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State-Dependent Impulsive Observer Design for Nonlinear Time-Delay Systems

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Abstract— **This paper has proposed a new state-dependent impulsive observer (SDIO) for nonlinear time-delay systems. This observer is based on extended pseudo-linearization, and its parameters are state-dependent. The SDIO is capable to estimate system states continuously by using system output that is just available at discrete impulse times. The stability of the proposed observer is proved by using time-varying Lyapunov function, and comparison system theory of impulsive differential equation systems. By new theorem, it is guaranteed that the estimation error asymptotically converges to zero under well-defined, and less-conservative sufficient conditions. Furthermore, the stability theorem gave an upper bound on the maximum allowable time interval between consequent impulses. The simulation results show effectiveness, and good performance of the proposed observer, for a wider classes of nonlinear time-delay systems.**

Keywords- state-sependent impulsive observer; nonlinear time-delay systems; extended pseudo-linearization; comparison system theory; time-varying Lyapunov function

1 INTRODUCTION

During recent decades, impulsive systems have been considered by many researches in this area. Impulsive systems have two continuous and discontinuous dynamical behaviors. The continuous dynamical behavior is presented by continuous differential equations between impulses intervals. While discontinuous dynamic behavior is described by difference equations defined at impulses times, when states of system are suddenly jumping. Due to the hybrid characteristics, impulsive systems are very appropriate to describe many processes in the real world [1]. In many practical applications of the control, only discrete-time measurements are available for continuous-time system control. For example, in the chemical and economic processes, biological applications, such as distribution of the drug in the human body, and impulsive vaccination, chaos communication systems, renewable resources management, biological neural networks, population ecology, rhythmic models of pathology, the modulated frequency signal processing systems, flying objects, and etc., measurement of output are available at discrete instance time, and the time interval between samples is not necessarily constant or regular [2].

The design of impulsive functional observer for linear systems is proposed in [3]. The stability analysis is presented by considering the piecewise differentiable Lyapunov function. The sufficient conditions of exponential stability of the proposed observer are derived in terms of LMIs. Then the authors proposed impulsive observer deign for uncertain linear systems in [4]. A time-varying Lyapunov function is used for stability analysis, and sufficient conditions are derived in terms of LMIs. In [5], continuous-discrete time interval observer is offered for linear systems with the existence of additive disturbances. The piecewise continuous observer design is presented for linear systems with sampled and delayed output measurements, and variable periods [6]. The proposed observer is based on the theory of piecewise continuous hybrid systems, which are a particular class of hybrid systems characterized with autonomous switching and controlled impulses.

The design of impulsive observer for nonlinear systems is investigated in [7]. The presented observer shares the structure as same as a Luenberger observer with an appropriate updating rule of the observer gain at every sampling instant. But, the considered nonlinear system should have a linear and nonlinear parts, separately. The proposed observer is simulated on a flexible joint robotic arm. In [8], an impulsive observer with time-varying gain is presented for nonlinear systems. The exponential convergence of the proposed observer is proved by using small gain arguments. But, the considered system equation is a special class of nonlinear systems because it has a separable linear part. In [9], an adaptive observer is designed for a class of uniformly observable nonlinear systems with nonlinear parametrization, and sampled output. Also, the considered nonlinear system must have two separable linear and nonlinear parts. The design of continuous-discrete observer is presented for continuous time nonlinear time-varying systems with discrete measurements [10]. It is shown, that the solution of the proposed observer by using the notion of cooperative systems, converge to the solution of origin system under sufficient conditions on the nonlinear terms. But, the considered nonlinear system should have a linear and nonlinear parts, separately. In [11], the authors presented an impulsive continuous-discrete time observer for a class of uncertain nonlinear systems. The performance of the proposed observer is shown by simulation on biochemical reactors. Like other articles, their considered system equation is a special class of nonlinear systems because it has a separable linear part. Adaptive impulsive observer (AIO) design is proposed for chaotic system synchronization in [12], [13]. The AIO is able to estimate both states and unknown parameters of the uncertain system. The comparison system theorem and LMIs are used to analysis the stability of AIO that leads to less-conservative sufficient conditions. The linear and nonlinear parts of system equation should be additive in the proposed method. Also the stochastic adaptive impulsive observer is developed in [14].

