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Design of Adaptive State-Dependent Impulsive Observer for Nonlinear Time-Delay Systems

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Abstract— **In current study, a recent adaptive state-dependent impulsive observer (ASDIO) is suggested for a wider category of nonlinear dynamics with delays in states. The suggested observer is on the basis of the extended pseudo-linearization technique. By applying this technique, the presented observer is utilized for nonlinear dynamics with several, distributed, and time-varying delays. Furthermore, the extended pseudo-linearization technique simplifies the procedure of impulsive observer layout for nonlinear time-delay dynamics. The constancy and convergence of the suggested observer are proven via a theorem utilizing the comparison system theory corresponding to impulsive systems. It is assured that the estimation error shows an asymptotic convergence to zero subject to adequate terms with less conservatism that are exhibited in the form of feasible linear matrix inequalities (LMIs). In addition, the stability theorem specifies a maximum value of the impulses time. The results are applied on Congo Ebola disease model which is an epidemic nonlinear time-delay system. Simulations confirm the efficiency of the presented ASDIO.**

Keywords- adaptive state-dependent impulsive observer; Congo ebola disease; extended pseudo-linearization; linear matrix inequality

I. INTRODUCTION

Recently, many researches are interested to impulsive systems and their challenges. A continuous dynamic and discontinuous comportment have been reported for impulsive systems. The nonstop dynamical activity is showed with nonstop differential equations. However discontinuous comportment is used to show the jump of states at special times. In numerous practical usages of the control, including chemical and economic procedures, biological usages, chaotic telecommunications, managing reproducible sources, society bionomics, and etc., just segregated output signal are accessible for time consecutive control process. Moreover, a constant or regular time interval between specimens is not essential [1]. In [2], the event-triggered technique was used for impulsive observer design for a flexible joint robot arm. It is assumed that the measured output is available for every constant sampling interval however, data transformation is done utilizing an event-triggered structure. In [3], the suggested impulsive observer provides the solution utilizing the idea of cooperative systems. It should be noted that in this method, the system dynamic should be modeled with two individual linear and nonlinear behaviors. In [4], an impulsive observer for a group of nonlinear dynamics with uncertainty was proposed. The efficiency of the suggested method was indicated using some simulations on biochemical reactors. In this approach, a particular group of nonlinear systems is the system equation since it has also a separable linear section.

In [5], for a chaotic synchronization problem, an adaptive impulsive observer (AIO) was outlined. The theory of comparison system in the impulsive differential equations were applied for analyzing the stability of the proposed observer. In the suggested technique the system dynamic is considered as two aggregated linear and nonlinear sections. Furthermore, in [6], the stochastic AIO was improved. The stability analysis of AIO was revised utilizing an improved Lyapunov candidate and extended Barbalat's Lemma [7]. In [8], an AIO was planned for a group of nonlinear dynamics contains discrete output signal. It should be notice that two segregated linear and nonlinear sections must be in the considered nonlinear system. In [9], an anadaptive state-feedback controller was suggested for a group of nonlinear impulsive dynamics contains time-delay with uncertainty. It is supposed that the delay values are time-varying but bounded. For confirming the performance of the presented controller, the approach was simulated on a complex-variable chaos dynamic. The routine of the impulsive observer plan was presented for a LTV system with output delay [10]. Two cases fixed and piecewise fixed delay value were considered. In [11], an impulsive observer by mutable distance between impulses was suggested in order to a group of nonlinear dynamics includes delay. The stability was analyzed using the discontinuous Lyapunov function, and LMIs were used to present sufficient conditions. In current study, the structure of nonlinear system is individually modeled as linear, nonlinear, linear include delay, and nonlinear contain delay sections. The continuous-discrete observer is planned for an impulsive nonlinear dynamic having variable delay [12]. A Lyapunov-Krasovskii candidate is taken into account for analyzing the stability, and the average dwell time method was used to establish sufficient conditions. The system dynamic is individually formulated as linear, nonlinear and linear include delay sections. In [13], an impulsive high-gain observer was suggested for a nonlinear process contains uncertainty. The

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state-dependent impulsive observer (SDIO) on the basis of the extended pseudo-linearization is suggested for an extensive group of nonlinear dynamics includes delay [14]. The comparison system theory is used forthe stability analysis of the proposed SDIO. Consequently, the obtained conditions have less conservatism for analyzing stability comparing to an existing impulsive observer.

