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Design of a Suboptimal Controller based on Riccati Equation and State-dependent Impulsive Observer for a Robotic Manipulator

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***Abstract*— In this paper, Riccati equation and state-dependent impulsive observer (SDIO) are combined to design a controller for the nonlinear dynamical system of a robotic manipulator. A state-dependent Riccati equation-based controller is similar to optimal linear quadratic regulator where its Riccati equation employs pseudo linearization. Pseudo-linearization is a technique to represent systems with a structure similar to a linear system while preserving all characteristics of the original nonlinear system. It is stronger than the linearization technique via Jacobin method and does not need to change the operating point. Here it is shown that, the impulsive observer is able to estimate system states even if they are not transmitted from the sensors with a regular sampling rate. Via the proposed design, the stability of the closed loop system is analyzed using comparison principle which results in less conservative stability criteria. Applying pseudo linearization technique to the system and combining the Riccati equation controller and SDIO, suboptimal observer based control is obtained for the nonlinear manipulator systems especially when the system output is not accessible at all moments. The simulation results indicate the performance and efficiency of the proposed method in controlling the system.**

***Index Terms*-- Comparison principle, manipulator, state-dependent impulsive observer, state-dependent Riccati equation, pseudo linearization.**

I. INTRODUCTION

Design of controller and observer for nonlinear systems is more complicated compared to linear systems. The designers have always been looking forward to obtain a simple design using the methods proposed for linear systems and generalizing them to nonlinear systems. Unfortunately, simple linearization methods, like Jacobin method, for systems in their equilibrium points cannot always describe the characteristics of the nonlinear system in all states. On the other hand, considering popularity of linear squares regulators (LQR) in state space, many efforts have been done to generalize them to nonlinear systems. As a result of these efforts, state-dependent control method was proposed which is based on employing pseudo linearization technique and solving state dependent Riccati equation (SDRE) [1]. SDRE technique is a method capable of including control effort and the value of states in the objective function and obtaining the suboptimal solution for designing the controller, observer and filter for nonlinear systems [2]. It has also been developed for time-delay systems as extended SDRE (ESDRE) [3]. ESDRE technique is based on the representation of the system via pseudo linearization method where the system, input and output matrices are all subject to some delays.

In [4], SDRE is used for tracking purposes in a hydraulic drive. Simulation and experimental results indicate proper efficiency of method. SDRE technique combined with various methods is used for robotic manipulators as well. State dependent differential equation (SDDRE) is designed for control and tracking a parallel manipulator with two degrees of freedom in [5]. In [6], this technique is utilized for cooperation of manipulators to avoid obstacles. Authors of [7] used SDRE to control a solid and flexible manipulator. In [8], a sliding mode algorithm is proposed using SDRE and simulated on Scout manipulator robot. Furthermore, SDRE is extended for chaos synchronization and control in [9]. In [10], SDRE is utilized to control a second-order nonlinear dynamics with focus on the design degree of freedom.

In the observer-based control framework, a state observer is first designed to estimate system states based on measured output. If the measured outputs are accessible with unknown and variable distances, the state estimation problem becomes more complicated. In many of the real systems, output can be accessed in discrete time instants which might not be constant or known. Thus, impulsive observers have been proposed for these systems. Impulsive observers are a combination of

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continuous dynamics with sudden jumps or impulses which estimate the system states continuously knowing the output at discrete time instants. Recently, impulsive observers have been designed based on the theory of impulsive dynamic systems for linear and nonlinear Lipschitz systems. Unlike continuous-time observers, impulsive observers update their states at discrete time instants. Advantages of the impulsive observers over classic observers are as follows: continuous state estimation with discrete outputs, reducing information transmission and cost, increasing bandwidth utilization and simple implementation [11].

Two types of stability have been presented for impulsive systems using Lyapunov theory. First one is the classic view in which derivative of the Lyapunov function should be non-positive. The second one is the comparison system in which the distance between impulses of the Lyapunov derivative might be ascending provided that value of the Lyapunov function at each impulse is smaller than the previous impulse which is known as "Ln" condition. This approach provides less conservative results for stability analysis [12]. In [13], impulsive observer is proposed for nonlinear systems where some parts of equations of this system are linear Lipschitz and some others are nonlinear Lipschitz. This observer is proposed for flexible manipulator of a specific class [14]. In [15], impulsive observers are designed for a specific class time-variant nonlinear systems. Impulsive observers have been proposed for bioreactor system for which the equations are described as an uncertain nonlinear set of equations [16]. Adaptive impulsive observers (AIO) are also presented for nonlinear systems of a specific class in [17]. In [18], an impulsive observer is proposed for a special class of second-order Lipschitz nonlinear dynamics. An AIO is suggested for linear dynamics with nonlinear and uncertain output in [19]. Moreover, These observers are presented for a group of uncertain nonlinear switched dynamics [20].