In [15], an impulsive observer with variable update interval is proposed for nonlinear time-delay system. The discontinuous Lyapunov function is applied to analysis the stability, and sufficient conditions are presented by using LMIs. The nonlinear system structure that is considered in this paper, has four parts, separately: linear, linear time-delay, nonlinear and nonlinear time-delay parts. In [16], the problem of observer design for discrete-time nonlinear impulsive switched systems with time-varying delay is investigated. A delay-dependent Lyapunov-Krasovskii function is considered for analysis the stability, and sufficient conditions are established by using the average dwell time approach and LMIs. The considered system has three parts, separately: linear, linear time-delay, nonlinear parts. This matter limits the proposed method to the special class of nonlinear time-delay systems.

As it is seen, in researches of impulsive observer design for nonlinear systems, only special and confined classes are discussed. Actually, for simplifying of routine, of stability analysis and calculation of sufficient conditions, the system equation is limitative to have a linear part. These conditions for nonlinear time-delay systems are more restrictive. In this paper, by using extended pseudo-linearization technique, this problem has been largely resolved. In the theory of impulsive systems, it has been shown, that it is not necessary that the Lyapunov function always be negative. This matter presented as comparison system theory, and its corollaries [1]. So, by considering this theory, the sufficient conditions for stability analysis are less-conservative than a current impulsive observer. In this paper, the state-dependent impulsive observer based on extended pseudo-linearization is presented for more general class of nonlinear time-delay systems. The stability analysis of the proposed observer is presented by using time-varying Lyapunov function, and comparison system theory of impulsive differential equation systems. By new theorem, it is shown, that under well-defined and less-conservative sufficient conditions, the estimation error converges to zero, exponentially. Also, the stability theorem gave an upper bound on the maximum possible distance of impulses.

The proposed paper is organized as follows: In section 2, basic concepts of impulsive systems are presented. In section 3, the state-dependent impulsive observer equation is offered. Also, the extended pseudo-linearization form for nonlinear time-delay system are explained. The sufficient conditions of the SDIO stability are presented by a new theorem. The simulation results of the SDIO design for Congo Ebola as nonlinear time-delay system, are shown in section 4. At the end, in section 5, the conclusion of this paper is presented.

2 Basic Concepts of Impulsive Systems

The impulsive differential equation can be described as

$$\begin{cases} \dot{x}(t) = f(t, x(t)); & t \neq t_k \\ \Delta x(t) = f_I(x(t)); & t = t_k \end{cases} \tag{1}$$

where, $x \in \mathbf{R}^n$ is state vector, t is the time variable and $f : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^n, f_I : \mathbf{R}^n \to \mathbf{R}^n$, are nonlinear functions with compatible dimensions. $t_k, k = 1, 2, \ldots$, are impulse times that $t_k > t_{k-1} > 0$. The state jump vector at impulse time is $\Delta x(t_k) = x(t_k) - x(t_k^-)$.

In the theory of stability of impulsive systems, it is not necessary that the time-derivative of Lyapunov function is non-positive. So, in some times, the Lyapunov function can increase but, stability of the whole system is guaranteed. The following Theorem is expressed this subject [1].

Definition 1 [1]: $a(x) \in C$ denotes that a is continuous and $a(x)$ is belong to class C^i if it is i times differentiable respect to x. Also a is belong to class κ if $a \in C\left[\mathbf{R}^+, \mathbf{R}^+\right]$, $a(0) = 0$, and $a(x)$ is strictly increasing in x.

Definition 2: For any $\rho \in \mathbf{R}^+$, $S_\rho = \left\{x \in \mathbf{R}^n \middle| \|x\| < \rho\right\}$ where $\|.\|$ is the Euclidean norm.