In current study, an adaptive state-dependent impulsive observer (ASDIO) is suggested for the state and parameter approximation of nonlinear time-delay systems. The proposed ASDIO provides a continuous approximation of both the states and unbeknown parameters. As it is mentioned, in the previous researches, the nonlinear dynamic is limited to have a linear section to simplify the routine, the stability resolution and computation of the observer gains. In the case of the time-delay dynamics, these conditions are rather limitative. In current study, the ASDIO on the basis of the extended pseudo-linearization technique is designed for a more universal type of nonlinear dynamics contains delay and it is the first advantage of the suggested ASDIO. The second benefit of the proposed ASDIO is, by considering theory of comparison system, the stability theorem leads to the sufficient terms with less conservatism comparing to an existing impulsive observer. The stability analysis of the suggested ASDIO is shown utilizing a Lyapunov candidate, and theory of comparison system of the impulsive dynamics. Using an exquisite theory, it is indicated that under several feasible and less-conservative LMIs, the states and unbeknown parameters approximation error shows an asymptotical close towards zero. Furthermore, a maximum value on the highest possible distance for impulses is calculated by the stability theorem.

The remnant of the current paper is presented as following. Section II performs the extended pseudo-linearization approach for delayed nonlinear dynamics. The theory of impulsive systems, formulation and comparison systems theory for the stability analysis are explained in Section III. The suggested ASDIO is designed in Section IV. The simulation outcomes for the ASDIO design corresponding to Congo Ebola are shown in Section V. Lastly, the conclusion of the paper is presented in Section VI.

II. Extended Pseudo-Linearization Approach

The pseudo-linearization technique is the factorization procedure of a nonlinear system into a linear structure. The control and observation strategies for linear systems can be utilized and extended for nonlinear systems by this approach [15]. The maintenance of nonlinear features of the system is the key advantage of the pseudo-linearization. This technique is extended for nonlinear time-delay systems. The variant issues are work out considering this solution; for example, the observer layout basis on the extended state-dependent Riccati equation [16], the design of a sliding mode controller [17], the dive plane controller design for an autonomous underwater vehicle [18] and design of blood glucose controller for diabetes disease [19]. In extended pseudo-linearization technique all time-delay values are located in the matrices that are dependent to the states. Therefore, the linear approach corresponding to the observer design can be extended to delayed nonlinear dynamics with maintaining all nonlinear features. The formulation of a nonlinear time-delay dynamic is taken into account as:

$$\begin{cases} \dot{x} = f\left(x(t), x(t-\tau_1), \ldots, x(t-\tau_m)\right) \\ y = Cx(t) \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ and $y \in \mathbf{R}^p$ refer to the state and output vectors, respectively. $f : \mathbf{R}^n \times \ldots \times \mathbf{R}^n \to \mathbf{R}^n$ is a differentiable continuous function and $C \in \mathbf{R}^{p \times n}$ refers to the output matrix. The time-delays $\tau_1, \ldots, \tau_m$ are positive values that are constants or affected by t and x and m refers to the number of delays. It is supposed, $f\left(0, x(t-\tau_1), \ldots, x(t-\tau_m)\right) = 0$. The pseudo-linearization factorization of system (1) is as follows:

$$\dot{x} = A\left(x(t), x(t-\tau_1), \ldots, x(t-\tau_m)\right)x \tag{2}$$

where $A : \overbrace{\mathbf{R}^n \times \ldots \times \mathbf{R}^n}^{m+1} \to \mathbf{R}^{n \times n}$ is state-dependent system matrix [16]. For a system with an affine input, there exists an extended pseudo-linearization as follows [16]:

$$A(x_t) = \int_0^1 \left.\frac{\partial f(x_t)}{\partial x(t)}\right|_{x=\lambda x} d\lambda \tag{3}$$

where λ is dummy variable for integration. Hereinafter, for simplifying, $x_t = \left(x(t), x(t-\tau_1), \ldots, x(t-\tau_m)\right)$ is defined. If the system has multi states, infinite pseudo-linearization forms will be exist.

III. Theory of Impulsive Systems

In the impulsive systems theory, it has been indicated that negative time-derivative for Lyapunov function is not essential always. This matter shown as theory of comparison system, and some corollaries [1]. The impulsive system dynamic is defined as

$$\begin{cases} \dot{x} = f_x(t, x); & t \neq t_k \\ \Delta x = f_I(x); & t = t_k \end{cases} \tag{4}$$

where $f_x : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^n, f_I : \mathbf{R}^n \to \mathbf{R}^n$, refer to nonlinear functions. t_k represents time of the impulses where $t_k > t_{k-1} > 0$ and k is an integer number. The vector of states jump is determined as $\Delta x = x\left(t_k^+\right) - x(t_k)$.