In all of the above studies, classic view has been used to prove convergence of the impulsive observer. In [21-25], adaptive SDIO is used to estimate the states of the time-delay nonlinear systems. Stability of the observer presented in this paper is analyzed through comparative approach which offers stability conditions with less conservatism. This observer is able to estimate states of nonlinear system with a wider range of classes and different delays. Finally, simulation is applied on Congo Ebola and HIV delay nonlinear equations.

Considering the merits of SDRE based controllers and the impulsive observers, this study aims to extend the controller design based on these methods. For the proof of concept, the manipulators are preferred here. Manipulators are one of the most applicable robots in industry, military and services. Dynamic equations of these robots are severely nonlinear and have various operating points, thus linearizing them about equilibrium points using methods such as Jacobin method cannot preserve all properties of the system. Consequently, in this paper, state dependent Riccati equation controller using pseudo linearization is employed for suboptimal control of the manipulator system. Here, SDIO is used for state estimation and offers a continuous estimation of states at each sampling time without requiring the output. Remarkably, combining the state dependent Riccati equation controller and the impulsive observer and design of this observer based on pseudo linearization for manipulator is the innovation of this paper. The stability analysis of the proposed controller and observer is addressed here via Lyapunov theory along with the comparison system theory. It is worth mentioning that, while the conventional impulsive observers for nonlinear systems are limited to a specific class of such systems, for examples nonlinear systems which have an additive linear part or a nonlinear part with Lipschitz condition or a system with bilinear equations, the pseudo linearization based impulsive observers can be designed and applied to a wider class of nonlinear systems.

The rest of this paper is organized as follows. Section 2 presents the dynamical equations of the manipulator system. Pseudo linearization SDRE are described in section 3. Section 4 designs the SDIO. Simulation results of the proposed method are analyzed in section 5. Finally, the paper is concluded in section 6.

II. Manipulator Dynamic

Equations of the manipulator are as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

in which, $\ddot{q},\dot{q},q \in \mathrm{R}^{n}$ are angle, velocity and angular acceleration vectors of the joints. $M, C \in \mathrm{R}^{n\times n}$ are the mass, inertia, Coriolis force and centrifugal force matrices. G, τ are gravity acceleration and torque vector of the joints. These matrices and vectors for a manipulator with two degrees of freedom are defined as follows [26]:

$$M = \begin{bmatrix} a_1 + a_3 + 2a_2\cos(q_2) & a_3 + a_2\cos(q_2) \\ a_3 + a_2\cos(q_2) & a_3 \end{bmatrix}$$

$$a_1 = I_1 + \left(\frac{m_1}{4} + m_2\right) l_1^2$$

$$a_2 = \frac{m_2 l_1 l_2}{2};\ a_3 = I_2 + \frac{m_2 l_2^2}{4}$$

$$C = \begin{bmatrix} -a_2\sin(q_2)\dot{q}_2 & -a_2\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ a_2\sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \tag{2}$$

$$G = \begin{bmatrix} g_a + \frac{m_2 l_2}{2} g\cos(q_1 + q_2) \\ \frac{m_2 l_2}{2} g\cos(q_1 + q_2) \end{bmatrix}$$

$$g_a = \left(\frac{m_1 l_1}{2} + m_2 l_1\right) g\cos(q_1)$$

Parameters of the robot are given in Table I. Reference trajectories are considered as follows.

$$q_{1d} = 0.5\cos(t) + 0.2\sin(3t) + 1$$

$$q_{2d} = -0.5\cos(t) - 0.2\sin(2t) + 0.5 \tag{3}$$

TABLE I. PARAMETERS OF THE MANIPULATOR [26]

	$m_i\left(kg\right)$	$l_i\left(m\right)$	$I_i\left(kgm^2\right)$
Link 1	8	0.5	0.4
Link 2	8	0.5	0.4

