{ SI})$$

Where:

d = effective depth (in, mm)

P_u/b = applied vertical load per unit length of footing (lb/ft, kN/m)

b = footing width (in, mm)

c = wall width (in, mm)

ϕ = resistance factor = 0.85

f_c' = 28-day compressive strength of concrete (lb/in^2, MPa)

Then, compute the footing thickness, T, using the criterion described earlier.

Designing for Flexure

Nearly all continuous footings should have longitudinal reinforcing steel (i.e., running parallel to the wall). This steel helps the footing resist flexural stresses from non-uniform loading, soft spots in the soil, or other causes. Temperature and shrinkage stresses also are a concern. Therefore, place a nominal amount of longitudinal steel in the footing (0.0018 A_g to 0.0020 A_g) with at least two #4 bars (2 metric #13 bars). If large differential heaves or settlements are likely, we may need to use additional longitudinal reinforcement. Chapter 19 includes a discussion of this issue.

Transverse steel (that which runs perpendicular to the wall) is another issue. Most continuous footings are narrow enough so the entire base is within a 45° frustum, as shown in Figure 9.14. Thus, they do not need transverse steel. However, wider footings should include transverse steel designed to resist the flexural stresses at the critical section as defined in Table 9.2. The factored moment at this section is:

$$M_{uc}/b = \frac{(P_u/b)l^2}{2\,B} + \frac{2\,(M_u/b)l}{B} \qquad (9.21)$$

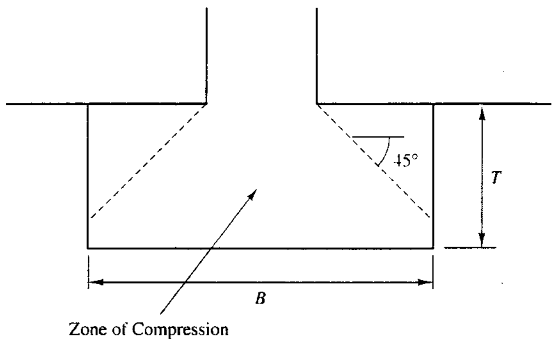


Figure 9.14 Zone of compression in lightly-loaded footings.

Where:

M_{uc}/b = factored moment at critical section per unit length of footing

P_u/b = factored applied compressive load per unit length of footing

M_u/b = factored applied moment load perpendicular to wall per unit length of footing

l = cantilever distance (from Figure 9.11 or Table 9.2)

Compute the required transverse steel area per unit length, A_s/b, using Equation 9.13, as demonstrated in Example 9.2.

Example 9.2

A 200-mm wide concrete block wall carries a vertical dead load of 120 kN/m and a vertical live load of 88 kN/m. It is to be supported on a continuous spread footing that is to be founded at a depth of at least 500 mm below the ground surface. The allowable bearing pressure of the soil beneath the footing is 200 kPa, and the groundwater table is at a depth of 10 m. Develop a structural design for this footing using $f_c' = 15$ MPa and $f_y = 300$ MPa.

Solution

Unfactored load—Equation 2.2 governs

$$P/b = (P/b)_D + (P/b)_L + \ldots = 120 \text{ kN/m} + 88 \text{ kN/m} + 0 = 208 \text{ kN/m}$$

Per Table 8.1, minimum D = 400 mm, but problem statement says use D = 500 mm

$$W_f/b = B\,D\,\gamma_c = B(0.5 \text{ m})(23.6 \text{ kN/m}^3) = 11.8\,B$$

$$B = \frac{P/b + W_f/b}{q_A - u_D} = \frac{208 + 11.8\,B}{200 - 0} = 1.1 \text{ m}$$

Factored load—Equation 2.7 governs

$$P_u/b = 1.4P_D/b + 1.7P_L/b = 1.4(120) + 1.7(88) = 318 \text{ kN/m}$$

Compute the required thickness using a shear analysis

$$d = \frac{1500\,(P_u/b)(B - c)}{500\,\phi\,B\sqrt{f_c'} + 3\,P_u/b}$$

$$= \frac{(1500)(318 \text{ kN/m})(1100 \text{ mm} - 200 \text{ mm})}{(500)(0.85)(1100 \text{ mm})\sqrt{15 \text{ MPA}} + (3)(318 \text{ kN/m})}$$

$$= 237 \text{ mm}$$

For ease of construction, place the longitudinal steel below the lateral steel. Assuming metric #13 bars (diameter = 12.7 mm), the footing thickness, T, is:

$$T = d + (1/2)(\text{diam. of lat. steel}) + \text{diam. of long steel} + 70 \text{ mm}$$

$$= 237 + 12.7/2 + 12.7 + 70$$

$$= 326 \text{ mm} \rightarrow \text{Use 400 mm}$$

$$d = 400 - 12.7/2 - 12.7 - 70 = 311 \text{ mm}$$

In the square footing design of Example 9.1, we used an effective depth, d, as the distance from the top of the footing to the contact point of the two layers of reinforcing bars (as shown in Figure 9.4. We used this definition because square footings have two-way bending, this is the average d of the two sets of rebar. However, with continuous footings we are designing only the lateral steel, so d is measured from the top of the footing to the center of the lateral bars. The longitudinal bars will be designed separately.

Design the lateral steel

$$l = \frac{B - c/2}{2} = \frac{1.1 - 0.2/2}{2} = 0.50 \text{ m} = 500 \text{ mm}$$

$$M_{uc}/b = \frac{(P_u/b)l^2}{2B} + 0 = \frac{(318)(0.50)^2}{2(1.1)} = 36.1 \text{ kN-m/m}$$

$$A_s/b = \left(\frac{f_c'\,b}{1.176 f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353\,M_u}{\phi f_c'\,b}}\right)$$

$$= \left(\frac{(15 \text{ MPa})(1 \text{ m})}{(1.176)(300 \text{ MPa})}\right)\left(0.311 \text{ m} - \sqrt{(0.311 \text{ m})^2 - \frac{2.353\,(36.1 \text{ kN-m})}{(0.9)(15{,}000 \text{ kPa})(1 \text{ m})}}\right)\left(\frac{10^3 \text{ mm}}{\text{m}}\right)^2$$

$$= 437 \text{ mm}^2/\text{m}$$

Check minimum steel

$$A_s/b \geq 0.0020\,(460)(1000)$$
$$\geq 800 \text{ mm}^2/\text{m}$$

Use metric #13 bars @ 150 mm OC [A_s = (129 mm²/bar)(6.67 bars/m) = 860 mm²/m > 800 mm²/m]

Check development length

$$(l_d)_{supplied} = l - 75 = 500 - 75 = 425 \text{ mm}$$

$$\frac{c + K_{tr}}{d_b} = \frac{70 + 0}{12.7} = 5.5 > 2.5 \quad \text{use } 2.5$$

$$\frac{l_d}{d_b} = \frac{9}{10}\frac{f_y}{\sqrt{f_c'}}\frac{\alpha\,\beta\,\gamma\,\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} = \frac{9}{10}\frac{300}{\sqrt{15}}\frac{(1)(1)(1)(1)}{2.5} = 28$$

$$l_d = 28\,d_b = (28)(12.7) = 355 \text{ mm}$$

$l_d < (l_d)_{supplied}$, so development length is OK.
Design the longitudinal steel

$$A_s = \rho\,b\,d = (0.0020)(1100)(400) = 880 \text{ mm}^2$$

Use 7 metric #13 bars (A_s = 903 mm² ≈ > 880 mm²)

The final design is shown in Figure 9.15.

QUESTIONS AND PRACTICE PROBLEMS

9.9 A 12-in wide concrete block wall carries vertical dead and live loads of 13.0 and 12.1 k/ft, respectively. It is to be supported on a continuous footing made of 2000 lb/in² concrete and 40 k/in² steel. The soil has an allowable bearing pressure of 4000 lb/ft², and the groundwater table is at a great depth. The local building code requires that the bottom of this footing be at least 24 inches below the ground surface. Determine the required footing thickness, and design the lateral and longitudinal steel. Show your design in a sketch.

9.10 A 200-mm wide concrete block wall carries vertical dead and live loads of 50 and 70 kN/m, respectively. It is to be supported on a continuous footing made of 18 MPa concrete and 420 MPa steel. The soil has an allowable bearing pressure of 150 kPa, and the groundwater table is at a great depth. The local building code requires that the bottom of this footing be at least 500 mm below the ground surface. Determine the required footing thickness, and design the lateral and longitudinal steel. Show your design in a sketch.

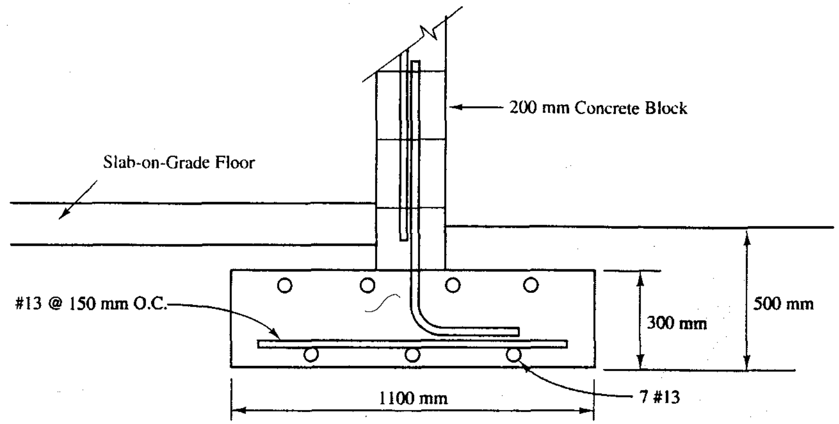


Figure 9.15 Final design for Example 9.2.