Definition 1: $l(x) \in C$ signifies that l is continuous and belongs to C^i if it refers to i times differentiable with regard to x. Moreover, l belongs to κ if $l(0) = 0$, $l \in C\left[\mathbf{R}^+, \mathbf{R}^+\right]$, and $l(x)$ is remarkably enhancing in x.

Definition 2: For each $\rho \in \mathbf{R}^+$, $S_\rho = \left\{x \in \mathbf{R}^n \,\middle|\, \|x\| < \rho\right\}$ that $\|.\|$ presents the Euclidean norm.

Definition 3: $V:\mathbf{R}^+\times\mathbf{R}^n\rightarrow\mathbf{R}^+$ belongs to V_0 if V is continuous-time in $(t_{k-1},t_k]\times\mathbf{R}^n$, Lipschitz in x locally for any $x\in\mathbf{R}^n$ and $\lim_{(t,z)\rightarrow(t_k^+,x)} V(t,z)=V(t_k^+,x)$ exists.

Definition 4: For $(t,x)\in(t_{k-1},t_k]\times\mathbf{R}^n$, Dini's derivative is presented as follows:

$$D^+V(t,x)=\lim_{h\rightarrow 0^+}\sup\frac{1}{h}\Big(V\big(t+h,x+hf(t,x)\big)-V(t,x)\Big) \tag{5}$$

Definition 5: The comparison system for the main system (4) is described as following [1]:

$$\begin{cases}\dot{w}=g(t,w); & t\neq t_k\\ w(t_k^+)=\psi_k\big(w(t_k)\big)\end{cases} \tag{6}$$

where $g:\mathbf{R}^+\times\mathbf{R}^+\rightarrow\mathbf{R}$ is continuous-time and complies Definition 3, $\psi_k:\mathbf{R}^+\rightarrow\mathbf{R}^+$ is non-reducing and

$$\begin{cases}D^+V(t,x)\leq g\big(t,V(t,x)\big); & t\neq t_k\\ V(t,x+\Delta x)\leq\psi_k\big(V(t,x)\big); & t=t_k\end{cases} \tag{7}$$

Theorem 1: Suppose that subsequent terms are established [1]:

1) $D^+V(t,x)\leq g\big(t,V(t,x)\big)$ where $V:\mathbf{R}^+\times S_\rho\rightarrow\mathbf{R}^+$, $V\in V_0$, $\rho>0$ and $t\in(t_{k-1},t_k]$.

2) $\rho_0>0$ exists so that $x\in S_{\rho_0}$ denotes $x+\Delta x\in S_{\rho_0}$ for every k and $V(t,x+\Delta x)\leq\psi_k\big(V(t,x)\big)$ in $t=t_k$.

3) $b(\|x\|)\leq V(t,x)\leq a(\|x\|)$ on $\mathbf{R}^+\times S_\rho$ where $a,b\in\kappa$.

Thus, the stability characteristics of the natural respond of the comparison system (6) lead the related stability specifications of the natural respond of the main system (4).

Corollary 1: Suppose that $g(t,V)=\dot{\xi}(t)V$, that $\xi\in C^1\big[\mathbf{R}^+,\mathbf{R}^+\big]$ and $\psi_k(V)=d_kV$. The main system (4) has asymptotic stability if the subsequent terms are established [1]:

$$\begin{cases}\dot{\xi}(t)\geq 0\\ d_k\geq 0\\ \xi(t_{k+1})-\xi(t_k)+\ln(\gamma d_k)\leq 0; & \gamma>1\end{cases} \tag{8}$$

In order to prove the stability of the suggested ASDIO, the following Lemma is utilized.

Lemma 1: For any matrix $P,D\in\mathbf{R}^{n\times n}$ and n-dimentional vectors x,w, the subsequent inequality is satisfied for any positive scalar ε, [20]:

$$x^{\mathrm{T}}P^{\mathrm{T}}Dw+w^{\mathrm{T}}D^{\mathrm{T}}Px\leq\varepsilon w^{\mathrm{T}}w+\frac{1}{\varepsilon}x^{\mathrm{T}}P^{\mathrm{T}}DD^{\mathrm{T}}Px$$