III. OPTIMAL CONTROL BASED ON STATE DEPENDENT RICCATI EQUATION

State dependent Riccati equation technique was first proposed by Pearson in 1962 and attracted attentions since 1990. In this method, the nonlinear system is first converted to a pseudo linear system without any approximations. This representation results in a structure similar to linear structure of the matrices dependent on system states and advantages of linear representation are used for designing the controller, observer and nonlinear filter. SDRE regulators are in fact the nonlinear form of optimal LQR with squared performance index in which state space matrices of the system and weight matrices are functions of the system states. In brief, main advantages of this method are as follows [1-2]:

a) Possibility of using this method in the design of the controller, observer and filter for nonlinear system with various and wider classes
b) Possibility of compromising between control and performance, simply.
c) Pseudo linearization of the nonlinear system is not unique as a result of which number of degrees of freedom of the design are increased.

A. Impulsive Observer and Comparison System

In general, time-invariant nonlinear system can be described as follows:

$$\dot{x}=h\left(x,u\right) \tag{4}$$

Using the pseudo linearization method, these equations can be written as follows [1]:

$$\dot{x}=A\left(x\right)x+B\left(x\right)u \tag{5}$$

in which, $A\left(x\right)$ and $B\left(x\right)$ are state dependent coefficient matrices. Main advantage of the pseudo linearization method is that it preserves all nonlinear characteristics of the system unlike Jacobin linearization method [1]. In practice, any nonlinear system cannot be represented in pseudo linear form. Consider the nonlinear system with the following state space representations:

$$\dot{x}=h\left(x\right)+B\left(x\right)u \tag{6}$$

The following theorem describes sufficient condition for representation of nonlinear systems with the above equations in pseudo linear form.

Theorem 1: Function $h:\Omega\to R^n$ is continuous and differentiable and $\Omega\subseteq R^n$ is an open and limited set including the origin. Then, for each $x\in\Omega$, the following pseudo linearization exists for system (6) [1].

$$A\left(x\right)=\int_0^1 \left.\frac{\partial h\left(x\right)}{\partial x}\right|_{x=\lambda x} d\lambda \tag{7}$$

If there exists a pseudo linear form and the system has more than one state variable, an infinite number of pseudo linear forms can be written [1-2]. State space representation of the manipulator equations (1) are as follows:

$$\dot{x}=\begin{bmatrix} x_3 \\ x_4 \\ -M^{-1}\left(C+G\right)\end{bmatrix}+\begin{bmatrix}0\\0\\M^{-1}\tau\end{bmatrix} \tag{8}$$

in which, $x=\left[q_1,q_2,\dot{q}_1,\dot{q}_2\right]^{\mathrm{T}}$. One pseudo linearization is considered as follows:

$$\dot{x}=\begin{bmatrix}0 & \mathrm{I}\\ F & -M^{-1}C\end{bmatrix}x+\begin{bmatrix}0\\M^{-1}\end{bmatrix}\tau=A\left(x\right)x+B\left(x\right)u$$

$$F=\begin{bmatrix}-M^{-1}G\left(1\right)/x_1 & 0\\ 0 & -M^{-1}G\left(2\right)/x_2\end{bmatrix} \tag{9}$$

where $M^{-1}G\left(i\right)$ is the i[th] element of this matrix.

B. State Dependent Riccati Equation with Tracking Approach

In order to design a SDRE controller for tracking the reference trajectory r, the following infinite horizon cost function is defined [8]:

$$\mathrm{J}=\frac{1}{2}\int_0^\infty\left(\left(C_y x-r\right)^{\mathrm{T}}Q\left(C_y x-r\right)+u^{\mathrm{T}}Ru\right)dt \tag{10}$$

where C_y is the output matrix, R and Q are positive definite and positive semi-definite matrices describing weight coefficients of the inputs and states. The suboptimal control input is obtained as follows:

$$\begin{aligned}u&=-R^{-1}B\left(x\right)^{\mathrm{T}}P\left(x\right)x+R^{-1}B\left(x\right)^{\mathrm{T}}\bar{\eta}\left(x\right)r\\&=-k\left(x\right)x+v\left(x\right)r\end{aligned} \tag{11}$$

$$\bar{\eta}\left(x\right)=-\left(\left(A\left(x\right)-B\left(x\right)R^{-1}B\left(x\right)^{\mathrm{T}}P\left(x\right)\right)^{\mathrm{T}}\right)^{-1}C_y{}^{\mathrm{T}}Q$$

where its first part is used for stabilization and regulation and its second part is used to track the reference trajectory and P is the state dependent Riccati equation solution.