Definition 3 [1]: $V : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^+$ is belong to class V_0 if

a) V is continuous in $(t_{k-1}, t_k] \times \mathbf{R}^n$ and for each $x \in \mathbf{R}^n$, $\lim_{(t,y) \to (t_k^+, x)} V(t, y) = V(t_k^+, x)$ exists;

b) V is locally Lipschitz in x .

Definition 4: For $(t, x) \in (t_{k-1}, t_k] \times \mathbf{R}^n$, $D^+V(t, x)$ is defined as

$$D^+V(t,x) = \lim_{h \to 0^+} \sup \frac{1}{h}\left(V(t+h, x+hf(t,x)) - V(t,x)\right)$$
$$D^-V(t,x) = \lim_{h \to 0^-} \sup \frac{1}{h}\left(V(t+h, x+hf(t,x)) - V(t,x)\right) \tag{2}$$

Note 1: If $V \in C^1\left[\mathbf{R}^+ \times \mathbf{R}^n, \mathbf{R}^+\right]$, then

$$D^+V(t,x) = D^-V(t,x) = \frac{\partial V(t,x)}{\partial t} + \frac{\partial V(t,x)}{\partial x} f(t,x).$$

Definition 5 [1]: The comparison system of (1) is given by

$$\begin{cases} \dot{w}(t) = g(t, w(t)); & t \neq t_k \\ w(t_k^+) = \psi_k(w(t_k)) \end{cases} \tag{3}$$

where, $V \in V_0$, $g : \mathbf{R}^+ \times \mathbf{R}^+ \to \mathbf{R}$ is continuous and satisfies definition 3.a, $\psi_k : \mathbf{R}^+ \to \mathbf{R}^+$ is non-decreasing and with considering following assumption

$$\begin{cases} D^+V(t,x) \le g(t, V(t,x)); & t \neq t_k \\ V(t, x+\Delta x) \le \psi_k(V(t,x)); & t = t_k \end{cases} \tag{4}$$

Assumption 1: Assume that $f(t,0) = 0, f_I(0) = 0$, $g(t,0) = 0$ for all k and $t > 0$, then the trivial solutions of main system (1), and comparison systems (3) are equal in $(t_{k-1}, t_k]$.

Theorem 1 [1]: Assume that following conditions are satisfied.

a) $V \in V_0$, $\rho > 0$, $V : \mathbf{R}^+ \times S_\rho \to \mathbf{R}^+$ and in $t \neq t_k$ $D^+V(t,x) \le g(t, V(t,x))$.

b) There exists a $\rho_0 > 0$, such that $x \in S_{\rho_0}$ implies that $x + \Delta x \in S_{\rho_0}$ for all k and in $t = t_k$ $V(t, x+\Delta x) \le \psi_k(V(t,x))$.

c) $b(\|x\|) \le V(t,x) \le a(\|x\|)$ on $\mathbf{R}^+ \times S_\rho$ where $a, b \in \kappa$.

then, the stability properties of the trivial solution of comparison systems (3), imply the corresponding stability properties of the trivial solution of main system (1).

Corollary 1 [1]: Let $g(t, V(t,x)) = \dot{\xi}(t) V(t,x)$ where, $\xi \in C^1[\mathbf{R}^+, \mathbf{R}^+]$, $\psi_k(V(t,x)) = d_k V(t,x)$. The origin of (1) is asymptotically stable if following conditions are satisfied:

$$\begin{cases} \dot{\xi}(t) \ge 0 \\ d_k \ge 0; & k = 1,2,\ldots \\ \xi(t_{k+1}) - \xi(t_k) + \ln(\gamma d_k) \le 0; & \gamma > 1 \end{cases} \tag{5}$$

The following lemmas have been used during the proof of the SDIO stability theorem.

