Shallow Foundations
Mat

Reference:
Foundation Design, Principles and Practices, Donald P. Coduto, Part B, Chapter 10

MAT (RAFT) Foundations

- Structural loads are too high or soil conditions are too weak; spread footings cover more than one-third (or half?) of the building footprint
- Soil conditions or loading is so erratic that special design is needed to control differential settlements

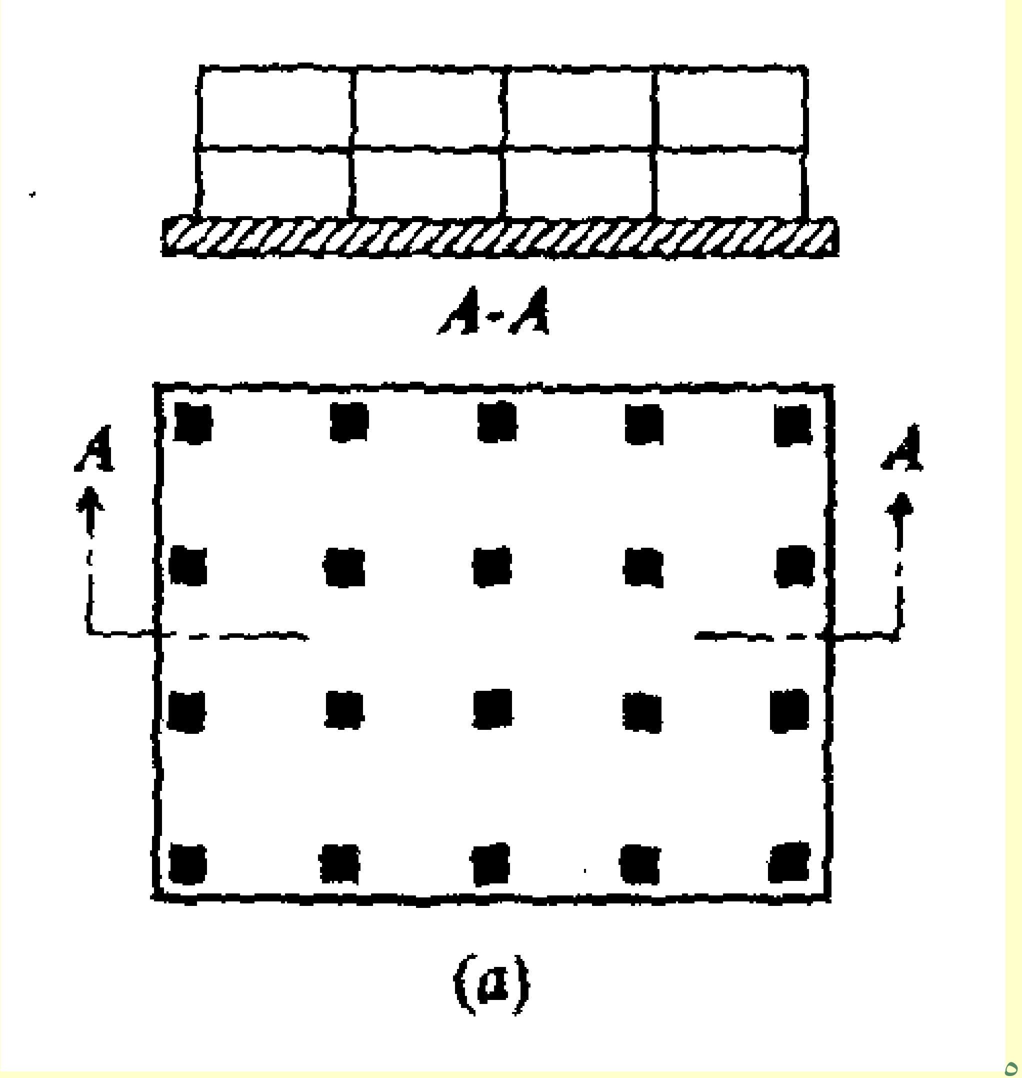
MAT (RAFT) Foundations

- Usually large concrete slab supporting many columns
- Commonly used as foundation for silos, chimneys, large machinery
- Provides larger FOS against bearing failure:
  - reduces bearing pressure
  - at the same time increases bearing capacity

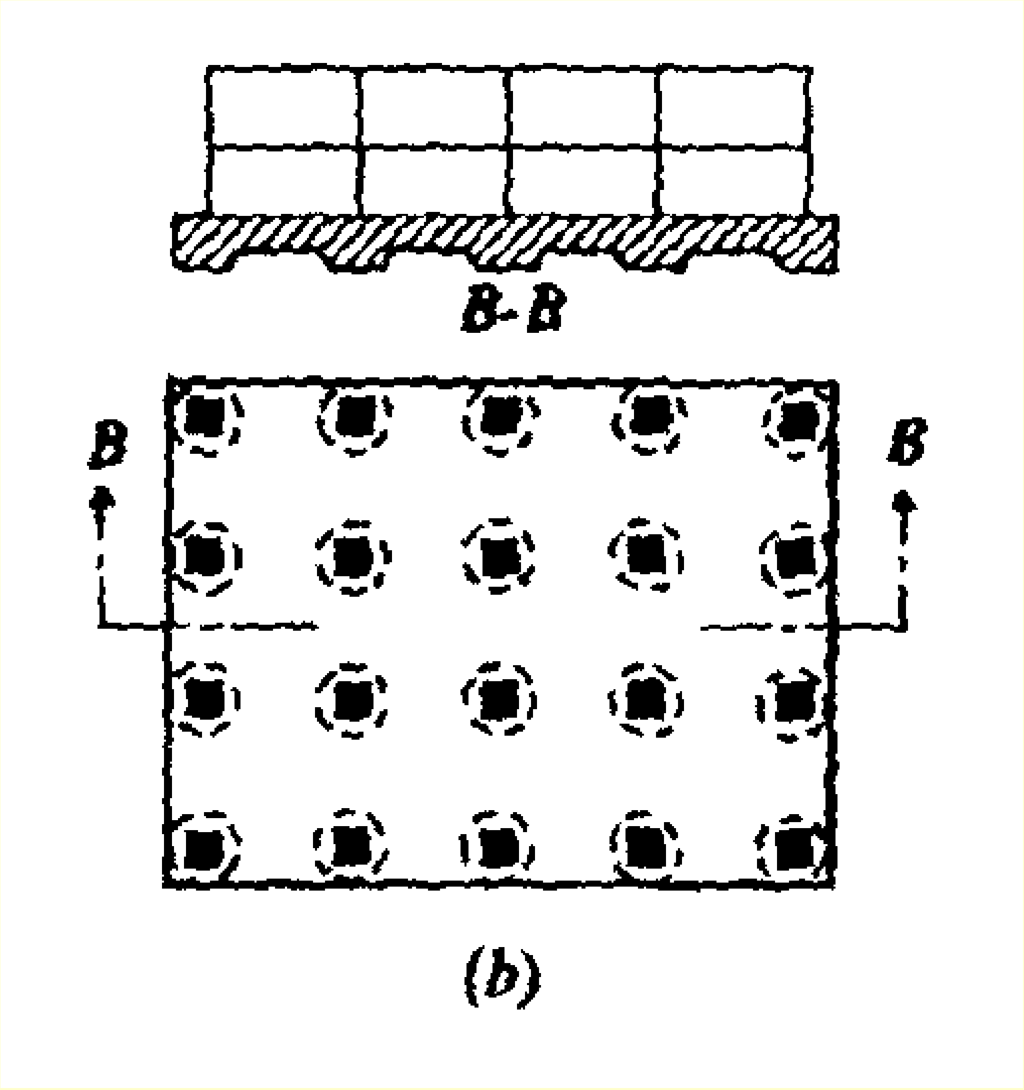
Types of Mat Foundations

- Flat concrete slab of uniform thickness
- Slab thickened under larger column loads
- Slabs with pedestals to support heavier column loads
- Slabs with two-way beams
- Cellular structures
- Rigid frames consisting of slabs and basement walls