$$\begin{aligned}&A\left(x\right)^{\mathrm{T}}P\left(x\right)+P\left(x\right)A\left(x\right)-P\left(x\right)B\left(x\right)R^{-1}B\left(x\right)^{\mathrm{T}}P\left(x\right)\\&+C_y{}^{\mathrm{T}}QC_y=0\end{aligned} \tag{12}$$

IV. IMPULSIVE OBSERVER AND COMPARISON PRINCIPLE

Equations of the impulsive system are considered as follows:

$$\begin{cases}\dot{x}=h\left(x\right) & t\neq\tau_k\\ \Delta x=h_I\left(x\right) & t=\tau_k\end{cases} \tag{13}$$

where, $\tau_k, k=1,2,\ldots$ are the impulse times such that $\tau_k>\tau_{k-1}>0$ and Δx is the impulse vector of the states defined as follows:

$$\Delta x\left(\tau_k\right)=x\left(\tau_k^+\right)-x\left(\tau_k\right) \tag{14}$$

The comparison system for system (13) is as follows:

$$\begin{cases}\dot{z}=s\left(z,t\right) & t\neq\tau_k\\ z\left(\tau_k^+\right)=\zeta_k\left(z\left(\tau_k\right)\right) & t=\tau_k\end{cases} \tag{15}$$

in which, $s:\mathrm{R}^{+}\times\mathrm{R}^{+}\to\mathrm{R}$ is a continuous and locally Lipschitz function. $\zeta_k:\mathrm{R}^{+}\to\mathrm{R}^{+}$ is a non-decreasing function and the following conditions are satisfied for a continuous right-handed differentiable and locally Lipschitz function V [27]:

$$\begin{cases}\dot{V}(x,t)\le s\left(V(x,t),t\right) & t\neq\tau_k\\ V(x+\Delta x,t)\le\zeta_k\left(V(x,t)\right) & t=\tau_k\end{cases}\tag{16}$$

Theorem 2: If $s\left(V(x,t),t\right)=\dot{\eta}(t)V(x,t)$ such that η is continuous and first order differentiable and $\zeta_k\left(V(x,t)\right)=\varsigma_k V(x,t)$. Origin of system (13) would be asymptotically stable if the following conditions are satisfied [27]:

$$\begin{cases}\dot{\eta}(t)\ge 0\\ \varsigma_k\ge 0;\\ \eta(t_k)-\eta(t_{k-1})+\ln(\gamma\varsigma_k)\le 0;\ \gamma>1\end{cases}\tag{17}$$

Lemma 1: for each vector a and b and arbitrary matrix $X>0$, the following relationship is established:

$$a^{\mathrm{T}}b+b^{\mathrm{T}}a\le a^{\mathrm{T}}Xa+b^{\mathrm{T}}X^{-1}b\tag{18}$$

the SDIO equations are considered as follows:

$$\begin{cases}\dot{\hat{x}}=A(\hat{x})\hat{x}+B(\hat{x})u & t\neq\tau_k\\ \hat{y}=C_y\hat{x} & \\ \Delta\hat{x}=L(\hat{x})(y-\hat{y}) & t=\tau_k\end{cases}\tag{19}$$

in which, $\hat{x}$ and $\hat{y}$ are state and estimated output vectors and $\Delta\hat{x}$ is the estimated state impulse. L which is the gain of the observer is determined according to Theorem 3 which describes closed loop stability.

Theorem 3: Estimation error of the proposed SDIO $e=x-\hat{x}$ with SDRE controller converges to zero asymptotically with the following assumptions and conditions.

$$\begin{bmatrix}\Sigma & 2A_c^{\mathrm{T}}(\hat{x})M\\ MA_c(\hat{x}) & -M\end{bmatrix}\le 0\tag{20}$$

$$\begin{aligned}\Sigma=&A_c^{\mathrm{T}}(\hat{x})M+MA_c(\hat{x})+(M_1-M_2)/\Delta_k\\&+\left(2+2K_A^2+K_B^2-\delta\right)M\end{aligned}\tag{21}$$

$$\begin{bmatrix}-\beta M_1 & \left(I-L(\hat{x})C_y\right)^{\mathrm{T}}M_2\\ M_2\left(\mathrm{I}-L(\hat{x})C_y\right) & -M_2\end{bmatrix}\le 0\tag{22}$$

$$\delta\Delta_k+\ln(\gamma\beta)\le 0\tag{23}$$

where $\delta\ge 0$, $\beta\ge 0$ and $\gamma\sigma\le 1$.