Lemma 1: For any vector a, b, and arbitrary matrix $X > 0$ with compatible dimension

$$a^{\mathrm{T}} b + b^{\mathrm{T}} a \le a^{\mathrm{T}} X a + b^{\mathrm{T}} X^{-1} b$$

Lemma 2: Consider A, B, C, D are matrices with compatible dimensions, and $X = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ then

$$X < 0 \Leftrightarrow A < 0, S_A = C - DA^{-1}B < 0$$

$$X < 0 \Leftrightarrow C < 0, S_C = A - BC^{-1}D < 0$$

where, A, C are invertible and S_A, S_C are called Schur complement of A, C, respectively.

3 State Dependent Impulsive Observer Design

The nonlinear time-delay system equation is considered as:

$$\begin{cases} \dot{x}(t) = f\left(x(t), x(t-\tau_1(\theta)), \ldots, x(t-\tau_k(\theta))\right) \\ y(t) = Cx(t) \\ x(t) = \varphi_0(t); & -\max\limits_{\theta, i=1:k}(\tau_i(\theta)) \le t \le 0 \end{cases} \tag{6}$$

where, $x \in \mathbf{R}^n$, is the continuous state vector, $y \in \mathbf{R}^p$, is output vector, $C \in \mathbf{R}^{p \times n}$ is output matrix. The time-delays $\tau_1(\theta), \ldots, \tau_k(\theta)$ are positive functions that can be dependent to t, x, or both. $\varphi_0(t)$ is continuous function for initial conditions of systems. It is assumed, $f\left(0, x(t-\tau_1(\theta)), \ldots, x(t-\tau_k(\theta))\right) = 0$ that is satisfied by states augmentation. For simplifying, x_t is defined as $x_t = \left(x(t), x(t-\tau_1(\theta)), \ldots, x(t-\tau_k(\theta))\right)$.

3.1 *Extended Pseudo-Linearization*

For nonlinear time-delay system, pseudo-linearization form of (1), is presented as:

$$\dot{x}(t) = A(x_t) x(t) \tag{7}$$

where, $A(x_t)$ is state-dependent system matrix [17]. Unlike Jacobian method, the main advantage of extended pseudo-linearization is that nonlinear characteristics of the system are maintained. Also, in this method all delayed parts are placed in state-dependent system and input matrices. Thus, it is possible to extend the linear method of observer design for nonlinear time-delay systems with maintenance of all nonlinear characteristics of the system. Unfortunately, there is not pseudo-linearization form for all nonlinear time-delay systems but, for affine system there is pseudo-linearization as following form [18]:

$$A(x_t) = \int_0^1 \left. \frac{\partial f(x_t)}{\partial x(t)} \right|_{x(t) = \lambda x(t)} d\lambda \tag{8}$$

If the system has more than one state, there are infinite pseudo-linearization forms. This matter causes degrees of freedom in observer design. For design of state-dependent impulsive observer following assumption is considered [18].

Assumption 2: $f(x_t)$ satisfies the following Lipschitz condition:

$$\|f(x_t) - f(\hat{x}_t)\| \le K_f \|x - \hat{x}\| \tag{9}$$

where, $K_f \in \mathbf{R}^+$ is Lipschitz constant. Also, it is assumed that

$$\left\| A\left(x_t\right) \right\| \le K_A \qquad (10)$$

where, $K_A \in \mathbf{R}^+$.

3.2 State-Dependent Impulsive Observer

The state-dependent impulsive observer is proposed as:

$$\begin{cases} \dot{\hat{x}}(t) = A(\hat{x}_t)\hat{x}(t) & \\ \hat{y}(t) = C\hat{x}(t) & t \ne t_k \\ \Delta\hat{x}(t) = F(\hat{x}_t)\left(y(t) - \hat{y}(t)\right) & t = t_k \end{cases} \qquad (11)$$

where, $\hat{x}(t), \hat{y}(t)$ are estimated state and output vectors, and $F(\hat{x}_t) \in \mathbf{R}^{n\times p}$ is states impulses gain matrix.