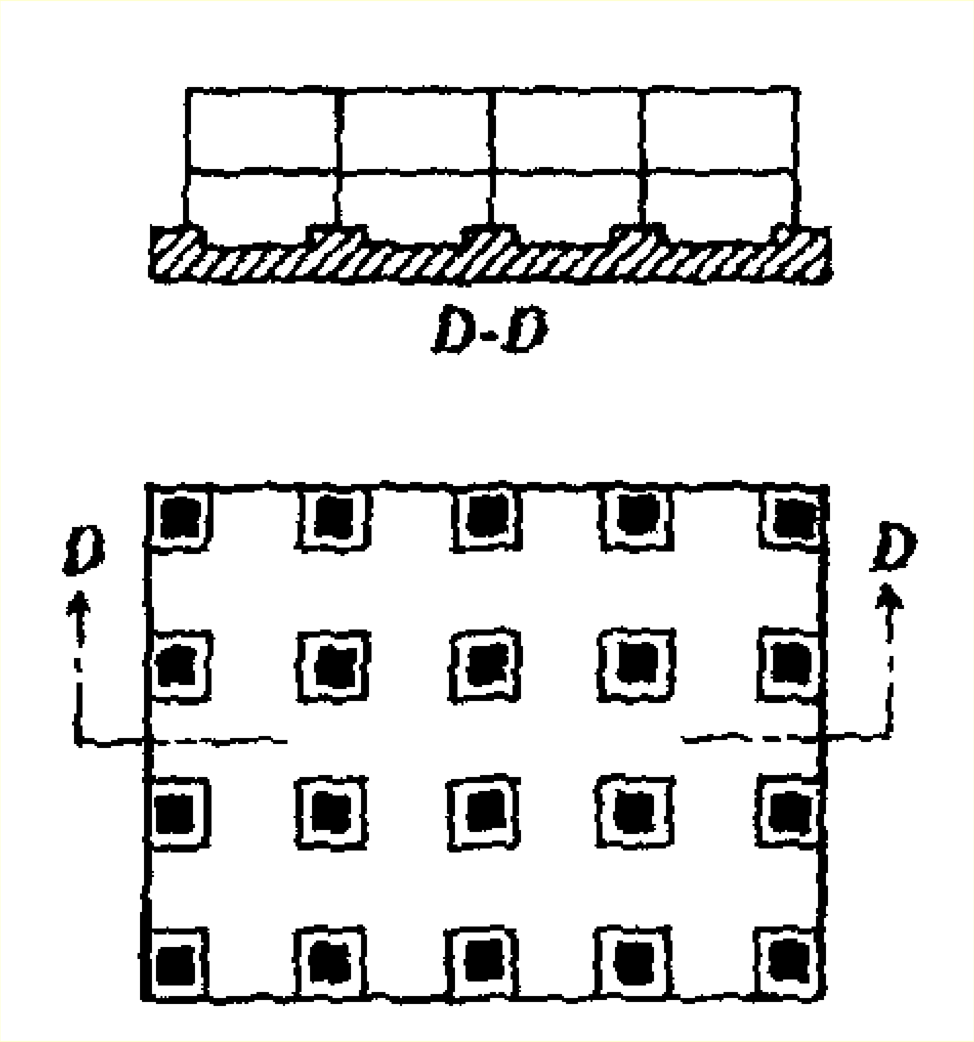
Concrete Slab of Uniform Thickness



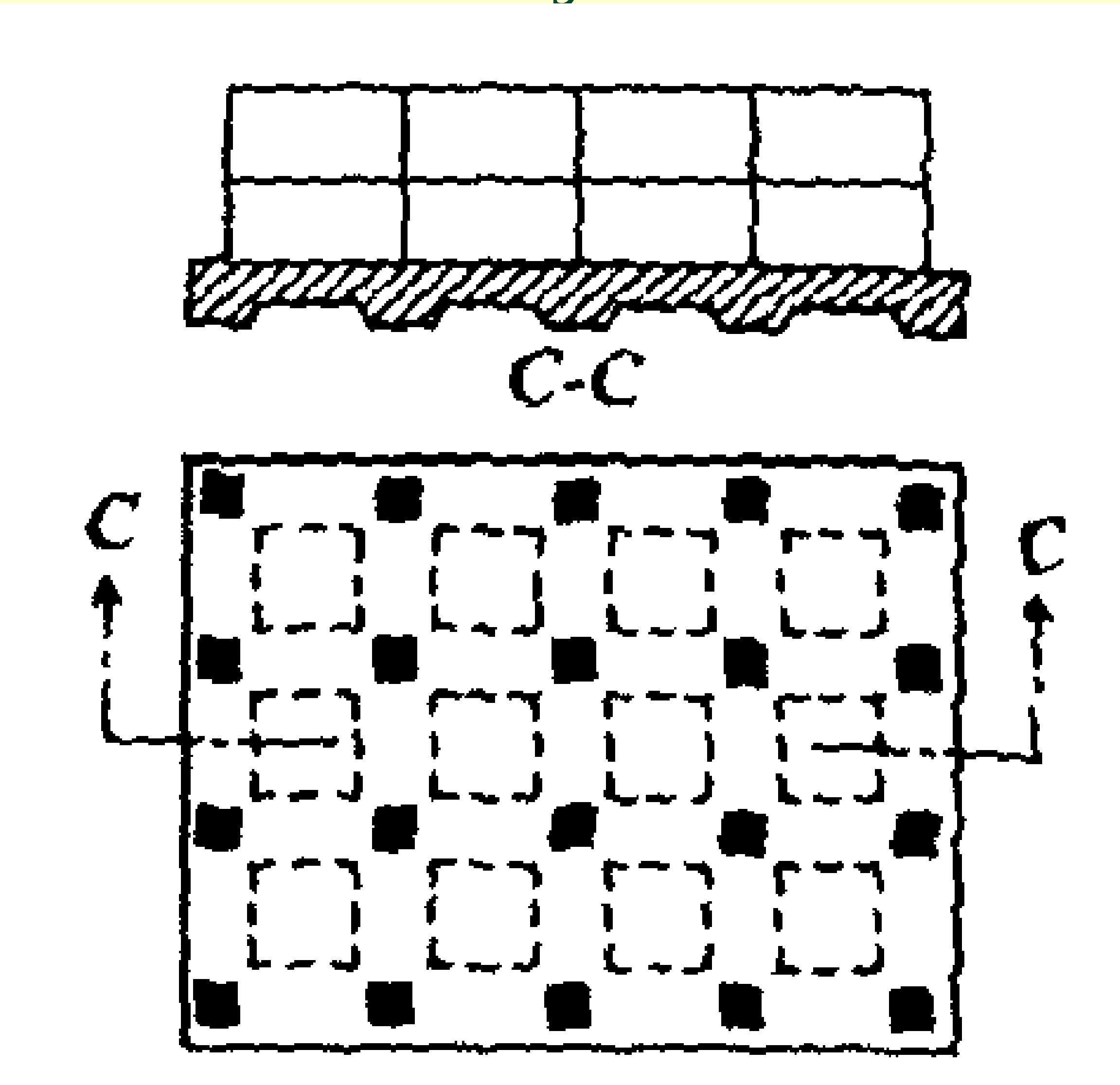
Slab Thickened under Heavier Columns



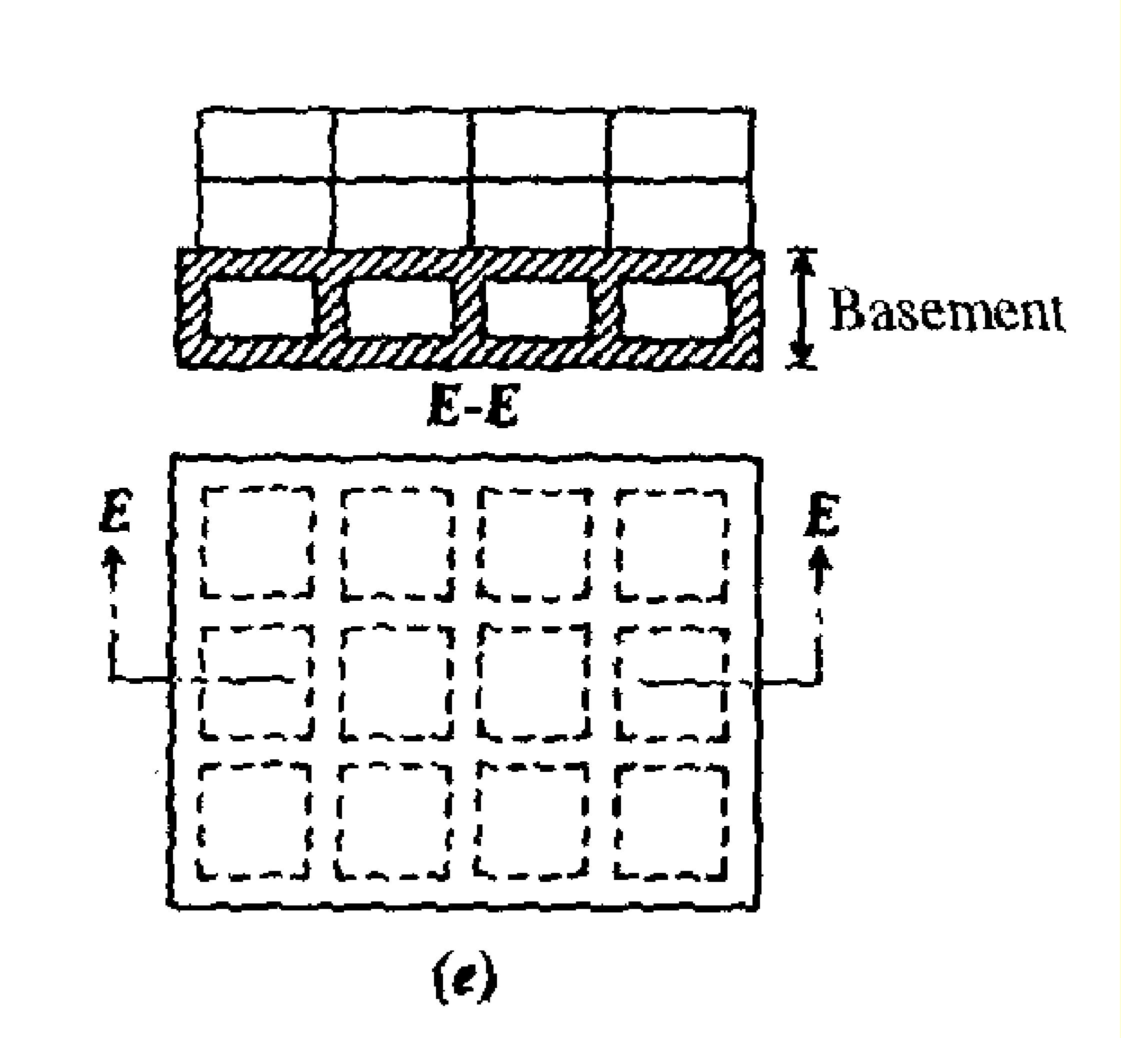
Slabs with Pedestals to Support Heavier Columns