9.7 RECTANGULAR FOOTINGS

Rectangular footings with width B and length L that support only one column are similar to square footings. Design them as follows:

1. Check both one-way shear (Equation 9.9) and two-way shear (Equation 9.6) using the critical shear surfaces shown in Figure 9.16a. Determine the minimum required d and T to satisfy both.
2. Design the long steel (see Figure 9.16b) by substituting L for B in Table 9.2 and Equation 9.16, and using Equation 9.17 with no modifications. Distribute this steel evenly across the footing as shown in Figure 9.16c.
3. Design the short steel (see Figure 9.16b) using Table 9.2 and Equation 9.16 with no modifications, and substituting L for B in Equation 9.17.
4. Since the central portion of the footing takes a larger portion of the short-direction flexural stresses, place more of the short steel in this zone [15.4.4]. To do so, divide the footing into inner and outer zones, as shown in Figure 9.16c. The portion of the total short steel area, A_s, to be placed in the inner zone is E:

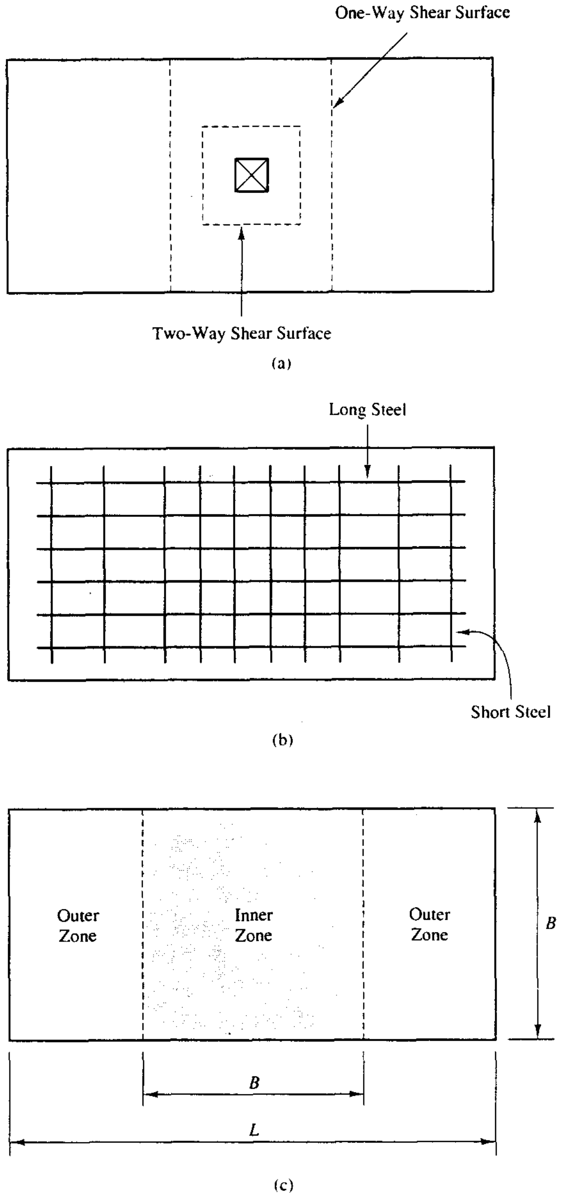


Figure 9.16 Structural design of rectangular footings: (a) critical shear surfaces: (b) long steel and short steel: (c) distribution of short steel.

$$E = \frac{2}{L/B + 1} \tag{9.22}$$

Distribute the balance of the steel evenly across the outer zones.

9.8 COMBINED FOOTINGS

Combined footings are those that carry more than one column. Their loading and geometry is more complex, so it is appropriate to conduct a more rigorous structural analysis. The rigid method, described in Chapter 10, is appropriate for most combined footings. It uses a soil bearing pressure that varies linearly across the footing, thus simplifying the computations. Once the soil pressure has been established, MacGregor (1996) suggests designing the longitudinal steel using idealized beam strips ABC, as shown in Figure 9.17. Then, design the transverse steel using idealized beam strips AD. See MacGregor (1996) for a complete design example.

Large or heavily loaded combined footings may justify a beam on elastic foundation analysis, as described in Chapter 10.

9.9 LIGHTLY-LOADED FOOTINGS

Although the principles described in Sections 9.5 to 9.8 apply to footings of all sizes, some footings are so lightly loaded that practical minimums begin to govern the design. For example, if P_u is less than about 400 kN (90 k) or P_u/b is less than about 150 kN/m (10 k/ft), the minimum d of 150 mm (6 in) [ACI 15.7] controls. Thus, there is no need to conduct a shear analysis, only to compute a T smaller than the minimum. In the same vein, if P_u is less than about 130 kN (30 k) or P_u/b is less than about 60 kN/m (4 k/ft), the minimum steel requirement ($\rho = 0.0018$) governs, so there is no need to conduct a flexural analysis. Often, these minimums also apply to footings that support larger loads.

In addition, if the entire base of the footing is within a 45° frustum, as shown in Figure 9.14, we can safely presume that very little or no tensile stresses will develop. This is often the case with lightly loaded footings. Technically, no reinforcement is required in such cases. However, some building codes [ICBO 1806.7] have minimum reinforcement requirements for certain footings, and it is good practice to include at least the following reinforcement in all footings:

Square footings

- If bottom of footing is completely within the zone of compression—no reinforcement required
- If bottom of footing extends beyond the zone of compression—as determined by a flexural analysis, but at least #4 @ 18 in o.c. each way (metric #13 @ 500 mm o.c. each way)

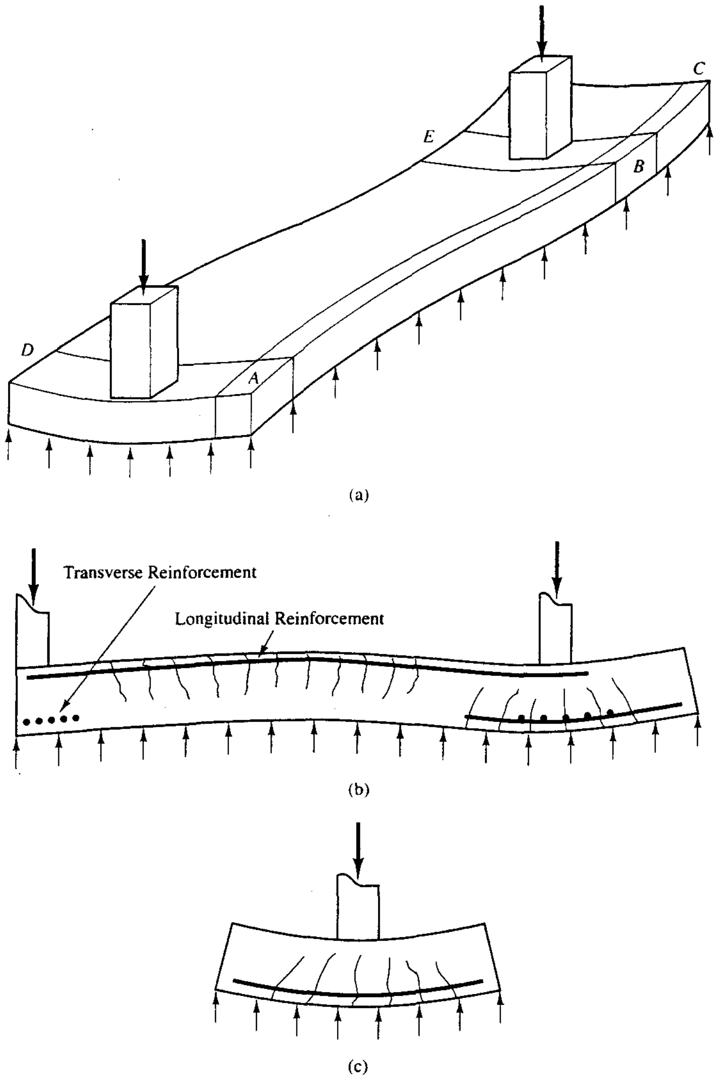


Figure 9.17 Structural design of combined footings: (a) idealized beam strips; (b) longitudinal beam strip; (c) transverse beam strip (Adapted from MacGregor, 1996).

Continuous footings

Longitudinal reinforcement

- Minimum two #4 bars (metric #13)

Lateral reinforcement

- If bottom of footing is completely within the zone of compression—no lateral reinforcement required
- If bottom of footing extends beyond the zone of compression—as determined by a flexural analysis, but at least #4 @ 18 in o.c. (metric #13 @ 500 mm o.c.)

This minimum reinforcement helps accommodate unanticipated stresses, temperature and shrinkage stresses, and other phenomena.

9.10 CONNECTIONS WITH THE SUPERSTRUCTURE

One last design feature that needs to be addressed is the connection between the footing and the superstructure. Connections are often the weak link in structures, so this portion of the design must be done carefully. A variety of connection types are available, each intended for particular construction materials and loading conditions. The design of proper connections is especially important when significant seismic or wind loads are present (Dowrick, 1987).

Connections are designed using either ASD (with the unfactored loads) or LRFD (with the factored loads) depending on the design method used in the superstructure.

Connections with Columns

Columns may be made of concrete, masonry, steel, or wood, and each has its own concerns when designing connections.

Concrete or Masonry Columns

Connect concrete or masonry columns to their footing [ACI 15.8] using *dowels*, as shown in Figure 9.18. These dowels are simply pieces of reinforcing bars that transmit axial, shear, and moment loads. Use at least four dowels with a total area of steel, A_s, at least equal to that of the column steel or 0.005 times the cross-sectional area of the column, whichever is greater. They may not be larger than #11 bars [ACI 15.8.2.3] and must have a 90° hook at the bottom. Normally, the number of dowels is equal to the number of vertical bars in the column.

Design for Compressive Loads

Check the bearing strength of the footing [ACI 10.17] to verify that it is able to support the axial column load. This is especially likely to be a concern if the column carries large compressive stresses that might cause something comparable to a bearing capacity failure

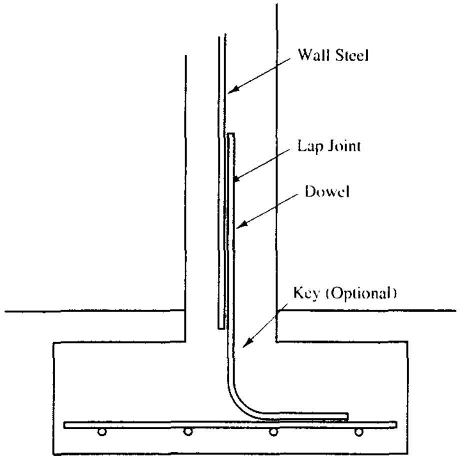


Figure 9.18 Use of dowels to connect a concrete or masonry column to its footing.

inside the footing. To check this possibility, compute the factored column load, P_u, and compare it to the nominal column bearing capacity, P_{nb}:

$$\boxed{P_{nb} = 0.85 f'_c A_1 s} \tag{9.23}$$

Then, determine whether the following statement is true:

$$\boxed{P_u \leq \phi P_{nb}} \tag{9.24}$$

Where:

P_u = factored column load

P_{nb} = nominal column bearing capacity (i.e., bearing of column on top of footing)

f'_c = 28-day compressive strength of concrete

s = $(A_2/A_1)^{0.5} \leq 2$ if the frustum in Figure 9.19 fits entirely within the footing (i.e., if $c + 4d \leq B$)

s = 1 if the frustum in Figure 9.19 does not fit entirely within the footing

A_1 = cross-sectional area of the column = c^2

A_2 = $(c + 4d)^2$ as shown in Figure 9.19

c = column width or diameter

ϕ = resistance factor = 0.7 [ACI 9.3.2.4]

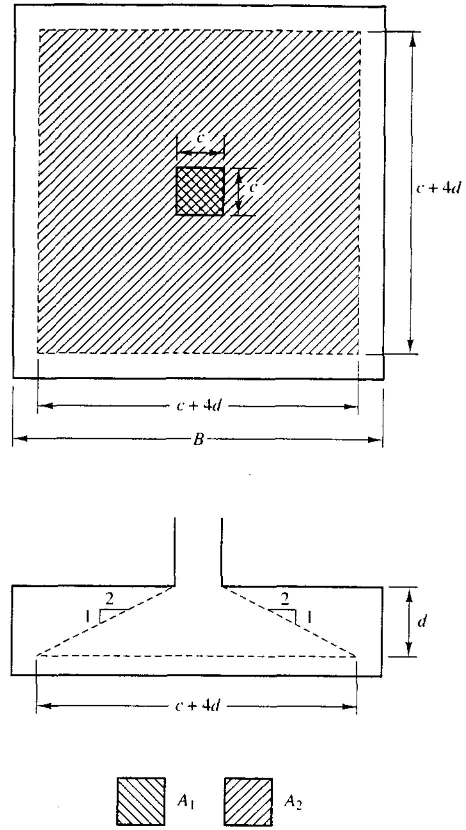


Figure 9.19 Application of a frustrum to find s and A_2.