Theorem 2: The state estimation error $e(t) = x(t) - \hat{x}(t)$ of presented SDIO by (11) asymptotically converges to zero if following conditions are satisfied:

$$\frac{\bar{\lambda}(\Sigma)}{\underline{\lambda}(P)} \ge 0 \qquad (12)$$

$$\begin{bmatrix} -\sigma\mu P_1 & \left(\mathbf{I} - F(\hat{x}_t)C\right)P_2 \\ P_2\left(\mathbf{I} - F(\hat{x}_t)C\right) & -P_2 \end{bmatrix} \le 0 \qquad (13)$$

$$\frac{\bar{\lambda}(\Sigma)}{\underline{\lambda}(P)}\Delta_k + \ln(\gamma\sigma) \le 0 \qquad (14)$$

where, $\gamma > 1$, $0 \le \sigma \le 1$ and $0 < \mu \le 1$ are design parameters and

$$\varphi(t) = \mu^{1-\rho_1(t)};\quad \rho_1(t) = \frac{(t_{k+1} - t)}{\Delta_k}; t \in \left[t_k, t_{k+1}\right)$$
$$P(t) = \left(1 - \rho_1(t)\right)P_1 + \rho_1(t)P_2 \qquad (15)$$

where $\Delta_k = t_{k+1} - t_k$ is the k th –impulse interval, P_1, P_2 are symmetric positive definite matrices that obtained by (12) and

$$\Sigma = \left(\frac{\bar{\lambda}(P)}{\underline{\lambda}(P)}(K_A + K_f)^2 + 1\right)P + A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t)$$
$$+ \left(P_1 - P_2 + \ln(\mu)P\right)/\Delta_k \qquad (16)$$

Proof: By using (7) and (11), the dynamic system and jump of the state estimation error are

$$\begin{cases} \dot{e}(t) = A(\hat{x}_t)e + \left(A(x_t) - A(\hat{x}_t)\right)x(t) & t \ne t_k \\ \Delta e(t) = -F(\hat{x}_t)\left(y(t) - \hat{y}(t)\right) = -F(\hat{x}_t)Ce(t); & t = t_k \end{cases} \qquad (17)$$

For simplifying, it is defined $\tilde{A} = A(x_t) - A(\hat{x}_t)$. So, the dynamic of discrete part at impulse times is given:

$$\Delta e(t_k) = e(t_k) - e\left(t_k^-\right) = -F(\hat{x}_t)Ce\left(t_k^-\right)$$
$$\rightarrow e(t_k) = \left(\mathbf{I} - F(\hat{x}_t)C\right)e\left(t_k^-\right) \qquad (18)$$

The Lyapunov function candidate is considered as:

$$V(t,x) = \varphi(t)e^{\mathrm{T}}(t)P(t)e(t) \qquad (19)$$

By this definitions, at $t = t_k^-$:

$$\rho_1\left(t_k^-\right) = 0 \rightarrow P\left(t_k^-\right) = P_1$$
$$\varphi\left(t_k^-\right) = \mu \rightarrow V\left(t_k^-\right) = \mu e^{\mathrm{T}}\left(t_k^-\right)P_1 e\left(t_k^-\right) \qquad (20)$$

Also, at $t = t_k$:

$$\rho_1(t_k) = 1 \rightarrow P(t_k) = P_2$$
$$\varphi(t_k) = 1 \rightarrow V(t_k) = e^{\mathrm{T}}(t_k)P_2 e(t_k) \qquad (21)$$

So, the time-derivative of Lyapunov function at $t \ne t_k$ is:

$$D^+V(t,x) = \varphi\left\{x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\tilde{A}x\right\}$$
$$+ \varphi e^{\mathrm{T}}\left\{A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t)\right\}e$$
$$+ \varphi e^{\mathrm{T}}\left\{\left(P_1 - P_2 + \ln(\mu)P\right)/\Delta_k\right\}e \qquad (22)$$