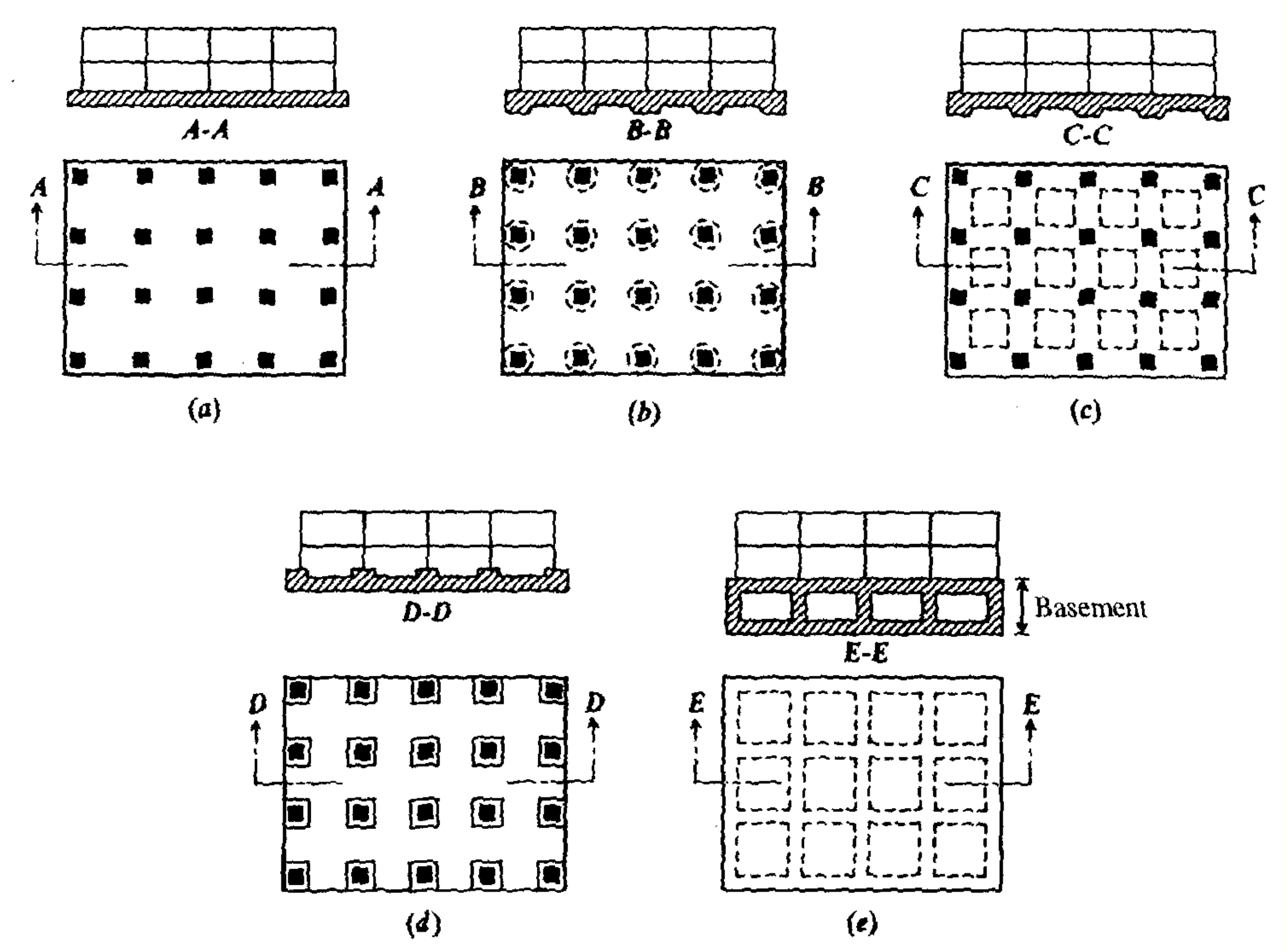
Slabs with Two-Way Beams



Foundation Mat with Cellular Structure; Walls act as stiffeners



Mat (or Raft) Foundation



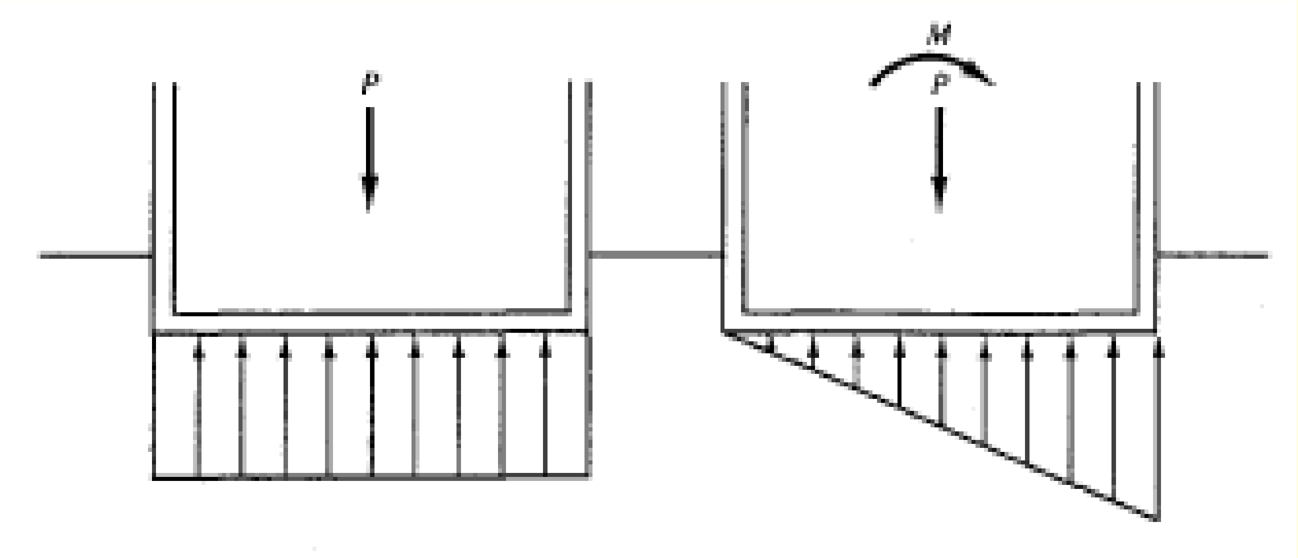
Methods used to design mat foundations

- **Rigid Methods**
- **Non-Rigid Methods**

Rigid Methods

- This method assumes the mat is much more rigid than the underlying soils.
- Therefore, the magnitude and distribution of bearing pressure depends only on the applied loads and the weight of the mat, and is either uniform across the bottom of the mat (if the normal load acts through