If Equation 9.24 is not satisfied, use a higher strength concrete (greater f'_c) in the footing or design the dowels as compression steel.

Design for Moment Loads

If the column imparts moment loads onto the footing, then some of the dowels will be in tension. Therefore, the dowels must be embedded at least one development length into the footing, as shown in Figure 9.20 and defined by the following equations [ACI 12.5]:

$$\boxed{l_{dh} = \frac{1200\, d_b}{\sqrt{f'_c}}} \tag{9.25 English}$$

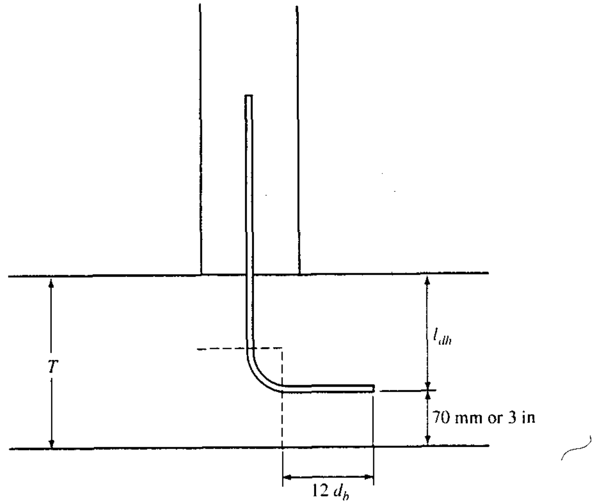


Figure 9.20 Minimum required embedment of dowels subjected to tension.

$$l_{dh} = \frac{100\, d_b}{\sqrt{f_c'}} \tag{9.25 SI}$$

Where:

T = footing thickness (in, mm)
l_{dh} = development length for 90° hooks, as defined in Figure 9.20 (in, mm)
d_b = bar diameter (in, mm)
f_c' = 28-day compressive strength of concrete (lb/in^2, MPa)

The development length computed from Equation 9.25 may be modified by the following factors [ACI 12.5.3][1] :

For standard reinforcing bars with yield strength other than 60,000 lb/in^2 : $f_y/60{,}000$
For metric reinforcing bars with yield strength other than 420 lb/in^2 : $f_y/420$
If at least 50 mm (2 in) of cover is present beyond the end of the hook: 0.7

Sometimes this development length requirement will dictate a footing thickness T greater than that required for shear (as computed earlier in this chapter).

[1]This list only includes modification factors that are applicable to anchorage of vertical steel in retaining wall footings. ACI 12.5.3 includes additional modification factors that apply to other situations.

As long as the number and size of dowels are at least as large as the vertical steel in the column, then they will have sufficient capacity to carry the moment loads.

Design for Shear Loads

If the column also imparts a shear load, V_u, onto the footing, the connection must be able to transmit this load. Since the footing and column are poured separately, there is a weak shear plane along the cold joint. Therefore, the dowels must transmit all of the applied shear load. The minimum required dowel steel area is:

$$A_s = \frac{V_u}{\phi f_y \mu} \tag{9.26}$$

Where:

A_s = minimum required dowel steel area
V_u = applied factored shear load
ϕ = 0.85 for shear
f_y = yield strength of reinforcing steel
μ = 0.6 if the cold joint not intentionally roughened or 1.0 if the cold joint is roughened by heavy raking or grooving [ACI 11.7.4.3]

However, the ultimate shear load, V_u, cannot exceed 0.2 $\phi f_c' A_c$, where f_c' is the compressive strength of the column concrete, and A_c is the cross-sectional area of the column.

Splices

Most designs use a lap splice to connect the dowels and the vertical column steel. However, some columns have failed in the vicinity of these splices during earthquakes, as shown in Figure 9.21. Therefore, current codes require much more spiral reinforcement in columns subjected to seismic loads. In addition, some structures with large moment loads, such as certain highway bridges, require mechanical splices or welded splices to connect the dowels and the column steel.

Steel Columns

Steel columns are connected to their foundations using base plates and anchor bolts, as shown in Figure 9.22. The base plates are welded to the bottom of the columns when they are fabricated, and the anchor bolts are cast into the foundation when the concrete is placed. The column is then erected over the foundation, and the anchor bolts are fit through predrilled holes in the base plate.

The top of the footing is very rough and not necessarily level, so the contractor must use special construction methods to provide uniform support for the base plate and to make the column plumb. For traffic signal poles, light standards, and other lightweight columns, the most common method is to provide a nut above and below the base plate, as

Figure 9.21 Imperial County Services Building, El Centro, California. The bases of these columns failed during the 1979 El Centro earthquake, causing the building to sag about 300 mm. As a result, this six-story building had to be demolished. (U.S. Geological Survey photo)

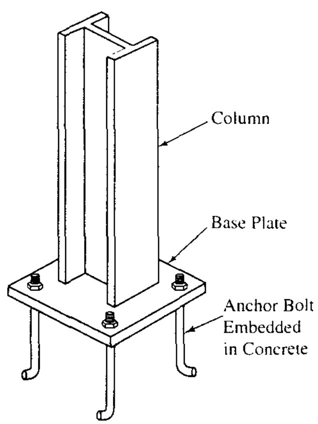


Figure 9.22 Base plate and anchor bolts to connect a steel column to its foundations.

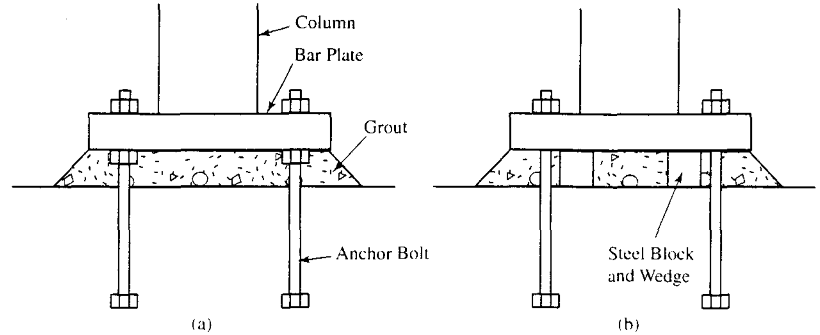


Figure 9.23 Methods of leveling the base plate: a) Double nuts, b) Blocks and shims.

shown in Figure 9.23a, and adjust these nuts as needed to make the column plumb. However, columns for buildings, bridges, and other large structures are generally too heavy for this method, so the contractor must temporarily support the base plate on steel blocks and shims, and clamp it down with a single nut on each anchor bolt, as shown in Figure 9.23b. These shims are carefully selected to produce a level base plate and a plumb column. Other construction methods also have been used.

Once the column is securely in place and the various members that frame into it have been erected, the contractor places a nonshrink grout between the base plate and the footing. This grout swells slightly when it cures (as compared to normal grout, which shrinks), thus maintaining continuous support for the base plate. The structural loads from the column are then transmitted to the footing as follows:

- Compressive loads are spread over the base plate and transmitted through the grout to the footing.
- Tensile loads pass through the base plate and are resisted by the anchor bolts.
- Moment loads are resisted by a combination of compression through the grout and tension in half of the bolts.
- Shear loads are transmitted through the anchor bolts, through sliding friction along the bottom of the base plate, or possibly both.

Design Principles

The base plate must be large enough to avoid exceeding the nominal bearing strength of the concrete (see earlier discussion under concrete and masonry columns). In addition, it must be thick enough to transmit the load from the column to the footing. The design of base plates is beyond the scope of this book, but it is covered in most steel design texts and in DeWolf and Ricker (1990).

Anchor bolts can fail either by fracture of the bolts themselves, or by loss of anchorage in the concrete. Steel is much more ductile than the concrete, and this ductility is important, especially when wind or seismic loads are present. Therefore, anchor bolts should be designed so the critical mode of failure is shear or tension of the bolt itself rather than failure of the anchorage. In other words, the bolt should fail before the concrete fails.

The following methods may be used to design anchor bolts that satisfy this principle. These methods are based on ACI and AISC requirements, but some building codes may impose additional requirements, or specify different design techniques, so the engineer must check the applicable code.

Selection and Sizing of Anchor Bolts

Five types of anchor bolts are available, as shown in Figure 9.24:

- *Standard structural steel bolts* may simply be embedded into the concrete to form anchor bolts. These bolts are similar to those used in bolted steel connections, except they are much longer. Unfortunately, these bolts may not be easily available in lengths greater than about 6 inches, so they often are not a practical choice.
- *Structural steel rods that have been cut to length and threaded* form anchor bolts that are nearly identical to a standard steel bolts and have the advantage of being more readily available. This is the most common type of anchor bolt for steel columns. If one nut is used at the bottom of each rod, it should be tack welded to prevent the rod from turning when the top nut is tightened. Alternatively, two nuts may be used.
- *Hooked bars* (also known as an *L-bolts* or a *J-bolts*) are specially fabricated fasteners made for this purpose. These are principally used for wood-frame structures, and are generally suitable for steel structures only when no tensile or shear loads are present.

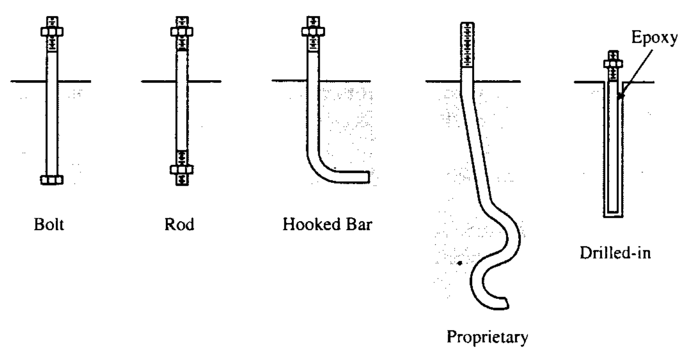


Figure 9.24 Types of anchor bolts.

- *Proprietary anchor bolts* are patented designs that often are intended for special applications, principally with wood-frame structures.
- *Drilled-in anchor bolts* are used when a cast-in-place anchor bolt was not installed during construction of the footing. They are constructed by drilling a hole in the concrete, then embedding a threaded rod into the hole and anchoring it using either epoxy grout or mechanical wedges. This is the most expensive of the five types and is usually required only to rectify mistakes in the placement of conventional anchor bolts.