Assumption 1: By defining $h_M^2=h^{\mathrm{T}}Mh$, where M is the symmetric positive definite matrix, the following Lipschitz conditions are considered:

$$\begin{aligned}A_c(x)x-A_c(\hat{x})\hat{x}_M^2&\le K_A^2e^TMe\\ B_c(x)r-B_c(\hat{x})r_M^2&\le K_B^2e^TMe\end{aligned}\tag{24}$$

Proof: the closed loop system with SDRE is:

$$\begin{aligned}\dot{x}&=A(x)x+B(x)\left(-k(x)x+v(x)r\right)\\&=\left(A(x)-B(x)k(x)\right)x+B(x)v(x)r\\&=A_c(x)x+B_c(x)r\end{aligned}\tag{25}$$

where $A_c(x)=A(x)-B(x)k(x)$ and $B_c(x)=B(x)v(x)$ are defined. At $t\neq\tau_k$, thus:

$$\begin{aligned}\dot{\hat{x}}&=\left(A(\hat{x})-B(\hat{x})k(\hat{x})\right)\hat{x}+B(\hat{x})v(\hat{x})r\\&=A_c(\hat{x})\hat{x}+B_c(\hat{x})r\end{aligned}\tag{26}$$

Thus, dynamic of the estimation error at $t\neq\tau_k$ is:

$$\begin{aligned}\dot{e}&=A_c(x)x+B_c(x)r-A_c(\hat{x})\hat{x}-B_c(\hat{x})r\\&=A_c(\hat{x})(x-\hat{x})+\left(A_c(x)-A_c(\hat{x})\right)x\\&+\left(B_c(x)-B_c(\hat{x})\right)r=A_c(\hat{x})e+\tilde{A}x+\tilde{B}r\end{aligned}\tag{27}$$

where $\tilde{A}=A_c(x)-A_c(\hat{x})$ and $\tilde{B}=B_c(x)-B_c(\hat{x})$ are defined. at $t=\tau_k$

$$\Delta e=L(\hat{x})C_ye(t_k)\to e\left(t_k^{+}\right)=\left(\mathrm{I}-L(\hat{x})C_y\right)e(t_k)\tag{28}$$

The following Lyapunov function is defined as:

$$V=e^{\mathrm{T}}Me;\quad M=(1-\rho)M_1+\rho M_2\tag{29}$$

$$\rho=(\tau_k-t)/\Delta_k;\quad \Delta_k=\tau_k-\tau_{k-1};\quad t\in\left(\tau_{k-1},\tau_k\right]\tag{30}$$

where M is a periodic function in $\left(\tau_{k-1},\tau_k\right]$ and M_1 and M_2 are symmetric positive definite matrices. Therefore

$$M(\tau_k)=M_1;\quad M\left(\tau_k^{+}\right)=M_2\tag{31}$$

Derivative of the Lyapunov function in $\left(\tau_{k-1},\tau_k\right]$ is

$$\begin{aligned}\dot{V}=&e^{\mathrm{T}}\left(A_c^{\mathrm{T}}(\hat{x})M+MA_c(\hat{x})+\frac{M_1-M_2}{\Delta_k}\right)e\\&+x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Me+e^{\mathrm{T}}M\tilde{A}x+r^{\mathrm{T}}\tilde{B}^{\mathrm{T}}Me+e^{\mathrm{T}}M\tilde{B}r\end{aligned}\tag{32}$$

Using Lemma1, the following is concluded:

$$\begin{aligned}x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}Me+e^{\mathrm{T}}M\tilde{A}x&\le e^{\mathrm{T}}Me+x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}M\tilde{A}x\\ r^{\mathrm{T}}\tilde{B}^{\mathrm{T}}Me+e^{\mathrm{T}}M\tilde{B}r&\le e^{\mathrm{T}}Me+r^{\mathrm{T}}\tilde{B}^{\mathrm{T}}M\tilde{B}r\end{aligned}\tag{33}$$

where the following is obtained according to assumption 1:

$$\begin{aligned}x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}M\tilde{A}x&\le 2A_c(x)x-A_c(\hat{x})\hat{x}_M^2\\&+2A_c(\hat{x})(x-\hat{x})_M^2\\&\le 2K_A^2e^TMe+2e^{\mathrm{T}}A_c^{\mathrm{T}}(\hat{x})MA_c(\hat{x})e\end{aligned}\tag{34}$$

$$r^{\mathrm{T}}\tilde{B}^{\mathrm{T}}M\tilde{B}r=\tilde{B}r_M^2\le K_B^2e^TMe\tag{35}$$