the centroid and no moment load is present) or varies linearly across the mat (if eccentric or moment loads are present).
- This simple distribution makes it easy to compute the flexural stresses and deflections (differential settlements) in the mat.

Rigid Methods



Rigid Methods

- Portions of a mat beneath columns and bearing walls settle more than the portions with less load, which means the bearing pressure will be greater beneath the heavily-loaded zones, as shown in Figure …

Rigid Methods

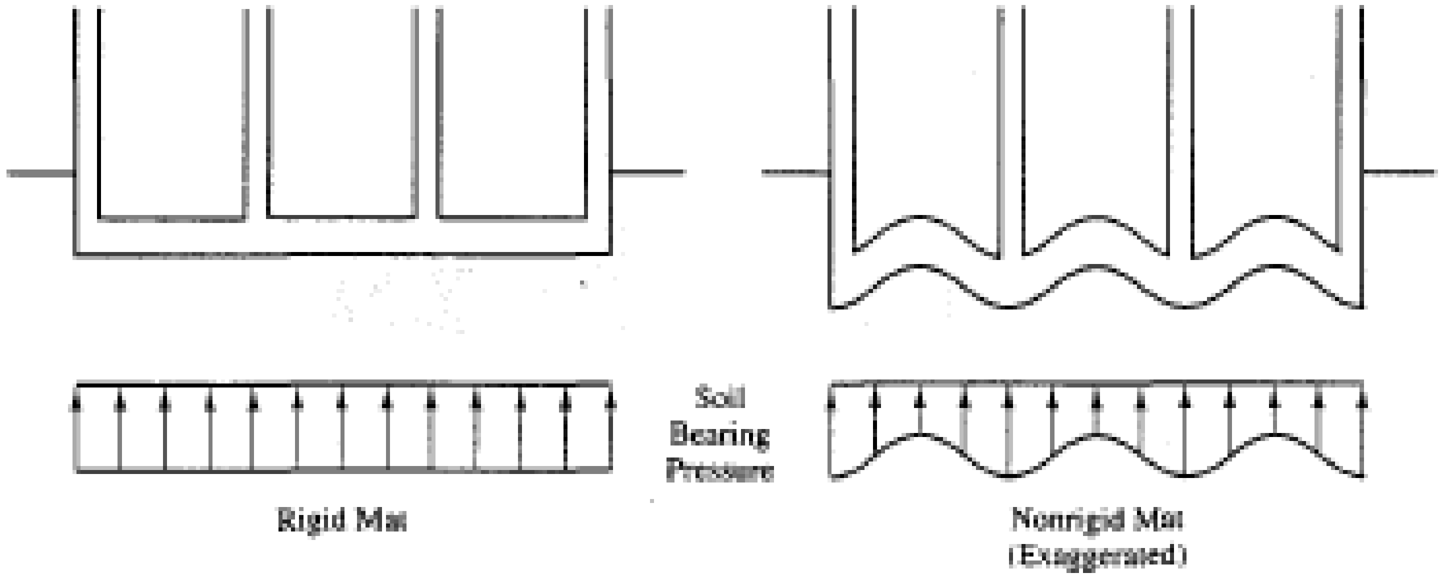
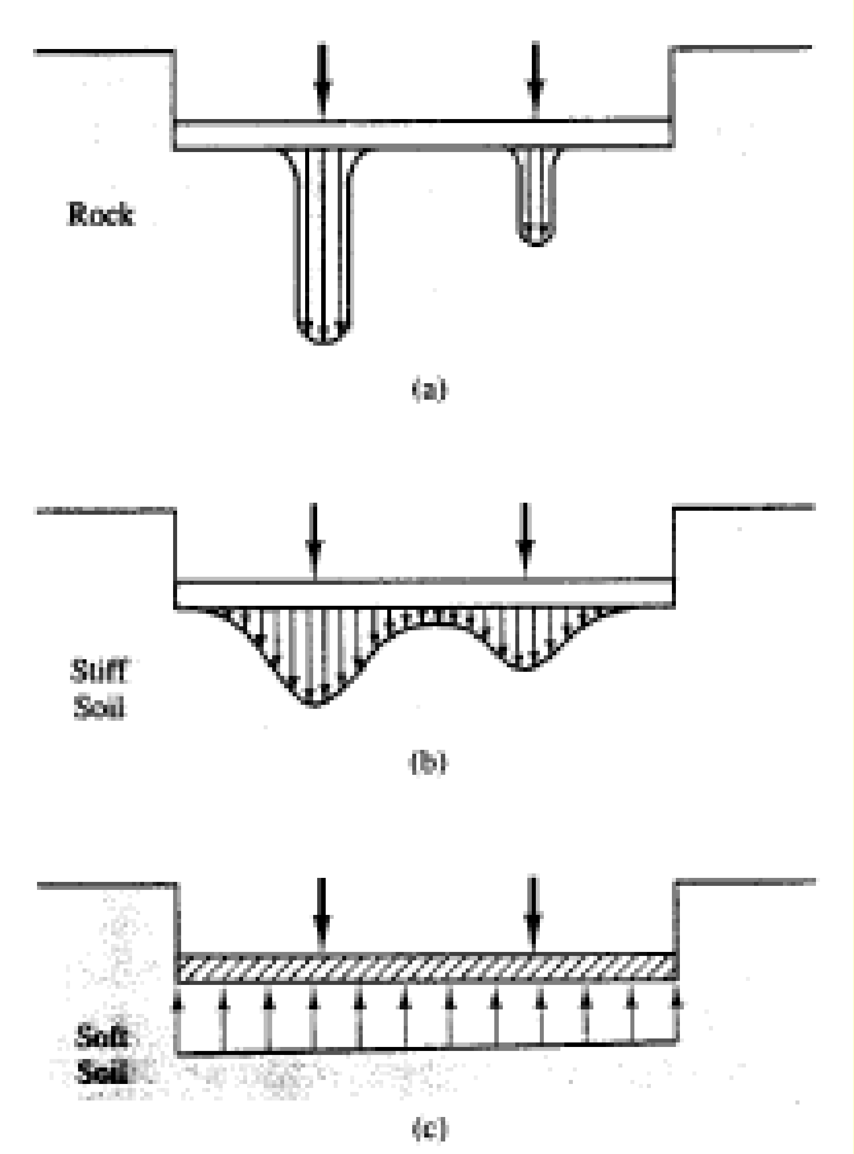