Most anchor bolts are made of steel that satisfies ASTM A36 or ASTM A307, both of which have $F_y = 36$ k/in^2 (250 MPa). However, higher strength steel may be used, if needed.

Each bolt must satisfy both of the following design criteria:

$$\boxed{P_u \leq \phi P_n} \tag{9.27}$$

$$\boxed{V_u \leq \phi V_n} \tag{9.28}$$

Where:

P_u = factored tensile force based on AISC load factors (Equations 2.18–2.23) expressed as a positive number
V_u = factored shear force based on AISC load factors (Equations 2.18–2.23)
ϕ = resistance factor
P_n = nominal tensile capacity
V_n = nominal shear capacity

In addition, the design must satisfy AISC requirements for interaction between shear and tensile stresses. Figure 9.25 presents the shear and tensile capacities for ASTM A36 and ASTM A307 bolts that satisfies Equations 9.27 and 9.28 and the interaction requirements, and may be used to select the required diameter.

Typically four anchor bolts are used for each column. It is best to place them in a square pattern to simplify construction and leave less opportunity for mistakes. Rectangular or hexagonal patterns are more likely to be accidentally built with the wrong orientation. More bolts and other patterns also may be used, if necessary.

If the design loads between the column and the footing consist solely of compression, then anchor bolts are required only to resist erection loads, accidental collisions during erection, and unanticipated shear or tensile loads. The engineer might attempt to estimate these loads and design accordingly, or simply select the bolts using engineering judgement. Often these columns simply use the same anchor bolt design as nearby columns, thus reducing the potential for mistakes during construction.

Anchorage

Once the bolt diameter has been selected, the engineer must determine the required depth of embedment into the concrete to provide the necessary anchorage. The required embedment depends on the type of anchor, the spacing between the anchors, the kind of loading,

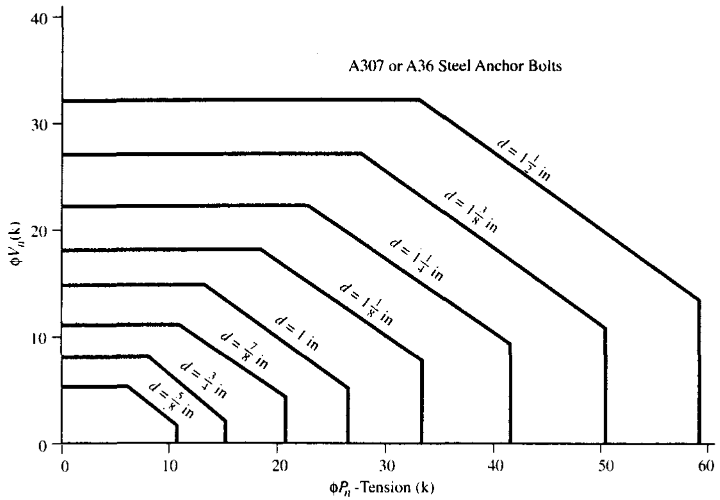


Figure 9.25 AISC tensile and shear capacities of A36 and A307 anchor bolts.

and the strength of the concrete. In addition, the bolt must be located at least a minimum horizontal distance from the edge of the concrete. Table 9.3 presents conservative design values for embedment depth and edge distance. Alternatively, the engineer may use the more refined procedure described by Marsh and Burdette (1985).

Shear Transfer

Shear forces may be transferred from the base plate to the foundation in two ways:

- Through shear in the anchor bolts
- By sliding friction along the bottom of the base plate

TABLE 9.3 ANCHORAGE REQUIREMENTS FOR BOLTS AND THREADED RODS (Shipp and Haninger, 1983) Copyright © American Institute of Steel Construction. Reprinted with permission.

Steel Grade	Minimum Embedment Depth	Minimum Edge Distance
A307, A36	12 *d*	5*d* or 100 mm (4 in), whichever is greater
A325, A449	17 *d*	7*d* or 100 mm (4 in), whichever is greater

d = nominal bolt diameter

Some engineers rely on both sources of shear resistance, while others rely only on one or the other.

When transferring shear loads through the anchor bolts, the engineer must recognize that the bolt holes in the base plate are oversized in order to simplify the erection of the column onto the footing. As a result, the resulting gap between the bolts and the base plate does not allow for efficient transfer of the shear loads. The base plate may need to move laterally before touching the bolts, and most likely only some of the bolts will become fully engaged with the plate. Therefore, when using this mode of shear transfer, it is probably prudent to assume the shear load is carried by only half of the bolts.

So long as the grout has been carefully installed between the base plate and the footing, and the base plate has not been installed using the double nut method as shown in Figure 9.23a, there will be sliding friction along the bottom of the base plate. AISC recommends using a coefficient of friction of 0.55 for conventional base plates, such as that shown in Figure 9.23b, and the resistance factor, ϕ, is 0.90. Thus, the available sliding friction resistance, ϕV_n, is:

$$\begin{aligned} \phi V_n &= \phi \mu P \\ &= (0.90)(0.55)\,P \\ &= 0.50\,P \end{aligned} \tag{9.29}$$

The value of P in Equation 9.29 should be the lowest unfactored normal load obtained from Equations 2.1 to 2.4. Usually Equation 2.1 governs, except when uplift wind or seismic loads are present, as described in Equation 2.4. It is good practice to ignore any normal stress produced by live loads or the clamping forces from the nuts.

Sometimes short vertical fins are welded to the bottom of the base plate to improve shear transfer to the grout. These fins may justify raising the coefficient of friction to 0.7.

Example 9.3

A steel wide flange column with a steel base plate is to be supported on a spread footing. The AISC factored design loads are: $P_u = 270$ k compression and $M_u = 200$ ft-k. Design an anchor bolt system for this column using four bolts arranged in a 15×15 inch square.

Solution

Reduce the applied loads to a couple separated by 15 in:

$$\begin{aligned} P &= \frac{270\text{ k}}{2} \pm \frac{200\text{ ft-k}}{(15/12)\text{ ft}} \\ &= 135 \pm 160\text{ k} \end{aligned}$$

There are two bolts on each side, so the maximum tensile force in each bolt is:

$$P = \frac{135 - 160}{2} = 12.5 \text{ k} \quad \text{tension}$$

The shear force is zero.
Per Figure 9.25, use 3/4 inch bolts.

The depth of embedment should be (12)(0.75) = 9 in.

Use four 3/4 inch diameter x 13 inch long A36 threaded rods embedded 9 inches into the footing. Firmly tighten two nuts at the bottom of each rod. ⇐ *Answer*

Wood Columns

Wood columns, often called *posts*, usually carry light loads and require relatively simple connections. The most common type is a metal bracket, as shown in Figure 9.26. These are set in the wet concrete immediately after it is poured. The manufacturers determine the allowable loads and tabulate them in their catalogs (for example, see www.strongtie.com).

It is poor practice to simply embed a wooden post into the footing. Although at first this would be a very strong connection, in time the wood will rot and become weakened.

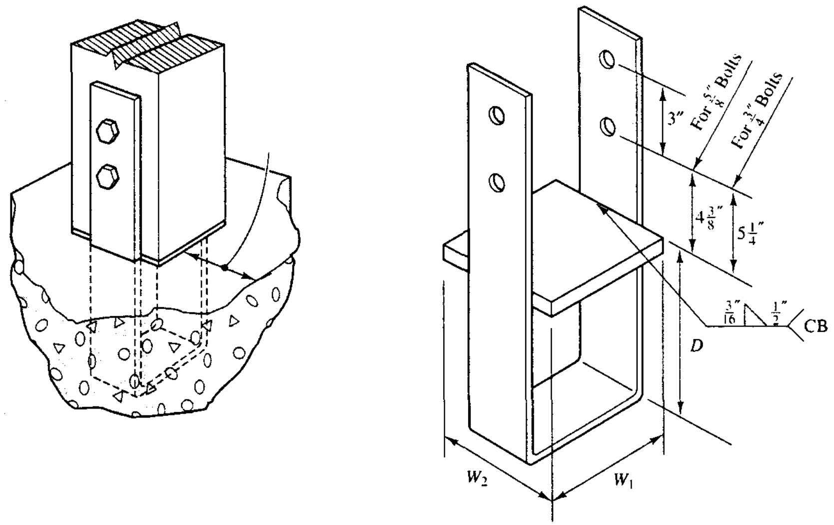


Figure 9.26 Steel post base for securing a wood post to a spread footing (Simpson Strong-Tie Co., Inc.).

Connections with Walls

The connection between a concrete or masonry wall and its footing is a simple one. Simply extend the vertical wall steel into the footing [ACI 15.8.2.2], as shown in Figure 9.27. For construction convenience, design this steel with a lap joint immediately above the footing.

The design of vertical steel in concrete retaining walls is discussed in Chapter 24. Design procedures for other walls are beyond the scope of this book.

Connect wood-frame walls to the footing using anchor bolts, as shown in Figure 9.28. Normally, standard building code criteria govern the size and spacing of these bolts. For example, the *Uniform Building Code* (ICBO, 1997) specifies 1/2 in (12 mm) nominal diameter bolts embedded at least 7 in (175 mm) into the concrete. It also specifies bolt spacings of no more than 6 ft (1.8 m) on center.

Sometimes we must supply a higher capacity connection between wood frame walls and footings, especially when large uplift loads are anticipated. Steel holdown brackets, such as that shown in Figure 9.29, are useful for these situations.

Many older wood-frame buildings have inadequate connections between the structure and its foundation. Figure 9.30 shows one such structure that literally fell off its foundation during the 1989 Loma Prieta Earthquake in Northern California.

Some wood-frame buildings have failed during earthquakes because the *cripple walls* collapsed. These are short wood-frame walls that connect the foundation to the floor. They may be retrofitted by installing plywood shear panels or by using diagonal steel bracing (Shepherd and Delos-Santos, 1991).

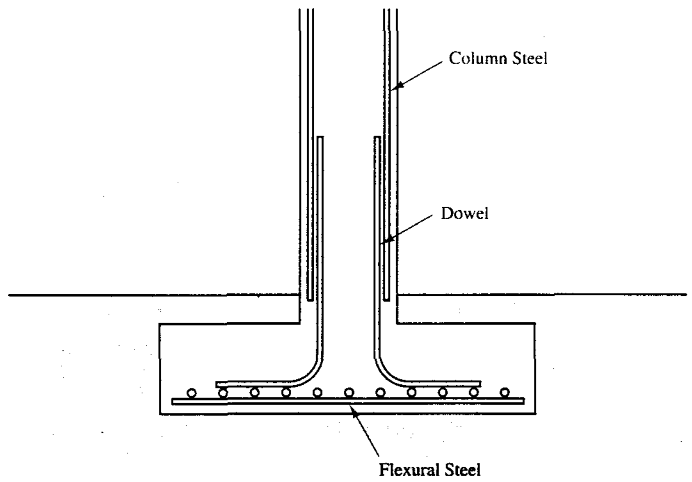


Figure 9.27 Connection between a concrete or masonry wall and its footing.