IV. ADAPTIVE STATE-DEPENDENT IMPULSIVE OBSERVER

The extended pseudo-linearization of a delayed nonlinear dynamic with unknown parameters is taken into account as:

$$\begin{cases}\dot{x}=A(x_t)x+Bf(x_t)\vartheta\\ y=Cx\end{cases} \tag{9}$$

that x, y, ϑ are the state, the output, and unbeknown parameter vectors, respectively. $f:\overbrace{\mathbf{R}^n\times\ldots\times\mathbf{R}^n}^{m+1}\rightarrow\mathbf{R}^{l\times q}$ is the state-dependent system matrix, q is the number of unknown parameters, and B is a $n\times l$ constant matrix. Two following Lipschitz conditions are considered:

$$\|A(x_{1t})x_1-A(x_{2t})x_2\|\leq K_A\|x_1-x_2\| \tag{10}$$

$$\|f(x_{1t})\vartheta_1-f(x_{2t})\vartheta_2\|\leq K_f\|x_1-x_2\| \tag{11}$$

where $P>0$ and $K_A,K_f\in\mathbf{R}^+$ are Lipschitz constants. The ASDIO for nonlinear time-delay system (9) is proposed as:

$$\begin{cases}\dot{\hat{x}}=A(\hat{x}_t)\hat{x}+Bf(\hat{x}_t)\hat{\vartheta} & t\neq t_k\\ \hat{y}=C\hat{x} & \\ \Delta\hat{x}=F_1(\hat{x}_t)(y-\hat{y}) & t=t_k\end{cases} \tag{12}$$

$$\begin{cases}\dot{\hat{\vartheta}}=\varphi^{-1}(t)f^{\mathrm{T}}(\hat{x}_t)HCe & t\neq t_k\\ \Delta\hat{\vartheta}=F_2(\hat{x}_t)(y-\hat{y}) & t=t_k\end{cases} \tag{13}$$

where $\hat{x}$, $\hat{y}$, $\hat{\vartheta}$ are the estimated state, the output, and parameter vectors, respectively. $F_1(\hat{x}_t)\in\mathbf{R}^{n\times p}$ and $F_2(\hat{x}_t)\in\mathbf{R}^{q\times p}$ are the states and parameters impulses gain matrices. φ,H are design parameters that are calculated by Theorem 2.

Theorem 2: The state estimation error $e=x-\hat{x}$ and the parameter estimation error $\tilde{\vartheta}=\vartheta-\hat{\vartheta}$ of the shown ASDIO using (12), (13) asymptotically converges to zero if the subsequent conditions are provided:

$$\begin{bmatrix}\Sigma & P\left(\mathbf{I}+BB^{\mathrm{T}}\right)\\ \left(\mathbf{I}+BB^{\mathrm{T}}\right)P & -\varepsilon\left(\mathbf{I}+BB^{\mathrm{T}}\right)\end{bmatrix}\leq 0$$

$$\begin{bmatrix}\dfrac{\varphi_1-\varphi_2}{\Delta k}-\alpha\varphi & f^{\mathrm{T}}(\hat{x}_t)\\ f(\hat{x}_t) & -\dfrac{1}{2\varepsilon}\mathbf{I}\end{bmatrix}\leq 0 \tag{14}$$

$$\begin{bmatrix}-\sigma\begin{bmatrix}P_1 & 0\\ 0 & \varphi_1\end{bmatrix} & \big(\mathbf{I}-F(\hat{x}_t)C\big)^T\begin{bmatrix}P_2 & 0\\ 0 & \varphi_2\end{bmatrix}\\ \begin{bmatrix}P_2 & 0\\ 0 & \varphi_2\end{bmatrix}\big(\mathbf{I}-F(\hat{x}_t)C\big) & -\begin{bmatrix}P_2 & 0\\ 0 & \varphi_2\end{bmatrix}\end{bmatrix}\leq 0 \tag{15}$$

$$\alpha\Delta_k+\ln(\gamma\sigma)\leq 0 \tag{16}$$

where

$$\Sigma = A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t) + \frac{P_1 - P_2}{\Delta k} + 2\varepsilon\left(k_A^2 + k_f^2\right)\mathrm{I} + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) - \alpha P \quad (17)$$

Also, $\alpha \geq 0$, $\gamma > 1$ and $\sigma \geq 0$ are constants that satisfy $\sigma\gamma \leq 1$ and

$$\rho(t) = \frac{t_k - t}{\Delta_k}; \qquad t \in (t_{k-1}, t_k] \quad (18)$$

$\Delta_k = t_k - t_{k-1}$ is the k[th] impulse interval, H_1, H_2 are arbitrary and P_i, φ_j for $i, j = 1,2$ are symmetric positive definite periodic matrices as

$$P = \left(1 - \rho(t)\right)P_1 + \rho(t)P_2 \quad (19)$$

$$\varphi = \left(1 - \rho(t)\right)\varphi_1 + \rho(t)\varphi_2 \quad (20)$$

$$H = \left(1 - \rho(t)\right)H_1 + \rho(t)H_2 \quad (21)$$

where

$$B^{\mathrm{T}}P_1 = H_1C, B^{\mathrm{T}}P_2 = H_2C \to B^{\mathrm{T}}P = HC \quad (22)$$