Figure 10.4 The rigid method assumes there are no flexural deflections in the mat, so the distribution of soil bearing pressure is simple to define. However, these deflections are important because they influence the bearing pressure distribution.

Rigid Methods

- This redistribution of bearing pressure is most pronounced when the ground is stiff compared to the mat, but is present to some degree in all soils.
- Because the rigid method does not consider this redistribution of bearing pressure, it does not produce reliable estimates of the shears, moments, and deformations in the mat.
- In addition, even if the mat was perfectly rigid, the simplified bearing pressure distributions are not correct in reality, the bearing pressure is greater on the edges and smaller in the center than shown in this figure.

Rigid Methods



Non-Rigid Methods

- We overcome the inaccuracies of the rigid method by using analyses that consider deformations in the mat and their influence on the bearing pressure distribution.
- These are called *non rigid methods*, and produce more accurate values of mat deformations and stresses.
- Unfortunately, non rigid analyses also are more difficult to implement because they require consideration of *soil-structure interaction* and because the bearing pressure distribution is not as simple.

Coefficient of Subgrade Reaction

- Because non rigid methods consider the effects of local mat deformations on the distribution of bearing pressure, we need to define the relationship between settlement and bearing pressure.
- This is usually done using the *coefficient of subgrade reaction*, *ks* (also known as the *modulus of subgrade reaction*, or the *subgrade modulus)*

$$Ks = q / \delta$$

- The coefficient *ks* has units of force per length cubed. Although we use the same units to express unit weight, *ks* is not the same as the unit weight and they are not numerically equal.
- The interaction between the mat and the underlying soil may then be represented as a "bed of springs," each with a stiffness *ks* per unit area, as shown in next figure.

Coefficient of Subgrade Reaction

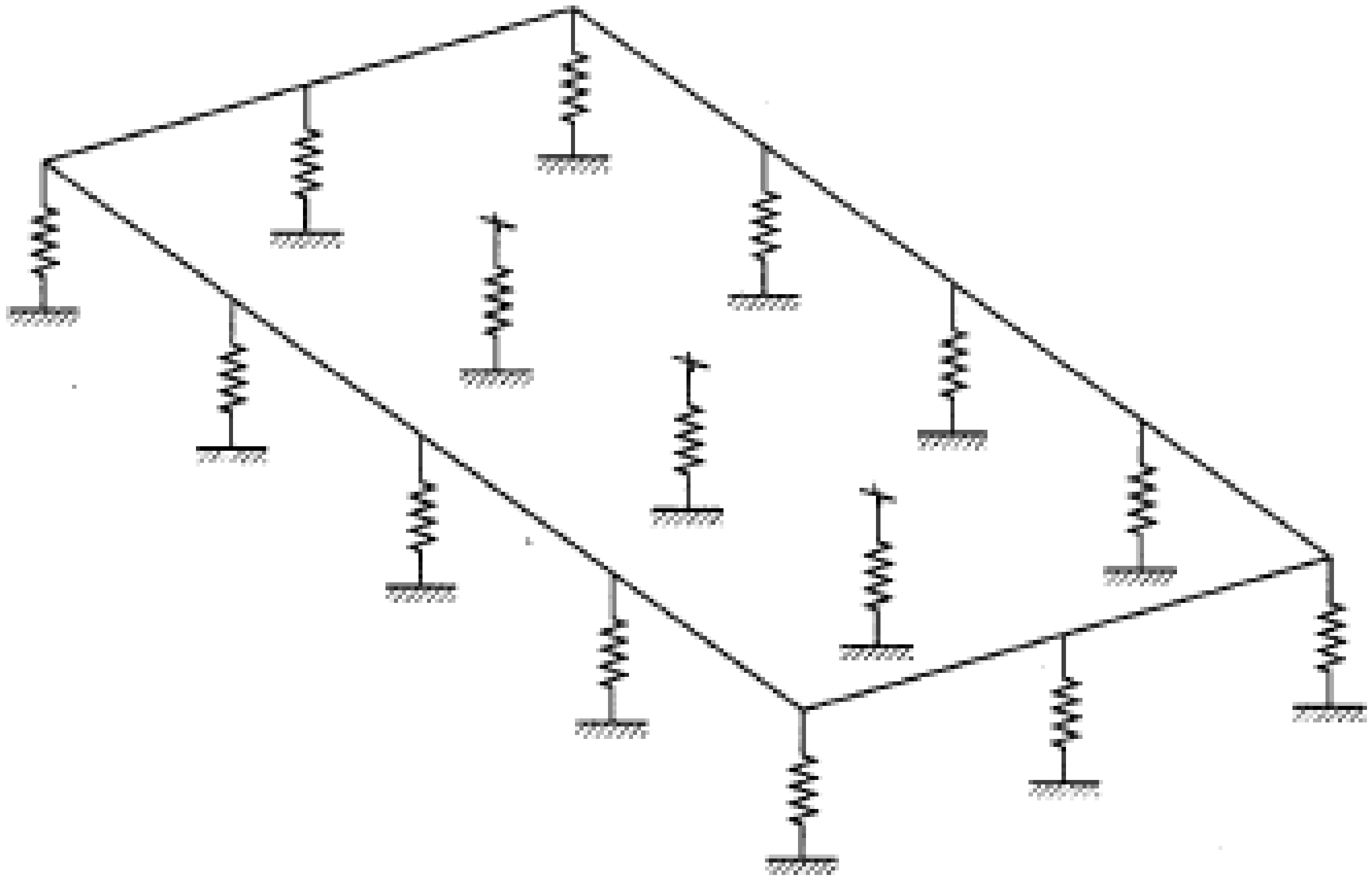


Figure 10.6 The coefficient of subgrade reaction forms the basis for a "bed of springs" analogy to model the soil-structure interaction in mat foundations.