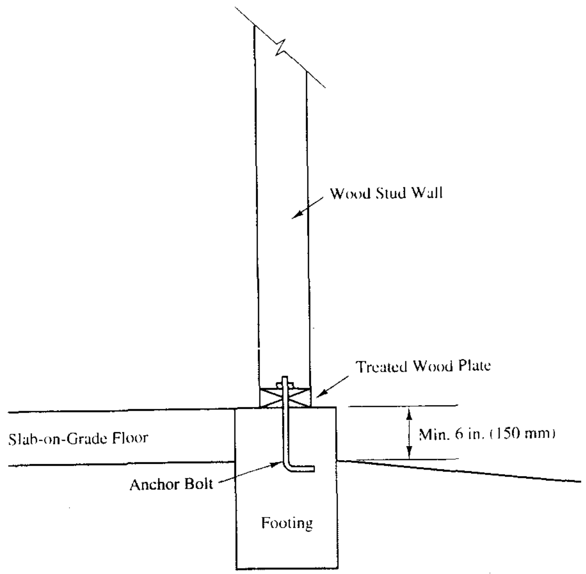


Figure 9.28 Use of anchor bolts to connect a wood-frame wall to a continuous footing.

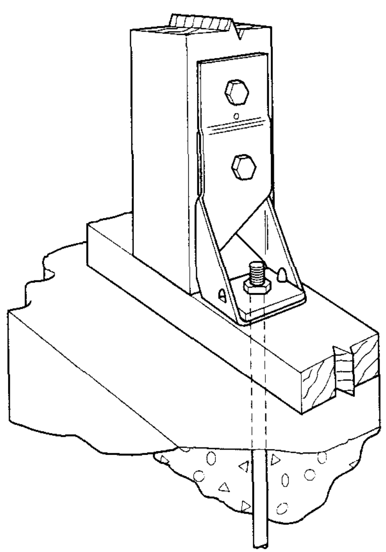


Figure 9.29 Use of steel holdown bracket to connect a wood-frame wall with large uplift force to a footing (Simpson Strong-Tie Co., Inc.).

Figure 9.30 House that separated from its foundation during the 1989 Loma Prieta Earthquake (Photo by C. Stover, U.S. Geological Survey).

QUESTIONS AND PRACTICE PROBLEMS

9.11 An 18-inch square concrete column carries dead and live compressive loads of 240 and 220 k, respectively. It is to be supported on a 8 ft wide 12 ft long rectangular spread footing. Select appropriate values for f_c' and f_y, then determine the required footing thickness and design the flexural reinforcing steel. Show the results of your design in a sketch.

9.12 The column described in Problem 9.11 is reinforced with 6 #8 bars. Design the dowels required to connect it with the footing, and show your design in a sketch.

9.13 A 400-mm diameter concrete column carrying a factored compressive load of 1500 kN is supported on a 600-mm thick, 2500-mm-wide square spread footing. It is reinforced with eight metric #19 bars. Using $f_c' = 18$ MPa and $f_y = 420$ MPa, design the dowels for this connection.

9.14 A 24-inch square concrete column carries a factored compressive load of 900 k and a factored shear load of 100 k. It is to be supported on a spread footing with $f_c' = 3000$ lb/ft^2 and $f_y = 60$ k/in^2. The column is reinforced with twelve #9 bars. Design the dowels for this connection.

9.15 A steel column with a square base plate is to be supported on a spread footing. The AISC factored design loads are: $P_u = 600$ k compression and $V_u = 105$ k. Design an anchor bolt system for this base plate and show your design in a sketch.

SUMMARY

Major Points

1. The plan dimensions and minimum embedment depth of a spread footing are governed by geotechnical concerns, and are determined using the unfactored loads.
2. The thickness and reinforcement of a spread footing are governed by structural concerns. Structural design is governed by the ACI code, which means these analyses are based on the factored load.
3. The structural design of spread footings must consider both shear and flexural failure modes. A shear failure consists of the column or wall punching through the footing, while a flexural failure occurs when the footing has insufficient cantilever strength.
4. Since we do not wish to use stirrups (shear reinforcement), we conduct the shear analysis first and select an effective depth, d, so the footing that provides enough shear resistance in the concrete to resist the shear force induced by the applied load. This analysis ignores the shear strength of the flexural steel.
5. Once the shear analysis is completed, we conduct a flexural analysis to determine the amount of steel required to provide the needed flexural strength. Since d is large, the required steel area will be small, and it is often governed by ρ_{min}.
6. For square footings, use the same flexural steel in both directions. Thus, the footing is reinforced twice.
7. For continuous footings, the lateral steel, if needed, is based on a flexural analysis. Use nominal longitudinal steel to resist nonuniformities in the load and to accommodate inconsistencies in the soil bearing pressure.
8. Design rectangular footings similar to square footings, but group a greater portion of the short steel near the center.
9. Practical minimum dimensions will often govern the design of lightly loaded footings.
10. The connection between the footing and the superstructure is very important. Use dowels to connect concrete or masonry structures. For steel columns and wood-frame walls, use anchor bolts. For wood posts, use specially manufactured brackets.

Vocabulary

Anchor bolts	Effective depth	Reinforcing bars
Critical section for bending	Factored load	Shear failure
Critical shear surface	Flexural failure	Two-way shear
Development length	Minimum steel	Unfactored load
Dowels	One-way shear	28-day compressive strength
	Post base	

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

9.16 A 400-mm square concrete column reinforced with eight metric #19 bars carries vertical dead and live loads of 980 and 825 kN, respectively. It is to be supported on a 2.0 m × 3.5 m rectangular footing. The concrete in the footing will have $f_c' = 20$ MPa and $f_y = 400$ MPa. The building will have a slab-on-grade floor, so the top of the footing must be at least 150 mm below the finish floor elevation. Develop a complete structural design, including dowels, and show it in a sketch.

9.17 A 12-in wide masonry wall carries dead and live loads of 5 k/ft and 8 k/ft, respectively and is reinforced with #6 bars at 24 inches on center. This wall is to be supported on a continuous footing with $f_c' = 2000$ lb/in^2 and $f_y = 60$ k/in^2. The underlying soil has an allowable bearing pressure of 3000 lb/ft^2. Develop a complete structural design for this footing, including dowels, and show your design in a sketch.

10

Mats

The mere formulation of a problem is far more often essential than its solution, which may be merely a matter of mathematical or experimental skill. To raise new questions, new possibilities, to regard old problems from a new angle requires creative imagination and marks real advances in science.

Albert Einstein

The second type of shallow foundation is the *mat foundation*, as shown in Figure 10.1. A mat is essentially a very large spread footing that usually encompasses the entire footprint of the structure. They are also known as *raft foundations*.

Foundation engineers often consider mats when dealing with any of the following conditions:

- The structural loads are so high or the soil conditions so poor that spread footings would be exceptionally large. As a general rule of thumb, if spread footings would cover more than about one-third of the building footprint area, a mat or some type of deep foundation will probably be more economical.
- The soil is very erratic and prone to excessive differential settlements. The structural continuity and flexural strength of a mat will bridge over these irregularities. The same is true of mats on highly expansive soils prone to differential heaves.
- The structural loads are erratic, and thus increase the likelihood of excessive differential settlements. Again, the structural continuity and flexural strength of the mat will absorb these irregularities.

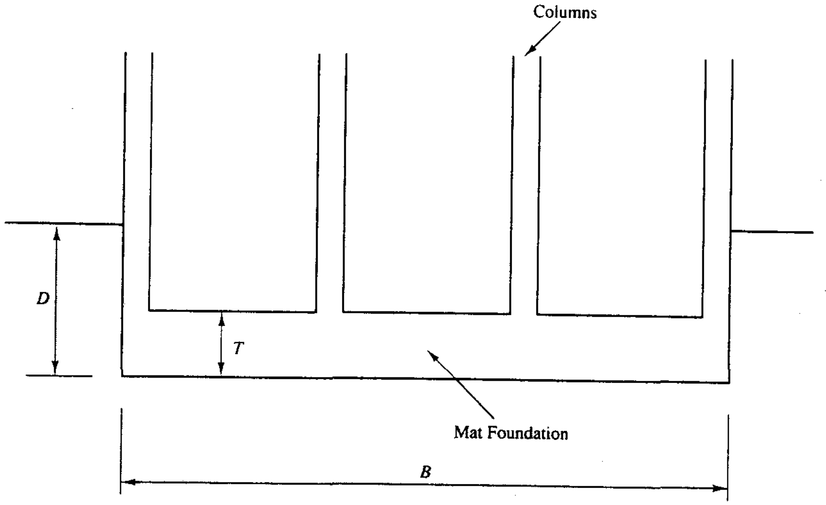


Figure 10.1 A mat foundation supported directly on soil.

- Lateral loads are not uniformly distributed through the structure and thus may cause differential horizontal movements in spread footings or pile caps. The continuity of a mat will resist such movements.
- The uplift loads are larger than spread footings can accommodate. The greater weight and continuity of a mat may provide sufficient resistance.
- The bottom of the structure is located below the groundwater table, so waterproofing is an important concern. Because mats are monolithic, they are much easier to waterproof. The weight of the mat also helps resist hydrostatic uplift forces from the groundwater.

Many buildings are supported on mat foundations, as are silos, chimneys, and other types of tower structures. Mats are also used to support storage tanks and large machines. Typically, the thickness, T, is 1 to 2 m (3–6 ft), so mats are massive structural elements. The seventy five story Texas Commerce Tower in Houston is one of the largest mat-supported structures in the world. Its mat is 3 m (9 ft 9 in) thick and is bottomed 19.2 m (63 ft) below the street level.

Although most mat foundations are directly supported on soil, sometimes engineers use pile- or shaft-supported mats, as shown in Figure 10.2. These foundations are often called *piled rafts*, and they are hybrid foundations that combine features of both mat and deep foundations. Pile- and shaft-supported mats are discussed in Section 11.9.

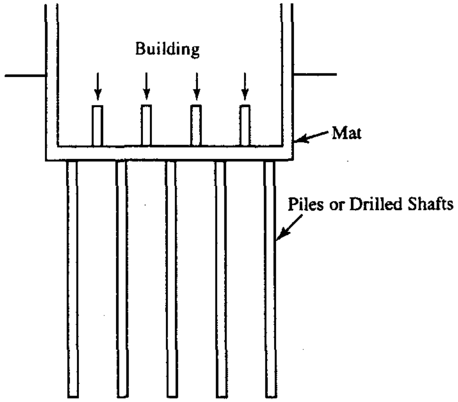


Figure 10.2 A pile- or shaft-supported mat foundation. The deep foundations are not necessarily distributed evenly across the mat.

Various methods have been used to design mat foundations. We will divide them into two categories: Rigid methods and nonrigid methods.