Proof: According to (9), (12) and (13), the dynamic equation and the impulse corresponding to the state and parameter estimation errors are as follows:

$$\begin{cases} \dot{e} = A(\hat{x}_t)e + \tilde{A}x + Bf(\hat{x}_t)\tilde{\vartheta} + B\tilde{f}\vartheta; & t \neq t_k \\ \Delta e = -F_1(\hat{x}_t)Ce; & t = t_k \end{cases} \quad (23)$$

$$\begin{cases} \dot{\tilde{\vartheta}} = -\varphi^{-1}f^{\mathrm{T}}(\hat{x}_t)HCe; & t \neq t_k \\ \Delta\tilde{\vartheta} = -F_2(\hat{x}_t)Ce; & t = t_k \end{cases} \quad (24)$$

where $\tilde{A} = A(x_t) - A(\hat{x}_t)$ and $\tilde{f} = f(x_t) - f(\hat{x}_t)$. The time-varying Lyapunov function candidate is taken into account as:

$$V = e^{\mathrm{T}}Pe + \tilde{\vartheta}^{\mathrm{T}}\varphi\tilde{\vartheta} \quad (25)$$

Thus, the Dini's derivative of Lyapunov candidate at $t \in (t_{k-1}, t_k]$ is calculated as:

$$\begin{aligned} D^+V = {} & e^{\mathrm{T}}\left\{A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t) + \frac{P_1 - P_2}{\Delta_k}\right\}e \\ & + e^{\mathrm{T}}\left\{PBf(\hat{x}_t) - C^{\mathrm{T}}H^{\mathrm{T}}f(\hat{x}_t)\varphi^{-1}\varphi\right\}\tilde{\vartheta} \\ & + \tilde{\vartheta}^{\mathrm{T}}\left\{f^{\mathrm{T}}(\hat{x}_t)B^{\mathrm{T}}P - \varphi\varphi^{-1}f^{\mathrm{T}}(\hat{x}_t)HC\right\}e \\ & + x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\tilde{A}x + \vartheta^{\mathrm{T}}\tilde{f}^{\mathrm{T}}B^{\mathrm{T}}Pe + e^{\mathrm{T}}PB\tilde{f}\vartheta \\ & + \tilde{\vartheta}^{\mathrm{T}}\left(\frac{\varphi_1 - \varphi_2}{\Delta_k}\right)\tilde{\vartheta} \end{aligned} \quad (26)$$

Considering (22), it is concluded:

$$\begin{aligned} D^+V = {} & e^{\mathrm{T}}\left(A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t) + \frac{P_1 - P_2}{\Delta_k}\right)e \\ & + x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\tilde{A}x + \vartheta^{\mathrm{T}}\tilde{f}^{\mathrm{T}}B^{\mathrm{T}}Pe + e^{\mathrm{T}}PB\tilde{f}\vartheta \\ & + \tilde{\vartheta}^{\mathrm{T}}\left(\frac{\varphi_1 - \varphi_2}{\Delta_k}\right)\tilde{\vartheta} \end{aligned} \quad (27)$$

Utilizing Lemma 1, it is achieved:

$$x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\tilde{A}x \leq \varepsilon x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}\tilde{A}x + \frac{1}{\varepsilon}e^{\mathrm{T}}P^2e \quad (28)$$

$$\vartheta^{\mathrm{T}}\tilde{f}^{\mathrm{T}}B^{\mathrm{T}}Pe + e^{\mathrm{T}}PB\tilde{f}\vartheta \leq \varepsilon\vartheta^{\mathrm{T}}\tilde{f}^{\mathrm{T}}\tilde{f}\vartheta + \frac{1}{\varepsilon}e^{\mathrm{T}}PBB^{\mathrm{T}}Pe \quad (29)$$

Considering assumptions (10) and (11),

$$\begin{aligned} x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}\tilde{A}x & = \left\|\tilde{A}x\right\|^2 = \left\|\tilde{A}x \pm A(\hat{x}_t)\hat{x}\right\|^2 \\ & \leq 2\left\|A(x_t)x - A(\hat{x}_t)\hat{x}\right\|^2 + 2\left\|A(\hat{x}_t)(x - \hat{x})\right\|^2 \\ & \leq 2K_A^2e^{\mathrm{T}}e + 2e^{\mathrm{T}}A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t)e \end{aligned} \quad (30)$$