Coefficient of Subgrade Reaction

- Portions of the mat that experience more settlement produce more compression in the "springs," which represents the higher bearing pressure, whereas portions that settle less do not compress the springs as far and thus have less bearing pressure.
- The sum of these spring forces must equal the applied structural loads plus the weight of the mat.
- This method of describing bearing pressure is called a *soil-structure interaction analysis* because the bearing pressure depends on the mat deformations, and the mat deformations depend on the bearing pressure.

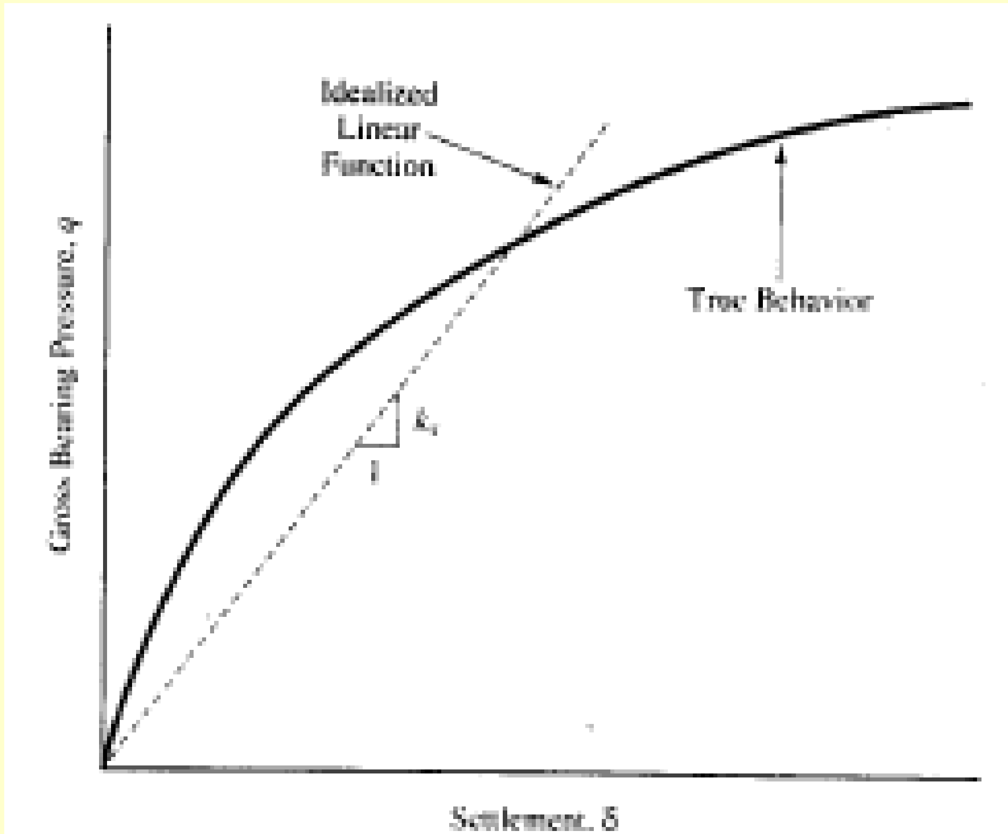
Winkler Method

- The "bed of springs" model is used to compute the shears, moments, and deformations in the mat, which then become the basis for developing a structural design.
- The earliest use of these "springs" to represent the interaction between soil and foundations has been attributed to Winkler (1867), so the analytical model is sometimes called a *Winkler foundation* or the *Winkler method*. It also is known as a *beam on elastic foundation* analysis.
- In its classical form the Winkler method assumes each "spring" is linear and acts independently from the others, and that all of the springs have the same *ks*.
- This representation has the desired effect of increasing the bearing pressure beneath the columns, and thus is a significant improvement over the rigid method.
- However, it is still only a coarse representation of the true interaction between mats and soil and suffers from many problems.

Winkler Method

- The problems, include the following:
- 1. The load-settlement behavior of soil is nonlinear, so the *ks* value must represent some equivalent linear function, as shown in next figure

Winkler Method



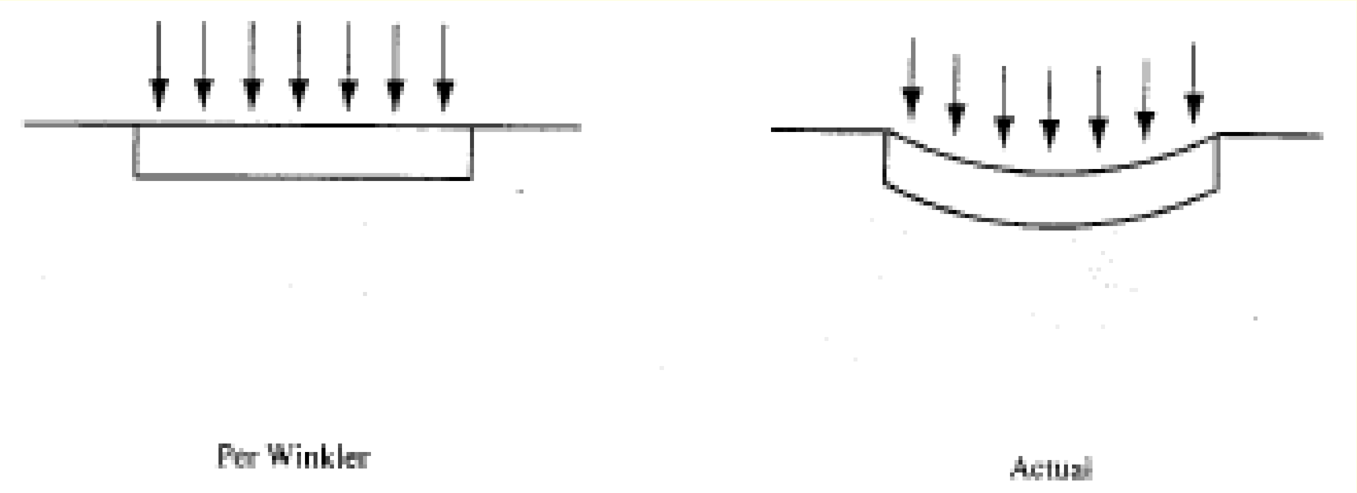
Winkler Method

2. According to this analysis, a uniformly loaded mat underlain by a perfectly uniform soil, as shown in the next Figure will settle uniformly into the soil (i.e., there will be no differential settlement) and all of the "springs" will be equally compressed.

In reality, the settlement at the center of such a mat will be greater than that along the edges.

This is because the $\Delta\sigma_z$ values in the soil are greater beneath the center.

Winkler Method



Winkler Method

3. The "springs" should not act independently. In reality, the bearing pressure induced at one point on the mat influences more than just the nearest spring.

- 4. Primarily because of items 2 and 3, there is no single value of k_s that truly represents the interaction between soil and a mat.