Finally, derivative of the Lyapunov function is as follows:

$$\begin{aligned}\dot{V}\le&e^{\mathrm{T}}\Big(A_c^{\mathrm{T}}(\hat{x})M+MA_c(\hat{x})+(M_1-M_2)/\Delta_k\\&+\left(2+2K_A^2+K_B^2\right)M+2A_c^{\mathrm{T}}(\hat{x})MA_c(\hat{x})\Big)e\end{aligned}\tag{36}$$

By adding $\pm\delta V$ to the derivative, we have:

$$\dot{V}\le e^{\mathrm{T}}\left(\Sigma+2A_c^{\mathrm{T}}(\hat{x})MA_c(\hat{x})\right)e+\delta V\tag{37}$$

as condition (20) is satisfied and $\dot{\eta}(t)=\delta\ge 0$ is defined, first condition of Theorem 2 is satisfied. At $t=\tau_k^+$, Lyapunov function is as follows

$$V\left(\tau_k^+\right)=e^{\mathrm{T}}\left(\tau_k^+\right)M\left(\tau_k^+\right)e\left(\tau_k^+\right)$$
$$=e^{\mathrm{T}}\left(\tau_k\right)\left(I-L\left(\hat{x}\right)C_y\right)^{\mathrm{T}}M_2\left(\mathrm{I}-L\left(\hat{x}\right)C_y\right)e\left(\tau_k\right) \tag{38}$$

as the following condition is satisfied

$$\left(I-L\left(\hat{x}\right)C_y\right)^{\mathrm{T}}M_2\left(\mathrm{I}-L\left(\hat{x}\right)C_y\right)\le\beta M_1 \tag{39}$$

which is the LMI condition (22) $V\left(\tau_k^+\right)\le\beta V\left(\tau_k\right)$ where $\varsigma_k=\beta\ge 0$. Therefore, second condition of Theorem 2 is satisfied. As the third condition is satisfied, upper bound of the distance between impulses is obtained:

$$\Delta_{k\max}=-\ln\left(\gamma\beta\right)/\delta \tag{40}$$

V. SIMULATION RESULTS

In this section, the simulation results of the SDIO and Riccati equation controller on the manipulator are represented. Outputs of the system are angles of joints and the objective is to bring them to the reference trajectories. The sample time is considered $T_s=10^{-3}$ in the simulations. The SDRE parameters are considered as $R=\mathrm{diag}(10^{-2},10^{-2})$, $Q=\mathrm{diag}(10^{2},10^{2})$. The design parameters of SDIO are chosen as $\delta=5$, $\beta=0.95$, $\gamma=1.01$. Considering the value of design parameters, the upper bound of the distance between impulses is calculated as $\Delta_{k\max}=\left(-\ln\left(1.01\times 0.95\right)\right)/5=0.0083$. Thus, the fixed impulse interval is chosen $\Delta_k=0.005$ where $\Delta_k<\Delta_{k\max}$. The initial conditions of the real and estimated states are considered as $x=\left[1.6,0,0,0\right]^{\mathrm{T}}, \hat{x}=\left[0,2,0,0\right]^{\mathrm{T}}$.

Angles of the joints are shown in Fig. 1 and Fig. 2 and the tracking and estimation errors of joint angles are shown in Fig. 3 to Fig. 6. As it is shown, although output of all 5 samples is known, a proper estimation of the states is obtained and the reference trajectory is tracked with proper speed and accuracy. State impulses which are created when the output information is introduced, are also shown in Fig. 7 and Fig. 8. At the initial stages, the amplitude of the impulses is higher due to higher difference between real states and the estimated states and as this difference is reduced, the amplitude of the impulses is also reduced. The torque of joint angles (the controller output) are plotted in Fig. 9 and Fig. 10. As it is shown the value of torques are in the allowed range. The signals are enough smooth to be produced by system actuators.