$$\begin{aligned} \vartheta^{\mathrm{T}}\tilde{f}^{\mathrm{T}}\tilde{f}\vartheta & = \left\|\tilde{f}\vartheta\right\|^2 = \left\|\tilde{f}\vartheta \pm f(\hat{x}_t)\hat{\vartheta}\right\|^2 \\ & \leq 2\left\|f(x_t)\vartheta - f(\hat{x}_t)\hat{\vartheta}\right\|^2 + 2\left\|f(\hat{x}_t)(\vartheta - \hat{\vartheta})\right\|^2 \\ & \leq 2K_f^2e^{\mathrm{T}}e + 2\tilde{\vartheta}^{\mathrm{T}}f^{\mathrm{T}}(\hat{x}_t)f(\hat{x}_t)\tilde{\vartheta} \end{aligned} \quad (31)$$

The Dini's derivative of Lyapunov nominated is obtained as:

$$\begin{aligned} D^+V \leq {} & e^{\mathrm{T}}\left(A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t) + \frac{P_1 - P_2}{\Delta_k}\right)e \\ & + 2\varepsilon e^{\mathrm{T}}\left(\left(k_A^2 + k_f^2\right)\mathrm{I} + A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t)\right)e \\ & + \frac{1}{\varepsilon}e^{\mathrm{T}}\left(P^2 + PBB^{\mathrm{T}}P\right)e \\ & + \tilde{\vartheta}^{\mathrm{T}}\left(\frac{\varphi_1 - \varphi_2}{\Delta_k} + 2\varepsilon f^{\mathrm{T}}(\hat{x}_t)f(\hat{x}_t)\right)\tilde{\vartheta} \end{aligned} \quad (32)$$

Now, the right side of (32) is added to $\pm\alpha V$

$$D^+V \leq e^{\mathrm{T}}\Omega_1e + \tilde{\vartheta}^{\mathrm{T}}\Omega_2\tilde{\vartheta} + \alpha V \quad (33)$$

where

$$\begin{aligned} \Omega_1 = {} & A^{\mathrm{T}}(\hat{x}_t)P + PA(\hat{x}_t) + \frac{P_1 - P_2}{\Delta_k} \\ & + 2\varepsilon\left(\left(k_A^2 + k_f^2\right)\mathrm{I} + A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t)\right) \\ & + \frac{1}{\varepsilon}\left(P^2 + PBB^{\mathrm{T}}P\right) - \alpha P \\ \Omega_2 = {} & \frac{\varphi_1 - \varphi_2}{\Delta_k} + 2\varepsilon f^{\mathrm{T}}(\hat{x}_t)f(\hat{x}_t) - \alpha\varphi \end{aligned} \quad (34)$$

Satisfying of two LMIs of (14) by Schur complement [5], $e^{\mathrm{T}}\Omega_1e \leq 0$ and $\tilde{\vartheta}^{\mathrm{T}}\Omega_2\tilde{\vartheta} \leq 0$ are obtained so, $D^+V \leq \alpha V$. It is considered $\dot{\xi}(t) = \alpha \geq 0$ thus, the first term of corollary 1 (8) is established. At impulse times, the errors are calculated as:

$$\begin{aligned} e\left(t_k^+\right) & = \left(\mathbf{I} - F_1(\hat{x}_t)C\right)e(t_k) \\ \tilde{\vartheta}\left(t_k^+\right) & = \tilde{\vartheta}(t_k) - F_2(\hat{x}_t)Ce(t_k) \end{aligned} \quad (35)$$

Due to (18), (19) and (20), Lyapunov nominated at $t = t_k^+$ is

$$V\left(t_k^+\right) = e^{\mathrm{T}}\left(t_k^+\right)P_2e\left(t_k^+\right) + \tilde{\vartheta}^{\mathrm{T}}\left(t_k^+\right)\varphi_2\tilde{\vartheta}\left(t_k^+\right) \quad (36)$$

Replacing (36) into (37)

$$V\left(t_k^+\right)=X^{\mathrm{T}}\left(\mathbf{I}-FC\right)^{\mathrm{T}}\begin{bmatrix}P_2 & 0\\ 0 & \varphi_2\end{bmatrix}\left(\mathbf{I}-FC\right)X \tag{37}$$

where $X=\begin{bmatrix}e & \tilde{\vartheta}\end{bmatrix}^{\mathrm{T}}$ and $FC=\begin{bmatrix}F_1(\hat{x}_t)C & 0\\ F_2(\hat{x}_t)C & 0\end{bmatrix}$. Satisfying the following condition by LMI (15) utilizing Schur complement [5], $V\left(t_k^+\right)\le \sigma V\left(t_k\right)$ is obtained, where $d_k=\sigma\ge 0$ and the second term corresponding to corollary 1 (8) is established.