Winkler Method

Items 2 and 3 are the primary sources of error, and this error is potentially non-conservative (i.e., the shears, moments, and deflections in the mat may be greater than those predicted by Winkler).

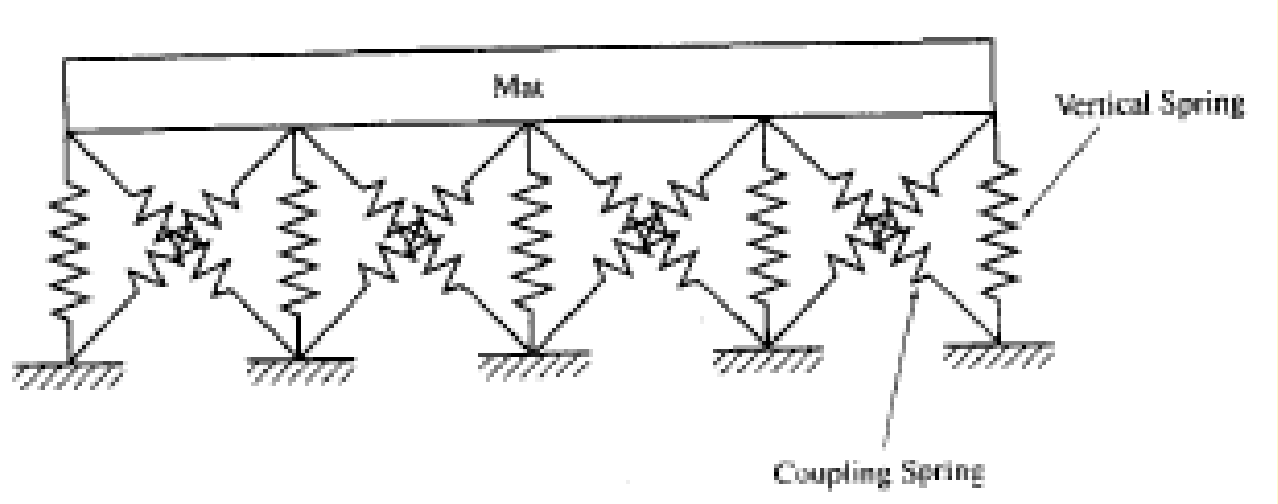
The heart of these problems is the use of independent springs in the Winkler model.

In reality, a load at one point on the mat induces settlement both at that point and in the adjacent parts of the mat, which is why a uniformly mat exhibits dish-shaped settlement, not the uniform settlement predicted by Winkler.

Coupled Method

- The next step up from a Winkler analysis is to use a *coupled method*, which uses additional springs as shown in next figure.
- This way the vertical springs no longer act independently, and the uniformly loaded mat exhibits the desired dish shape.
- In principle, this approach is more accurate than the Winkler method, but it is not clear how to select the *ks* values for the coupling springs, and it may be necessary to develop custom software to implement this analysis.

Coupled Method



Pseudo-Coupled Method

- The *pseudo-coupled method* is an attempt to overcome the lack of coupling in the Winkler method while avoiding the difficulties of the coupled method.
- It does so by using "springs" that act independently, but have different *ks* values depending on their location on the mat.
- To properly model the real response of a uniform soil, the "springs" along the perimeter of the mat should be stiffer than those in the center, thus producing the desired dish-shaped deformation in a uniformly-loaded mat.

Pseudo-Coupled Method

- If concentrated loads, such as those from columns, also are present, the resulting mat deformations are automatically superimposed on the dish-shape.
- Model studies indicate that reasonable results are obtained when *ks* values along the perimeter of the mat are about twice those in the center (ACI, 1993).
- We can implement this in a variety of ways, including the following:

Pseudo-Coupled Method

1. Divide the mat into two or more concentric zones, as shown in next figure. The innermost zone should be about half as wide and half as long as the mat.

2. Assign a *ks* value to each zone. These values should progressively increase from the center such that the outermost zone has a *ks* about twice as large at the innermost zone.

3. Evaluate the shears, moments, and deformations in the mat using the Winkler "bed of springs" analysis, as discussed later in this chapter.

4. Adjust the mat thickness and reinforcement as needed to satisfy strength and serviceability requirements.

Pseudo-Coupled Method

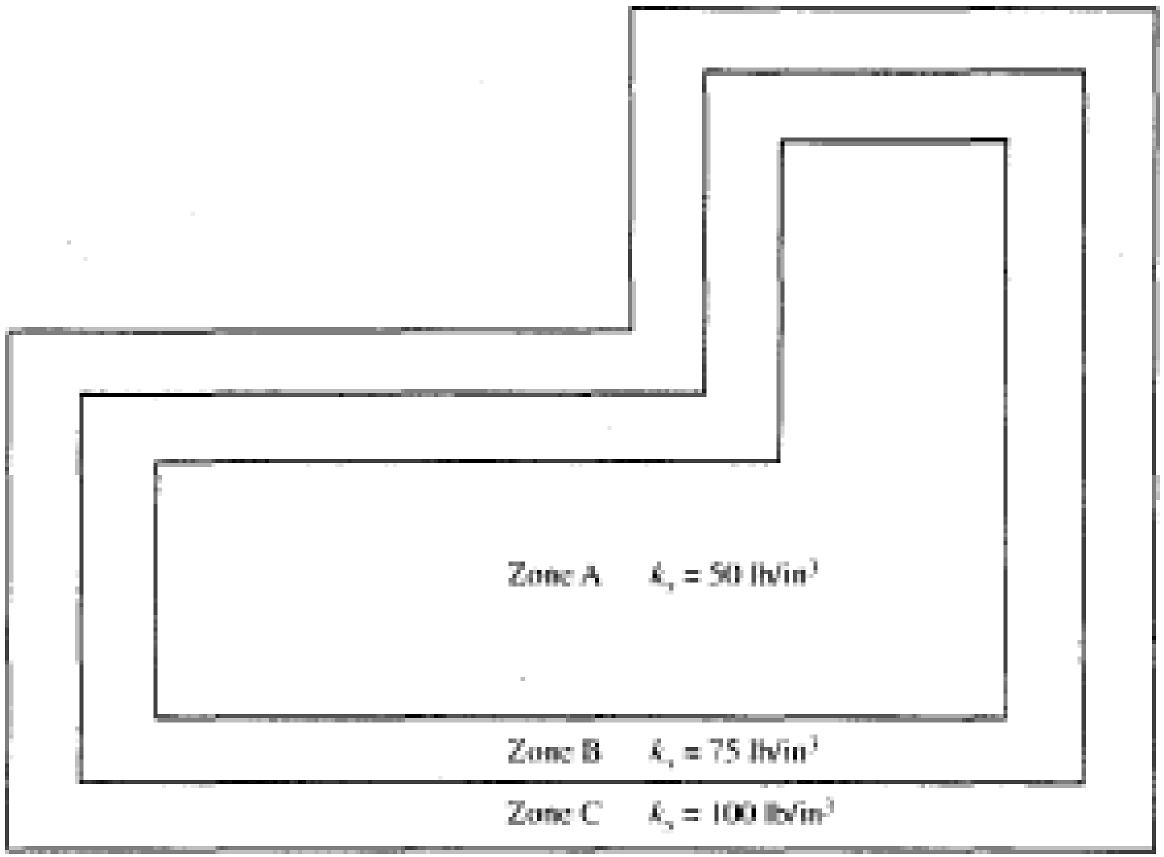


Figure 10.10 A typical mat divided into zones for a pseudo-coupled analysis. The coefficient of subgrade reaction, k_s, progressively increases from the innermost zone to the outermost zone.