10.1 RIGID METHODS

The simplest approach to structural design of mats is the *rigid method* (also known as the *conventional method* or the *conventional method of static equilibrium*) (Teng, 1962). This method assumes the mat is much more rigid than the underlying soils, which means any distortions in the mat are too small to significantly impact the distribution of bearing pressure. Therefore, the magnitude and distribution of bearing pressure depends only on the applied loads and the weight of the mat, and is either uniform across the bottom of the mat (if the normal load acts through the centroid and no moment load is present) or varies linearly across the mat (if eccentric or moment loads are present) as shown in Figure 10.3. This is the same simplifying assumption used in the analysis of spread footings, as shown in Figure 5.10e.

This simple distribution makes it easy to compute the flexural stresses and deflections (differential settlements) in the mat. For analysis purposes, the mat becomes an inverted and simply loaded two-way slab, which means the shears, moments, and deflections may be easily computed using the principles of structural mechanics. The engineer can then select the appropriate mat thickness and reinforcement.

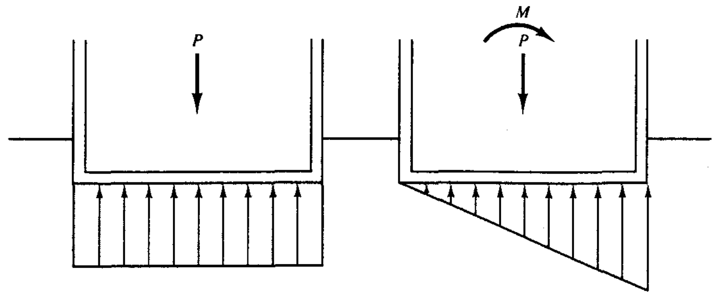


Figure 10.3 Bearing pressure distribution for rigid method.

Although this type of analysis is appropriate for spread footings, it does not accurately model mat foundations because the width-to-thickness ratio is much greater in mats, and the assumption of rigidity is no longer valid. Portions of a mat beneath columns and bearing walls settle more than the portions with less load, which means the bearing pressure will be greater beneath the heavily-loaded zones, as shown in Figure 10.4. This redistribution of bearing pressure is most pronounced when the ground is stiff compared to the mat, as shown in Figure 10.5, but is present to some degree in all soils.

Because the rigid method does not consider this redistribution of bearing pressure, it does not produce reliable estimates of the shears, moments, and deformations in the mat. In addition, even if the mat was perfectly rigid, the simplified bearing pressure distributions in Figure 10.3 are not correct—in reality, the bearing pressure is greater on the edges and smaller in the center than shown in this figure.

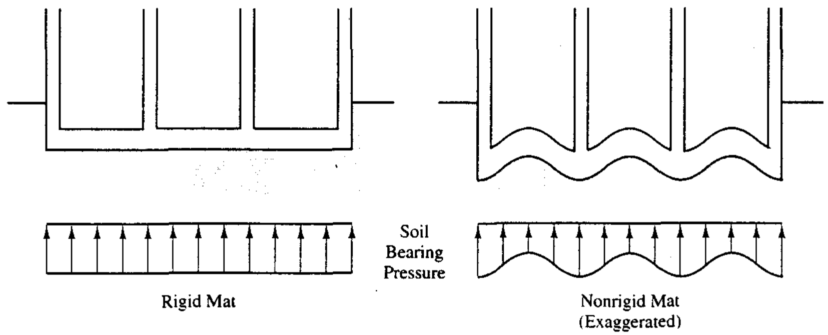


Figure 10.4 The rigid method assumes there are no flexural deflections in the mat, so the distribution of soil bearing pressure is simple to define. However, these deflections are important because they influence the bearing pressure distribution.

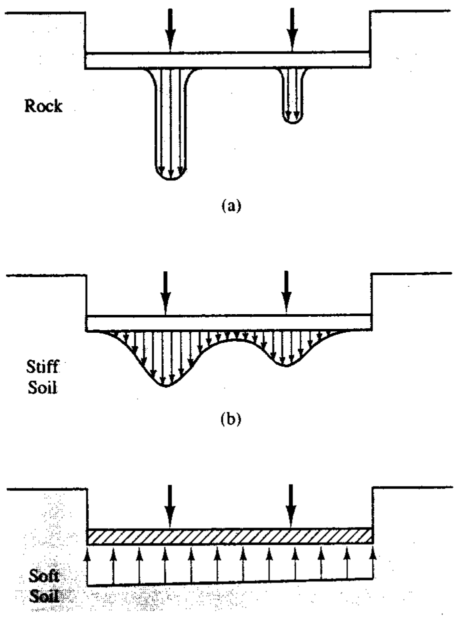


Figure 10.5 Distribution of bearing pressure under a mat foundation; (a) on bedrock or very hard soil; (b) on stiff soil; (c) on soft soil (Adapted from Teng, 1962).

10.2 NONRIGID METHODS

We overcome the inaccuracies of the rigid method by using analyses that consider deformations in the mat and their influence on the bearing pressure distribution. These are called *nonrigid methods*, and produce more accurate values of mat deformations and stresses. Unfortunately, nonrigid analyses also are more difficult to implement because they require consideration of *soil-structure interaction* and because the bearing pressure distribution is not as simple.

Coefficient of Subgrade Reaction

Because nonrigid methods consider the effects of local mat deformations on the distribution of bearing pressure, we need to define the relationship between settlement and bearing pressure. This is usually done using the *coefficient of subgrade reaction*, k_s (also known as the *modulus of subgrade reaction*, or the *subgrade modulus*):

$$k_s = \frac{q}{\delta} \quad (10.1)$$

Where:

k_s = coefficient of subgrade reaction
q = bearing pressure
δ = settlement

The coefficient k_s has units of force per length cubed. Although we use the same units to express unit weight, k_s is not the same as the unit weight and they are not numerically equal.

The interaction between the mat and the underlying soil may then be represented as a "bed of springs," each with a stiffness k_s per unit area, as shown in Figure 10.6. Portions of the mat that experience more settlement produce more compression in the "springs," which represents the higher bearing pressure, whereas portions that settle less do not compress the springs as far and thus have less bearing pressure. The sum of these spring forces must equal the applied structural loads plus the weight of the mat:

$$\Sigma P + W_f - u_D = \int q\,dA = \int \delta\, k_s\, dA \quad (10.2)$$

Where:

ΣP = sum of structural loads acting on the mat
W_f = weight of the mat

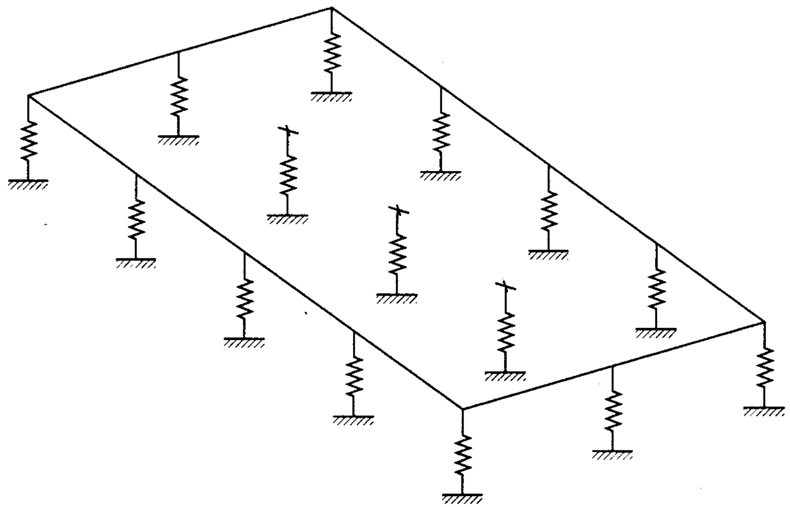


Figure 10.6 The coefficient of subgrade reaction forms the basis for a "bed of springs" analogy to model the soil-structure interaction in mat foundations.

u_D = pore water pressure along base of the mat
q = bearing pressure between mat and soil
A = mat–soil contact area
δ = settlement at a point on the mat

This method of describing bearing pressure is called a *soil-structure interaction analysis* because the bearing pressure depends on the mat deformations, and the mat deformations depend on the bearing pressure.

Winkler Method

The "bed of springs" model is used to compute the shears, moments, and deformations in the mat, which then become the basis for developing a structural design. The earliest use of these "springs" to represent the interaction between soil and foundations has been attributed to Winkler (1867), so the analytical model is sometimes called a *Winkler foundation* or the *Winkler method*. It also is known as a *beam on elastic foundation* analysis.

In its classical form the Winkler method assumes each "spring" is linear and acts independently from the others, and that all of the springs have the same k_s. This representation has the desired effect of increasing the bearing pressure beneath the columns, and thus is a significant improvement over the rigid method. However, it is still only a coarse representation of the true interaction between mats and soil (Hain and Lee, 1974; Horvath, 1983), and suffers from many problems, including the following:

1. The load-settlement behavior of soil is nonlinear, so the k_s value must represent some equivalent linear function, as shown in Figure 10.7.
2. According to this analysis, a uniformly loaded mat underlain by a perfectly uniform soil, as shown in Figure 10.8, will settle uniformly into the soil (i.e., there will be no differential settlement) and all of the "springs" will be equally compressed. In reality, the settlement at the center of such a mat will be greater than that along the edges, as discussed in Chapter 7. This is because the $\Delta\sigma_z$ values in the soil are greater beneath the center.
3. The "springs" should not act independently. In reality, the bearing pressure induced at one point on the mat influences more than just the nearest spring.
4. Primarily because of items 2 and 3, there is no single value of k_s that truly represents the interaction between soil and a mat.

Items 2 and 3 are the primary sources of error, and this error is potentially unconservative (i.e., the shears, moments, and deflections in the mat may be greater than those predicted by Winkler). The heart of these problems is the use of independent springs in the Winkler model. In reality, a load at one point on the mat induces settlement both at that point and in the adjacent parts of the mat, which is why a uniformly mat exhibits dish-shaped settlement, not the uniform settlement predicted by Winkler.

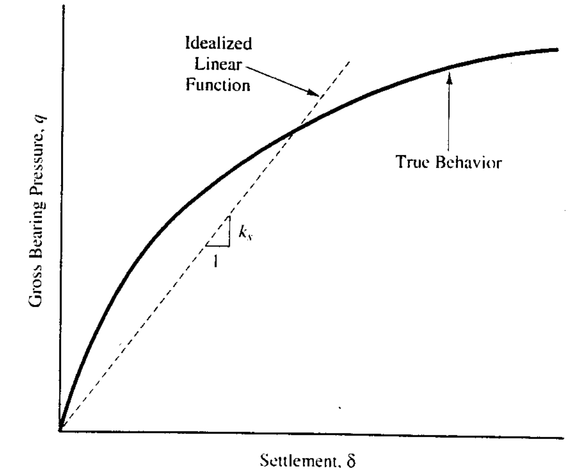


Figure 10.7 The q-δ relationship is nonlinear, so ks must represent some "equivalent" linear function.