$$\left(\mathbf{I}-FC\right)^{\mathrm{T}}\begin{bmatrix}P_2 & 0\\ 0 & \varphi_2\end{bmatrix}\left(\mathbf{I}-FC\right)\le\sigma\begin{bmatrix}P_1 & 0\\ 0 & \varphi_1\end{bmatrix} \tag{38}$$

At the end, based on the third term corresponding to corollary 1:

$$\left(\alpha t_k-\alpha t_{k-1}\right)+\ln\left(\gamma d_k\right)\le 0\rightarrow \ln\left(\gamma d_k\right)\le-\alpha\Delta_k \tag{39}$$

With regards to $\alpha\Delta_k\ge 0$ and $\gamma>1$, the argument of ln function should less than equal of 1 hence, it is concluded $\gamma d_k\le 1$ that is satisfied by $\sigma\gamma\le 1\Rightarrow\sigma\le 1$.

Remark 1. The maximum impulse interval is presented as

$$\Delta_k^{\max}=\max_{k=1,2,\ldots}\left(t_k-t_{k-1}\right)=\left|\frac{\ln\left(\gamma\sigma\right)}{\alpha}\right| \tag{40}$$

V. Simulation Results

In preventing the outbreak of contagious diseases, modeling is one of the important steps in identifying and treating the disease. The Susceptible-Infected-Recovered (SIR) is very effective for modeling of epidemic diseases. To show the demographic agents of the disease behavior in the SIR model, the birth and death rates are discussed as two parameters that are affiliated to the density. Furthermore, to achieve a SIR model close to the real disease behavior, the incubation time is synthesized while the contagious factors extend in the vector. Then, an SIR dynamic contains delay is presented. The SIR nonlinear time-delay dynamic is taken into account as [21]:

$$\begin{cases}\dot{S}=\left(b-\mu\dfrac{rN(t)}{K}\right)N(t)-\dfrac{\beta S(t)I(t-\tau)}{N(t-\tau)}\\ \qquad -\left(d+(1-\mu)\dfrac{rN(t)}{K}\right)S(t)\\ \dot{I}=\dfrac{\beta S(t)I(t-\tau)}{N(t-\tau)}-\left(d+(1-\mu)\dfrac{rN(t)}{K}+\lambda\right)I(t)\\ \dot{R}=\lambda I(t)-\left(d+(1-\mu)\dfrac{rN(t)}{K}\right)R(t)\end{cases} \tag{41}$$

that S, I and R refer to susceptible, infected and recovered persons, respectively. $N=S+I+R$ represents the amount of whole society. $\lambda>0$, $b>0$, $\beta>0$ and $d>0$ refer to the recovery, birth, contact and death rate, respectively. $r=b-d$ represents the rate of inherent increasing, μ refers constant of convex combination, K refers to the population shipment inclusion and τ represents a positive scalar showing a delay value on infected persons and whole society (I and N) over the disease extension. The state R is taken into account as the measured output so, $C=\begin{bmatrix}0 & 0 & 1\end{bmatrix}$. The values corresponding to the factors in a special disease Congo Ebola are $b=0.07$, $\mu=0.014$, $\beta=0.21$, $d=0.0123$, $\lambda=0.0476$, $\tau=10$ and $K=10900$ [21]. Assume that $\vartheta=\begin{bmatrix}\lambda & b\end{bmatrix}^{\mathrm{T}}$ is parameters vector so, the subsequent show is one of the indefinite number of extended pseudo-linearization form of (41)

$$A(x_t)=\begin{bmatrix}a_{11} & -\mu rN(t)/K & -\mu rN(t)/K\\ a_{21} & a_{22} & -(1-\mu)rI(t)/K\\ 0 & 0 & a_{33}\end{bmatrix}$$

$$B=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & 0\end{bmatrix}^{\mathrm{T}},\qquad f(x_t)=\begin{bmatrix}I(t) & 0\\ 0 & N(t)\end{bmatrix} \tag{42}$$

where

$$a_{11}=-\mu\frac{rN(t)}{K}-\frac{\beta I(t-\tau)}{N(t-\tau)}-\left(d+r(1-\mu)\frac{N(t)}{K}\right)$$