Pseudo-Coupled Method

ACI (1993) found the pseudo-coupled method produced computed moments 18 to 25 percent higher than those determined from the Winkler method, which is an indication of how un-conservative Winker can be.

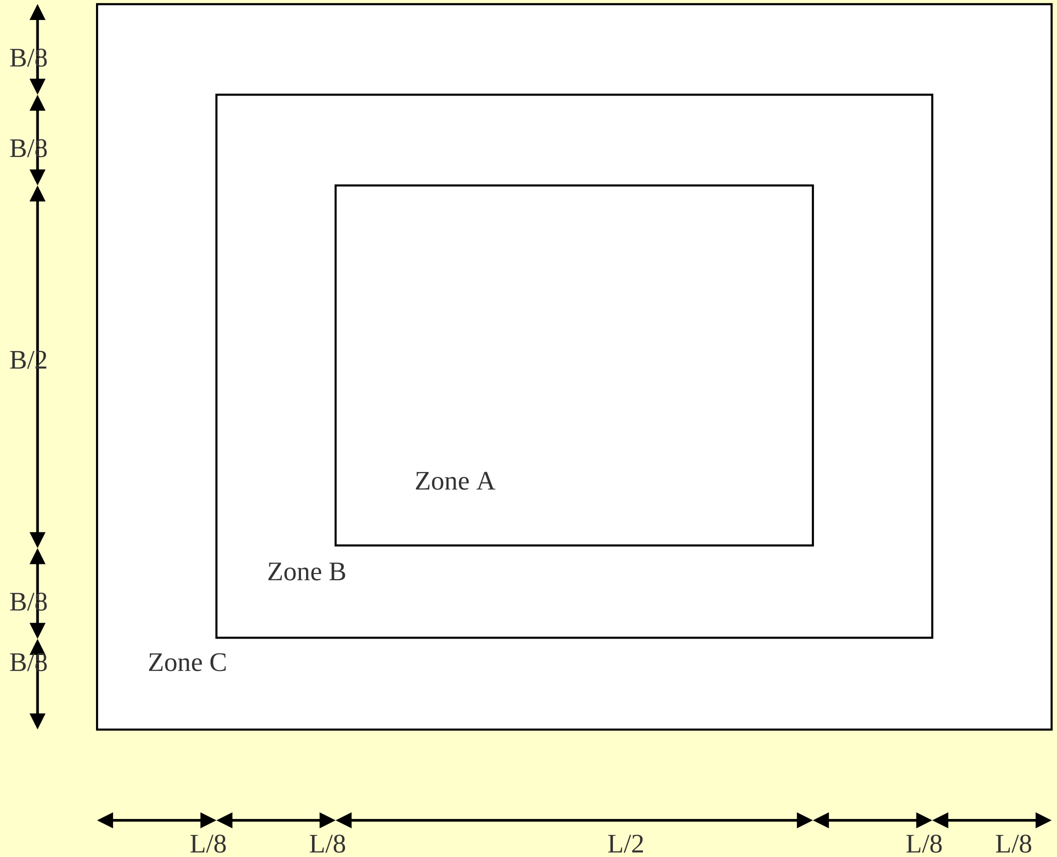
Most commercial mat design software uses the Winkler method to represent the soil-structure interaction, and these software packages usually can accommodate the pseudo-coupled method. Given the current state of technology and software availability, this is probably the most practical approach to designing most mat foundations.

ضریب واکنش بستر (ضریب فنریت) پی

- ضریب واکنش بستر از پارامترهای مورد نیاز طراحی پی بر روی بستر الاستیک است.
- ضریب واکنش بستر خاک مقدار ثابتی نیست.
- ضریب واکنش بستر (فنریت) خاک به ابعاد پی، عمق پی و سختی خاک زیر پی بستگی دارد.
- روش ارائه شده در این قسمت برای تخمین ضریب واکنش بستر از آیین نامه ACI 336.2R-88 اقتباس شده است.
- بدین منظور ابتدا یک مقدار میانگین بر اساس آزمایش بارگذاری صفحه و/یا نتایج تحلیل ظرفیت پی (فشار و نشست مجاز) در نظر گرفته می‌شود.

ضریب واکنش بستر میانگین = $(K_s)_{avg.}$

- با در نظر گرفتن Dishing Phenomena ، ضریب واکنش بستر در مرکز پی کوچکتر و در گوشه پی بزرگتر است.
- بدین منظور موسسه بتن آمریکا (ACI) پیشنهاد میکند پی به سه قسمت A و B و C تقسیم گردد و ضریب واکنش بستر جداگانه برای هر یک از نواحی نسبت به ضریب واکنش بستر میانگین در نظر گرفته شود.
- ضریب واکنش بستر هر ناحیه بر اساس ابعاد پی و ضریب واکنش بستر میانگین به صورت پارامتری در جدول ارائه گردیده است.



ضریب واکنش بستر	مساحت	ناحیه
$(K_s)_A = (K_s)_{avg.} / 1.595$	$A_A = B\,L / 4$	Zone A
$(K_s)_B = 1.5\,(K_s)_A$	$A_B = 5\,B\,L / 16$	Zone B
$(K_s)_C = 2.0\,(K_s)_A$	$A_C = 7\,B\,L / 16$	Zone C

Design of Mat Foundations

- Bearing Capacity Analysis follows the same approach as for spread footings

$$q_{ult} = c' N_c s_c d_c + \sigma'_{zD} N_q s_q d_q + 0.5 \gamma' B N_\gamma s_\gamma d_\gamma$$

- Factor of Safety (Das, 2004):

  Under normal D+L loads.. Minimum 3.0

  Under extreme loads …Minimum 1.75-2.0

Design of Mat Foundations

- Settlement Analysis
  - Deformation of the slab
  - Compression of the underlying soil
- Differential Settlement of Mat Foundations
  (American Concrete Institute Committee 336, 1988)

Modulus of Elasticity of Material used in Structure

Moment of inertia of structure per unit length at right angles to B

$$\boxed{K_r = \frac{E' I_b}{E_s B^3}}$$

Modulus of Elasticity of Soil

Width of raft

Design of Mat Foundations

§ Differential Settlement of Mat Foundations

(American Concrete Institute Committee 336, 1988)

$$E'I_b = E'\left(I_F + \sum I_{b'} + \sum \frac{ah^3}{12} \right)$$

$E'I_b$ = flexural rigidity of the superstructure and foundation per unit length at right angles to B

$\sum E'I_b'$ = flexural rigidity of the framed members at right angles to B

$\sum \left(E'ah^3/12\right)$ = flexural rigidity of shear walls

a = shear wall thickness

h = shear wall height

$E'I_F$ = flexibility of the foundation

If $K_r > 0.5$, then mat can be treated as rigid i.e. $(\delta_d/\delta) = 0$

If $K_r = 0.5$, then $(\delta_d/\delta) \approx 0.1$

If $K_r = 0$, then $(\delta_d/\delta) \approx 0.35$ (square mats) and $(\delta_d/\delta) \approx 0.5$ (long mats)

Structural Design of Mat Foundations

- Rigid method
  - Mat is extremely rigid
  - Contact pressure is planar
  - Same assumptions used in spread footing design
- Simplified Elastic Methods
  - Mat behaves like an elastic plate that is supported on a bed of elastic springs
- Finite Difference Method
- Finite Element Method