By using Lemma 1, and $X = P$ it is concluded:

$$x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\tilde{A}x \le x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}P\tilde{A}x + e^{\mathrm{T}}Pe \qquad (23)$$

With considering assumption 2, the following result is obtained:

$$\left\|\tilde{A}x\right\| = \left\|\left(A(x_t) - A(\hat{x}_t)\right)x \pm A(\hat{x}_t)\hat{x}\right\|$$
$$\le \left\|f(x_t) - f(\hat{x}_t)\right\| + \left\|A(\hat{x}_t)(x - \hat{x})\right\|$$
$$\le K_f\left\|x - \hat{x}\right\| + K_A\left\|x - \hat{x}\right\| = (K_f + K_A)\left\|e\right\| \qquad (24)$$

With regards to matrix P is a symmetric positive definite matrix:

$$x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}P\tilde{A}x \le \bar{\lambda}(P)\left\|\tilde{A}x\right\|^2 \le \bar{\lambda}(P)(K_A + K_f)^2\left\|e\right\|^2$$
$$\le \frac{\bar{\lambda}(P)}{\underline{\lambda}(P)}(K_A + K_f)^2 e^{\mathrm{T}}Pe \qquad (25)$$

where, $\bar{\lambda}(P), \underline{\lambda}(P)$ define maximum and minimum eigenvalues of P, respectively. So, (22) is rewritten as:

$$D^+V(t,x) \le \varphi\left\{\frac{\bar{\lambda}(P)}{\underline{\lambda}(P)}(K_A + K_f)^2 e^{\mathrm{T}}Pe + e^{\mathrm{T}}Pe\right\}$$
$$+ \varphi e^{\mathrm{T}}\left\{A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t)\right\}e$$
$$+ \varphi e^{\mathrm{T}}\left\{\left(P_1 - P_2 + \ln(\mu)P\right)/\Delta_k\right\}e$$
$$= \varphi e^{\mathrm{T}}\Sigma e \qquad (26)$$

By considering that Σ is a symmetric positive definite matrix, it is concluded:

$$D^+V(t,x) \le \varphi e^{\mathrm{T}}\Sigma e \le \varphi\bar{\lambda}(\Sigma)\left\|e\right\|^2$$
$$\le \bar{\lambda}(\Sigma)/\underline{\lambda}(P)\varphi e^{\mathrm{T}}Pe = \dot{\xi}(t)V(t,x) \qquad (27)$$

where, $\dot{\xi}(t) = \bar{\lambda}(\Sigma)/\underline{\lambda}(P) \geq 0$ and so, first condition of corollary 1 (5) is satisfied. Due to (21), the Lyapunov function at $t = t_k$ is

$$V\left(t_k, x\right) = e^{\mathrm{T}}\left(t_k^-\right)\left(I - F\left(\hat{x}_t\right)C\right)^{\mathrm{T}} P_2\left(I - F\left(\hat{x}_t\right)C\right)e\left(t_k^-\right) \quad (28)$$

With considering following condition

$$\left(I - F\left(\hat{x}_t\right)C\right)^{\mathrm{T}} P_2\left(I - F\left(\hat{x}_t\right)C\right) \leq \sigma\mu P_1 \quad (29)$$

By using Schur complement (Lemma 2), (29) can be rewritten as LMI as (13). So, the Lyapunov function is obtained as

$$V\left(t_k, x\right) \leq e^{\mathrm{T}}\left(t_k^-\right)\sigma\mu P_1 e\left(t_k^-\right) = \sigma V\left(t_k^-, x\right) = d_k V\left(t_k^-, x\right) \quad (30)$$

where, $d_k = \sigma \geq 0$ and so, second condition of corollary 1 (5) is satisfied. So, according to the third condition of corollary 1 (5)

$$\frac{\bar{\lambda}(\Sigma)}{\underline{\lambda}(P)}\Delta_k + \ln\left(\gamma d_k\right) \leq 0 \rightarrow \ln\left(\gamma d_k\right) \leq -\frac{\bar{\lambda}(\Sigma)}{\underline{\lambda}(P)}\Delta_k \quad (31)$$

with regards to $\left(\bar{\lambda}(\Sigma)/\underline{\lambda}(P)\right)\Delta_k \geq 0$, $\gamma > 1$ it is concluded $d_k \leq 1$ that is satisfied by $d_k = \sigma \leq 1$.□