$$a_{21}=\frac{\beta I(t-\tau)}{N(t-\tau)}-(1-\mu)\frac{rI(t)}{K}$$

$$a_{22}=-\left(d+(1-\mu)\frac{rI(t)}{K}\right)$$

$$a_{33}=-\left(d+(1-\mu)\frac{rN(t)}{K}\right)$$

The initial conditions and are taken into account as $x(0)=\begin{bmatrix}9126 & 315 & 59\end{bmatrix}^T$, $\hat{x}(0)=\begin{bmatrix}8000 & 800 & 400\end{bmatrix}^T$. The design parameters are considered as $\alpha=0.065$, $\sigma=0.97$, $\gamma=1.01$ and $\Delta=5$. In Fig. 1 and Fig. 2 the actual and estimated SIR states and amount of total society using the ASDIO are demonstrated. It is clearly that, expected states follow actual ones even the number of the measurement signal is only a fifth of the number of samples. The impulse amplitudes corresponding to expected states are demonstrated in Fig. 3. In Fig. 4, actual and approximated unbeknown parameters are drawn. The estimated parameters are converged to actual value in less than 100 samples.

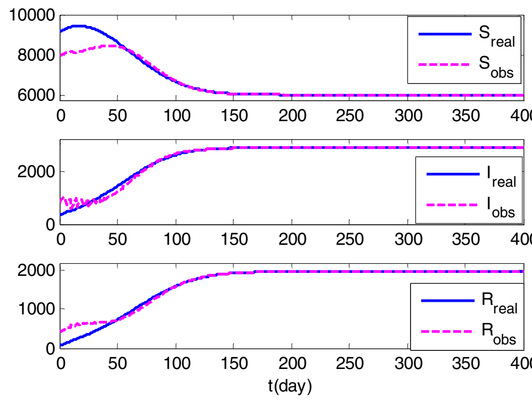

Figure 1. The states estimation by proposed ASDIO

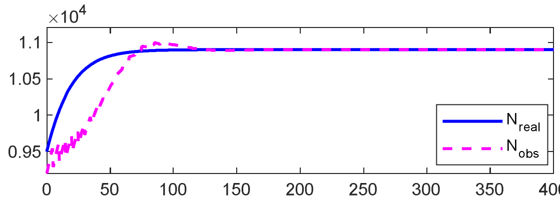

Figure 2. The total population estimation by proposed ASDIO

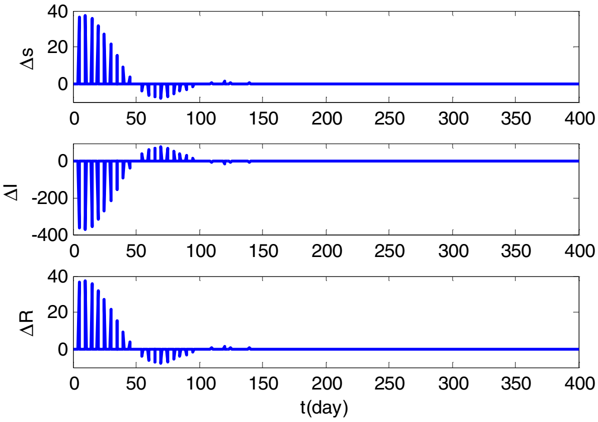

Figure 3. The impulses amplitude by proposed ASDIO

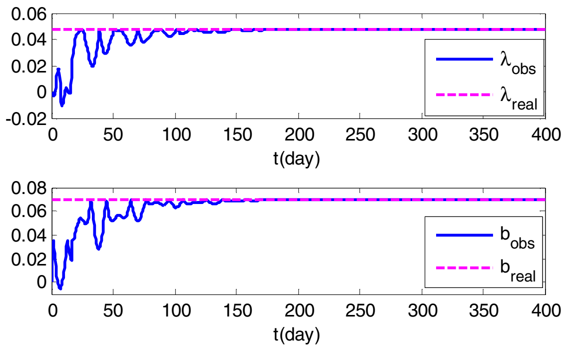

Figure 4. The parameters estimation by proposed ASDIO

VI. Conclusion

In current study, a novel ASDIO offered for nonlinear dynamics with time-delays on the basis of the extended pseudo-linearization presentation. The stability analysis of the proposed ASDIO investigated using theory of the comparison system corresponding to impulsive dynamics. There are two advantages of the suggested ASDIO: 1) applicability of the ASDIO for a more extensive type of nonlinear dynamics including different kinds of delays by extended pseudo-linearization technique. Thus, there was no requirement to exist a separated linear or delay structure in the original nonlinear model, 2) asymptotic convergence of the states and parameters estimation error towards zero under adequate terms with less conservatism that derived in terms of feasible LMIs based on comparison system theory. Moreover, the maximum distance between two impulses offered. The simulation outcomes corresponding to SIR nonlinear dynamic with time-delay of Congo Ebola authenticated the performance of the suggested ASDIO.

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