Coupled Method

The next step up from a Winkler analysis is to use a *coupled method*, which uses additional springs as shown in Figure 10.9. This way the vertical springs no longer act independently, and the uniformly loaded mat of Figure 10.8 exhibits the desired dish shape. In principle, this approach is more accurate than the Winkler method, but it is not clear how to select the k_s values for the coupling springs, and it may be necessary to develop custom software to implement this analysis.

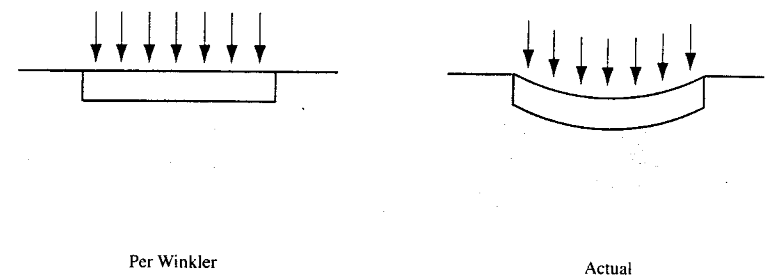


Figure 10.8 Settlement of a uniformly-loaded mat on a uniform soil: (a) per Winkler analysis, (b) actual.

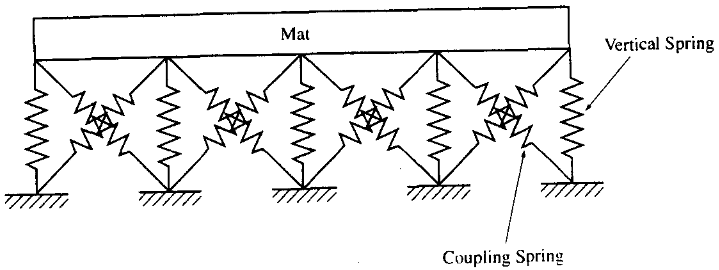


Figure 10.9 Modeling of soil-structure interaction using coupled springs.

Pseudo-Coupled Method

The *pseudo-coupled method* (Liao, 1991; Horvath, 1993) is an attempt to overcome the lack of coupling in the Winkler method while avoiding the difficulties of the coupled method. It does so by using "springs" that act independently, but have different k_s values depending on their location on the mat. To properly model the real response of a uniform soil, the "springs" along the perimeter of the mat should be stiffer than those in the center, thus producing the desired dish-shaped deformation in a uniformly-loaded mat. If concentrated loads, such as those from columns, also are present, the resulting mat deformations are automatically superimposed on the dish-shape.

Model studies indicate that reasonable results are obtained when k_s values along the perimeter of the mat are about twice those in the center (ACI, 1993). We can implement this in a variety of ways, including the following:

1. Divide the mat into two or more concentric zones, as shown in Figure 10.10. The innermost zone should be about half as wide and half as long as the mat.
2. Assign a k_s value to each zone. These values should progressively increase from the center such that the outermost zone has a k_s about twice as large at the innermost zone. Example 10.1 illustrates this technique.
3. Evaluate the shears, moments, and deformations in the mat using the Winkler "bed of springs" analysis, as discussed later in this chapter.
4. Adjust the mat thickness and reinforcement as needed to satisfy strength and serviceability requirements.

ACI (1993) found the pseudo-coupled method produced computed moments 18 to 25 percent higher than those determined from the Winkler method, which is an indication of how unconservative Winker can be.

Most commercial mat design software uses the Winkler method to represent the soil-structure interaction, and these software packages usually can accommodate the

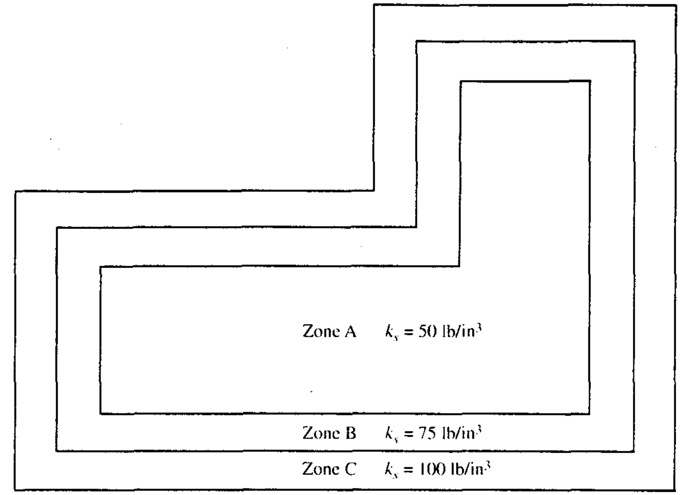


Figure 10.10 A typical mat divided into zones for a pseudo-coupled analysis. The coefficient of subgrade reaction, k_s, progressively increases from the innermost zone to the outermost zone.

pseudo-coupled method. Given the current state of technology and software availability, this is probably the most practical approach to designing most mat foundations.

Multiple-Parameter Method

Another way of representing soil-structure interaction is to use the *multiple parameter method* (Horvath, 1993). This method replaces the independently-acting linear springs of the Winkler method (a single-parameter model) with springs and other mechanical elements (a multiple-parameter model). These additional elements define the coupling effects.

The multiple-parameter method bypasses the guesswork involved in distributing the k_s values in the psuedo-coupled method because coupling effects are inherently incorporated into the model, and thus should be more accurate. However, it has not yet been implemented into readily-available software packages. Therefore, this method is not yet ready to be used on routine projects.

Finite Element Method

All of the methods discussed thus far attempt to model a three-dimensional soil by a series of one-dimensional springs. They do so in order to make the problem simple enough to perform the structural analysis. An alternative method would be to use a three-dimensional mathematical model of both the mat and the soil, or perhaps the mat, soil, and superstructure. This can be accomplished using the *finite element method.*

This analysis method divides the soil into a network of small elements, each with defined engineering properties and each connected to the adjacent elements in a specified way. The structural and gravitational loads are then applied and the elements are stressed and deformed accordingly. This, in principle, should be an accurate representation of the mat, and should facilitate a precise and economical design.

Unfortunately, such analyses are not yet practical for routine design problems because:

1. A three-dimensional finite element model requires tens of thousands or perhaps hundreds of thousands of elements, and thus place corresponding demands on computer resources. Few engineers have access to computers that can accommodate such intensive analyses.
2. It is difficult to determine the required soil properties with enough precision, especially at sites where the soils are highly variable. In other words, the analysis method far outweighs our ability to input accurate parameters.

Nevertheless, this approach may become more usable in the future, especially as increasingly powerful computers become more widely available.

This method should not be confused with structural analysis methods that use two-dimensional finite elements to model the mat and Winkler springs to model the soil. Such methods require far less computational resources, and are widely used. We will discuss this use of finite element analyses in Section 10.4.

10.3 DETERMINING THE COEFFICIENT OF SUBGRADE REACTION

Most mat foundation designs are currently developed using either the Winkler method or the pseudo-coupled method, both of which depend on our ability to define the coefficient of subgrade reaction, k_s. Unfortunately, this task is not as simple as it might first appear because k_s is not a fundamental soil property. Its magnitude also depends on many other factors, including the following:

- **The width of the loaded area**—A wide mat will settle more than a narrow one with the same q because it mobilizes the soil to a greater depth as shown in Figure 8.2. Therefore, each has a different k_s.
- **The shape of the loaded area**—The stresses below long narrow loaded areas are different from those below square loaded areas as shown in Figure 7.2. Therefore, k_s will differ.
- **The depth of the loaded area below the ground surface**—At greater depths, the change in stress in the soil due to q is a smaller percentage of the initial stress, so the settlement is also smaller and k_s is greater.

- **The position on the mat**—To model the soil accurately, k_s needs to be larger near the edges of the mat and smaller near the center.
- **Time**—Much of the settlement of mats on deep compressible soils will be due to consolidation and thus may occur over a period of several years. Therefore, it may be necessary to consider both short-term and long-term cases.

Actually, there is no single k_s value, even if we could define these factors because the q-δ relationship is nonlinear and because neither method accounts for interaction between the springs.

Engineers have tried various techniques of measuring or computing k_s. Some rely on plate load tests to measure k_s in situ. However, the test results must be adjusted to compensate for the differences in width, shape, and depth of the plate and the mat. Terzaghi (1955) proposed a series of correction factors, but the extrapolation from a small plate to a mat is so great that these factors are not very reliable. Plate load tests also include the dubious assumption that the soils within the shallow zone of influence below the plate are comparable to those in the much deeper zone below the mat. Therefore, plate load tests generally do not provide good estimates of k_s for mat foundation design.

Others have used derived relationships between k_s and the soil's modulus of elasticity, E (Vesić and Saxena, 1970; Scott, 1981). Although these relationships provide some insight, they too are limited.

Another method consists of computing the average mat settlement using the techniques described in Chapter 7 and expressing the results in the form of k_s using Equation 10.1. If using the pseudo-coupled method, use k_s values in the center of the mat that are less than the computed value, and k_s values along the perimeter that are greater. This should be done in such a way that the perimeter values are twice the central values, and the integral of all the values over the area of the mat is the same as the produce of the original k_s and the mat area. Example 10.1 describes this methodology.

Example 10.1

A structure is to be supported on a 30-m wide, 50-m long mat foundation. The average bearing pressure is 120 kPa. According to a settlement analysis conducted using the techniques described in Chapter 7, the average settlement, δ, will be 30 mm. Determine the design values of k_s to be used in a pseduo-coupled analysis.

Solution

Compute average k_s using Equation 10.1:

$$(k_s)_{avg} = \frac{q}{\delta} = \frac{120 \text{ kPa}}{0.030 \text{ m}} = 4000 \text{ kN/m}^3$$

Divide the mat into three zones, as shown in Figure 10.11, with $(k_s)_C = 2(k_s)_A$ and $(k_s)_B = 1.5(k_s)_A$

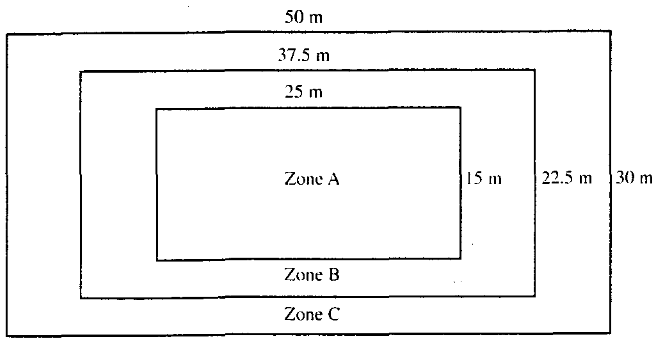


Figure 10.11 Mat foundation for Example 10.1.