Remark 1: By using third condition of theorem 2, the maximum distance of impulses is

$$\Delta_{k\ \max} = \max_k\left(t_{k+1} - t_k\right) = -\ln\left(\gamma\sigma\right)\left(\underline{\lambda}(P)/\bar{\lambda}(\Sigma)\right) \quad (32)$$

It is worth mentioning, (12) could be solved by the constrained nonlinear optimization methods like *fmincon* MATLAB function or the nonlinear programming methods.

4 Simulation Results

In this section, the effectiveness of the proposed the SDIO is illustrated by numerical simulations. The SIR epidemic time-delay nonlinear model is considered as [19], [20]:

$$\begin{cases} \dot{S}(t) = \left(b - a\frac{rN(t)}{K}\right)N(t) - \frac{\beta S(t)I(t-\tau)}{N(t-\tau)} \\ \qquad -\left(d + (1-a)\frac{rN(t)}{K}\right)S(t) \\ \dot{I}(t) = \frac{\beta S(t)I(t-\tau)}{N(t-\tau)} - \left(d + (1-a)\frac{rN(t)}{K} + \lambda\right)I(t) \\ \dot{R}(t) = \lambda I(t) - \left(d + (1-a)\frac{rN(t)}{K}\right)R(t) \end{cases} \quad (33)$$

where, S, I, R are susceptible, infective, and recovered individuals, respectively. $N(t) = S(t) + I(t) + R(t)$ is the number of total population. $b > 0, d > 0, \alpha > 0, \beta > 0$ are the birth, death, recovery, and contact rate, respectively. $r = b - d$ is the intrinsic growth rate, a is convex combination constant, K is the carrying capacity of the population and τ is a non-negative constant, represents a time delay on infected individuals I , and total individuals N during the spread of disease. The output of considered model is state R . The values of the parameters in a particular disease Congo Ebola are presented in table 1. The following form is one of the infinite extended pseudo-linearization forms of (31)

$$A\left(x_t\right) = \begin{bmatrix} a_{11} & b - arN(t)/K & b - arN(t)/K \\ a_{21} & a_{22} & -(1-a)rI(t)/K \\ 0 & \alpha & a_{33} \end{bmatrix} \quad (34)$$

where,

$$a_{11} = b - a\frac{rN(t)}{K} - \frac{\beta I(t-\tau)}{N(t-\tau)} - \left(d + r(1-a)\frac{N(t)}{K}\right)$$

$$a_{21} = \frac{\beta I(t-\tau)}{N(t-\tau)} - (1-a)\frac{rI(t)}{K}$$

$$a_{22} = -\left(d + (1-a)\frac{rI(t)}{K}\right) - \lambda; \quad a_{33} = -\left(d + (1-a)\frac{rN(t)}{K}\right)$$

The initial conditions and design parameters are considered as $x(0) = [9126 \quad 315 \quad 59]^T$, $\hat{x}(0) = [8500 \quad 500 \quad 200]^T$, $\gamma = 1.1$, $\sigma = 0.9$, $\mu = 0.98$. In Fig. 1, the real and estimated states by the SDIO are shown. It is obvious that, estimated states follow real states even the output is available only, in fifth samples once. The jump values of estimated states are shown in Fig. 2. The Lyapunov function and its time-derivative are plotted in Fig. 3 and Fig. 4

TABLE 1. SIR Parameters [20]

Parameters of SIR Model of Congo Ebola			
Parameter	***Value***	***Parameters***	***Value***
b	0.07 day^{-1}	d	0.0123 day^{-1}
a	0.014 day^{-1}	λ	0.0476 day^{-1}
β	0.21 day^{-1}	τ	10 day
K	10900		