Compute the area of each zone:

$$A_A = (25\text{ m})(15\text{ m}) = 375\text{ m}^2$$

$$A_B = (37.5\text{ m})(22.5\text{ m}) - (25\text{ m})(15\text{ m}) = 469\text{ m}^2$$

$$A_C = (50\text{ m})(30\text{ m}) - (37.5\text{ m})(22.5\text{ m}) = 656\text{ m}^2$$

Compute the design k_s values:

$$A_A\,(k_s)_A + A_B\,(k_s)_B + A_C\,(k_s)_C = (A_A + A_B + A_C)\,(k_s)_{avg}$$

$$375\,(k_s)_A + 469\,(1.5)(k_s)_A + 656\,(2)(k_s)_A = 1500\,(k_s)_{avg}$$

$$2390\,(k_s)_A = 1500\,(k_s)_{avg}$$

$$(k_s)_A = 0.627\,(k_s)_{avg}$$

$$(k_s)_A = (0.627)(4000\text{ kN/m}^3) = \mathbf{2510\text{ kN/m}^3} \quad \Leftarrow \textit{Answer}$$

$$(k_s)_B = (1.5)(0.627)(4000\text{ kN/m}^3) = \mathbf{3765\text{ kN/m}^3} \quad \Leftarrow \textit{Answer}$$

$$(k_s)_C = (2)(0.627)(4000\text{ kN/m}^3) = \mathbf{5020\text{ kN/m}^3} \quad \Leftarrow \textit{Answer}$$

Because it is so difficult to develop accurate k_s values, it may be appropriate to conduct a parametric studies to evaluate its effect on the mat design. ACI (1993) suggests varying k_s from one-half the computed value to five or ten times the computed value, and basing the structural design on the worst case condition.

This wide range in k_s values will produce proportional changes in the computed total settlement. However, we ignore these total settlement computations because they are not reliable anyway, and compute it using the methods described in Chapter 7. These changes in k_s have much less impact on the shears, moments, and deflections in the mat, and thus have only a small impact on the structural design.

10.4 STRUCTURAL DESIGN

General Methodology

The structural design of mat foundations must satisfy both strength and serviceability requirements. This requires two separate analyses, as follows:

Step 1: Evaluate the strength requirements using the factored loads (Equations 2.7–2.15) and LRFD design methods (which ACI calls *ultimate strength design*). The mat must have a sufficient thickness, T, and reinforcement to safely resist these loads. As with spread footings, T should be large enough that no shear reinforcement is needed.

Step 2: Evaluate mat deformations (which is the primary serviceability requirement) using the unfactored loads (Equations 2.1–2.4). These deformations are the result of concentrated loading at the column locations, possible non-uniformities in the mat, and variations in the soil stiffness. In effect, these deformations are the equivalent of differential settlement. If they are excessive, then the mat must be made stiffer by increasing its thickness.

Closed-Form Solutions

When the Winkler method is used (i.e., when all "springs" have the same k_s) and the geometry of the problem can be represented in two-dimensions, it is possible to develop closed-form solutions using the principles of structural mechanics (Scott, 1981; Hetényi, 1974). These solutions produce values of shear, moment, and deflection at all points in the idealized foundation. When the loading is complex, the principle of superposition may be used to divide the problem into multiple simpler problems.

These closed-form solutions were once very popular, because they were the only practical means of solving this problem. However, the advent and widespread availability of powerful computers and the associated software now allows us to use other methods that are more precise and more flexible.

Finite Element Method

Today, most mat foundations are designed with the aid of a computer using the *finite element method* (*FEM*). This method divides the mat into hundreds or perhaps thousands of elements, as shown in Figure 10.12. Each element has certain defined dimensions, a specified stiffness and strength (which may be defined in terms of concrete and steel properties) and is connected to the adjacent elements in a specified way.

The mat elements are connected to the ground through a series of "springs," which are defined using the coefficient of subgrade reaction. Typically, one spring is located at each corner of each element.

The loads on the mat include the externally applied column loads, applied line loads, applied area loads, and the weight of the mat itself. These loads press the mat downward,

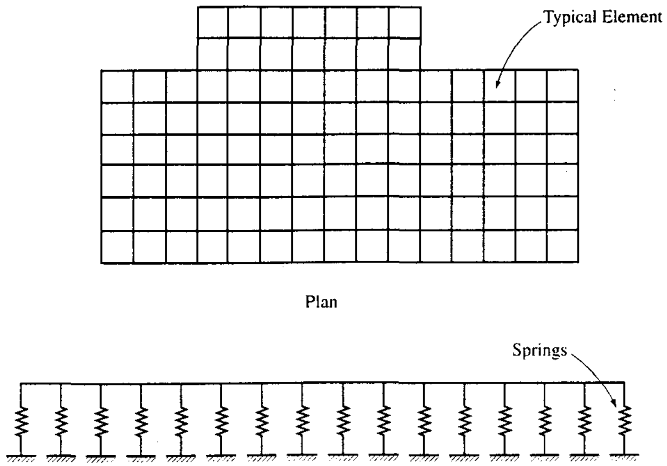


Figure 10.12 Use of the finite element method to analyze mat foundations. The mat is divided into a series of elements which are connected in a specified way. The elements are connected to the ground through a "bed of springs."

and this downward movement is resisted by the soil "springs." These opposing forces, along with the stiffness of the mat, can be evaluated simultaneously using matrix algebra, which allows us to compute the stresses, strains, and distortions in the mat. If the results of the analysis are not acceptable, the design is modified accordingly and reanalyzed.

This type of finite element analysis does not consider the stiffness of the superstructure. In other words, it assumes the superstructure is perfectly flexible and offers no resistance to deformations in the mat. This is conservative.

The finite element analysis can be extended to include the superstructure, the mat, and the underlying soil in a single three-dimensional finite element model. This method would, in principle, be a more accurate model of the soil-structure system, and thus may produce a more economical design. However, such analyses are substantially more complex and time-consuming, and it is very difficult to develop accurate soil properties for such models. Therefore, these extended finite element analyses are rarely performed in practice.

10.5 TOTAL SETTLEMENT

The bed of springs analyses produce a computed total settlement. However, this value is unreliable and should not be used for design. These analyses are useful only for computing shears, moments, and deformations (differential settlements) in the mat. Total settlement should be computed using the methods described in Chapter 7.

10.6 BEARING CAPACITY

Because of their large width, mat foundations on sands and gravels do not have bearing capacity problems. However, bearing capacity might be important in silts and clays, especially if undrained conditions prevail. The Fargo Grain Silo failure described in Chapter 6 is a notable example of a bearing capacity failure in a saturated clay.

We can evaluate bearing capacity using the analysis techniques described in Chapter 6. It is good practice to design the mat so the bearing pressure at all points is less than the allowable bearing capacity.

SUMMARY

Major Points

1. Mat foundations are essentially large spread footings that usually encompass the entire footprint of a structure. They are often an appropriate choice for structures that are too heavy for spread footings.
2. The analysis and design of mats must include an evaluation of the flexural stresses and must provide sufficient flexural strength to resist these stresses.
3. The oldest and simplest method of analyzing mats is the rigid method. It assumes that the mat is much more rigid than the underlying soil, which means the magnitude and distribution of bearing pressure is easy to determine. This means the shears, moment, and deformations in the mat are easily determined. However, this method is not an accurate representation because the assumption of rigidity is not correct.
4. Nonrigid analyses are superior because they consider the flexural deflections in the mat and the corresponding redistribution of the soil bearing pressure.
5. Nonrigid methods must include a definition of soil-structure interaction. This is usually done using a "bed of springs" analogy, with each spring having a linear force-displacement function as defined by the coefficient of subgrade reaction, k_s.
6. The simplest and oldest nonrigid method is the Winkler method, which uses independent springs, all of which have the same k_s. This method is an improvement over rigid analyses, but still does not accurately model soil–structure interaction, primarily because it does not consider coupling effects.
7. The coupled method is an extension of the Winkler method that considers coupling between the springs.
8. The pseudo-coupled method uses independent springs, but adjusts the k_s values to implicitly account for coupling effects.
9. The multiple parameter and finite element methods are more advanced ways of describing soil-structure interaction.
10. The coefficient of subgrade reaction is difficult to determine. Fortunately, the mat design is often not overly sensitive to global changes in k_s. Parametric studies are often appropriate.

11. If the Winkler method is used to describe soil–structure interaction, and the mat geometry is not too complex, the structural analysis may be performed using closed-form solutions. However, these methods are generally considered obsolete.
12. Most structural analyses are performed using numerical methods, especially the finite element method. This method uses finite elements to model the mat, and defines soil–structure interaction using the Winker or pseudo-coupled models. In principle, it also could use the multiple parameter model.
13. A design could be based entirely on a three-dimensional finite element analysis that includes the soil, mat, and superstructure. However, such analyses are beyond current practices, mostly because they are difficult to set up and require especially powerful computers.
14. The total settlement is best determined using the methods described in Chapter 7. Do not use the coefficient of subgrade reaction to determine total settlement.
15. Bearing capacity is not a problem with sands and gravels, but can be important in silts and clays. It should be checked using the methods described in Chapter 6.

Vocabulary

Beam on elastic foundation	Mat foundation	Rigid method
Bed of springs	Multiple parameter method	Shaft-supported mat
Coefficient of subgrade reaction	Nonrigid method	Soil–structure interaction
	Pile-supported mat	Winkler method
Coupled method	Pseudo-coupled method	
Finite element method	Raft foundation	

COMPREHENSIVE QUESTIONS AND PRACTICE PROBLEMS

10.1 Explain the reasoning behind the statement in Section 10.6: "Because of their large width, mat foundations on sands and gravels do not have bearing capacity problems."

10.2 How has the development of powerful and inexpensive digital computers affected the analysis and design of mat foundations? What changes do you expect in the future as this trend continues?

10.3 A mat foundation supports forty two columns for a building. These columns are spaced on a uniform grid pattern. How would the moments and differential settlements change if we used a nonrigid analysis with a constant k_s in lieu of a rigid analysis?

10.4 According to a settlement analysis conducted using the techniques described in Chapter 7, a certain mat will have a total settlement of 2.1 inches if the average bearing pressure is 5500 lb/ft^2. Compute the average k_s and express your answer in units of lb/in^3.

10.5 A 25-m diameter cylindrical water storage tank is to be supported on a mat foundation. The weight of the tank and its contents will be 50,000 kN and the weight of the mat will be 12,000 kN. According to a settlement analysis conducted using the techniques described in Chapter 7, the total settlement will be 40 mm. The groundwater table is at a depth of 5 m below the bottom of the mat. Using the pseudo-coupled method, divide the mat into zones and compute k_s for each zone. Then indicate the high-end and low-end values of k_s that should be used in the analysis.

10.6 An office building is to be supported on 150-ft × 300-ft mat foundation. The sum of the column loads plus the weight of the mat will be 90,000 k. According to a settlement analysis conducted using the techniques described in Chapter 7, the total settlement will be 1.8 inches. The groundwater table is at a depth of 10 ft below the bottom of the mat. Using the pseudo-coupled method, divide the mat into zones and composite each zone. Then indicate the high-end and low-end valves of k_s that should be used in the analysis.