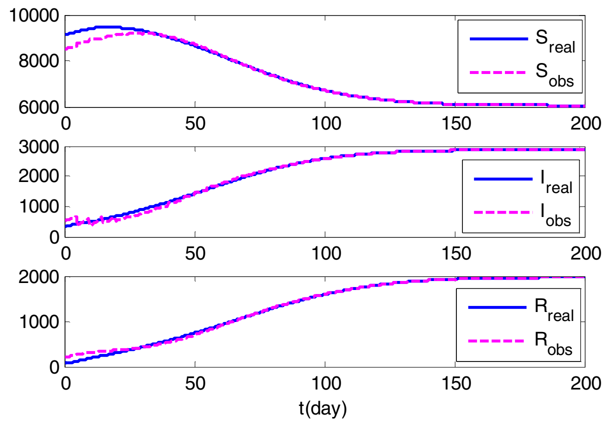

Figure 1. Real and estimated states by SDIO

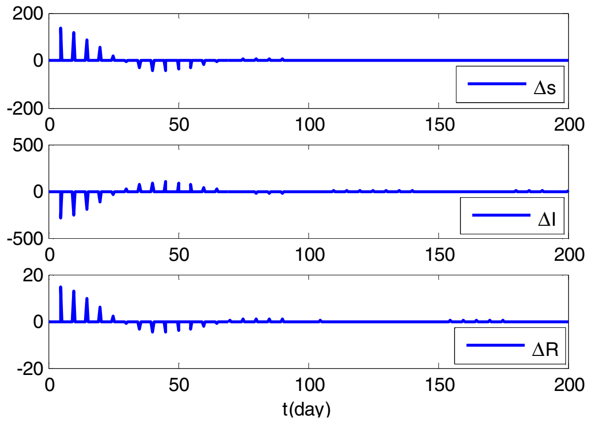

Figure 2. Jump of estimated states in impulse times

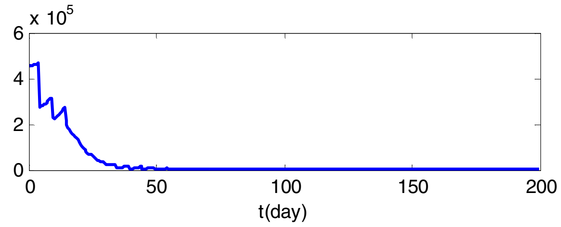

Figure 3. Lyapunov function

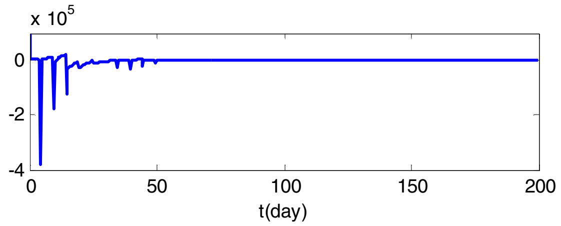

Figure 4. Time-derivative of Lyapunov function

As it is shown, the time-derivative of Lyapunov function is positive in some times but, the function is decreasing generally.

5 Coclusion

In this paper, a new state-dependent impulsive observer presented for nonlinear time-delay systems based on extended pseudo-linearization. The proposed observer estimated the system states, continuously by using system output that has been just available at, discrete and variable impulse instants. The stability analysis of the proposed observer is investigated by using time-varying Lyapunov function and comparison system theory of impulsive systems. Two design features of the proposed impulsive observer are worth to be emphasized: first, usability of the SDIO for a more general class of nonlinear time-delay systems by extended pseudo-linearization technique. Second, guaranty of asymptotical convergence of the estimation error to zero under well-defined and less-conservative sufficient conditions. Also, the upper bound on the maximum possible distance of impulses was presented. The simulation results on SIR epidemic nonlinear time-delay model for Congo Ebola showed effectiveness of the proposed impulsive observer.

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