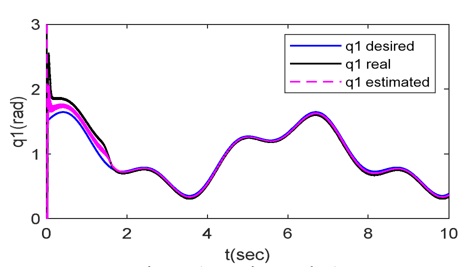

Figure 1. Joint angle 1

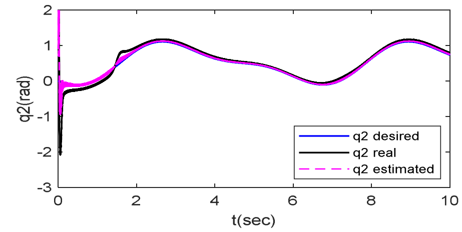

Figure 2. Joint angle 2

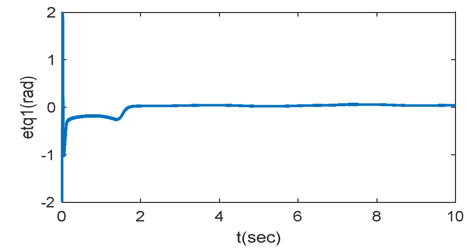

Figure 3. Tracking error of joint angle 1

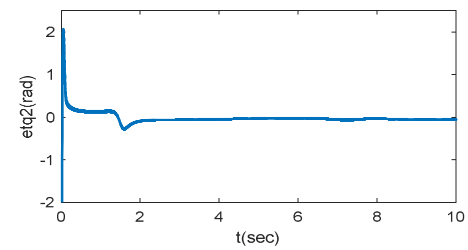

Figure 4. Tracking error of joint angle 2

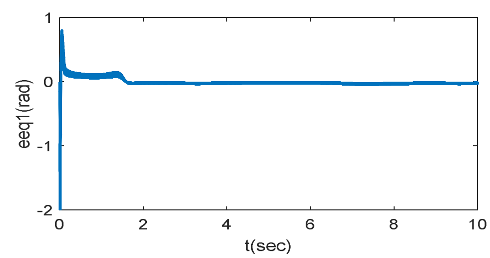

Figure 5. Estimation error of joint angle 1

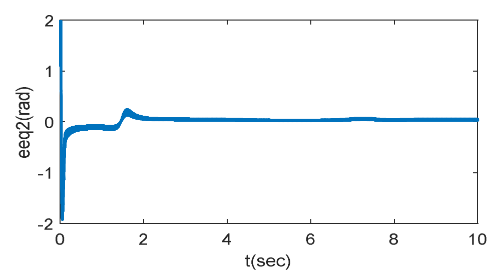

Figure 6. Estimation error of joint angle 2

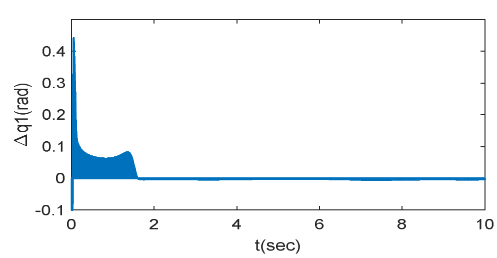

Figure 7. Jump of joint angle 1

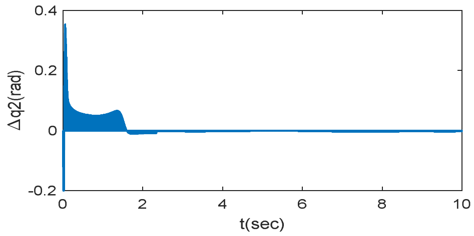

Figure 8. Jump of joint angle 2

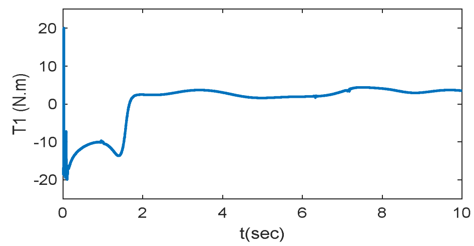

Figure 9. Torque of joint angle 1

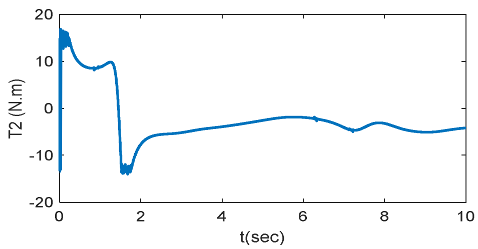

Figure 10. Torque of joint angle 2

VI. CONCLUSION

In this paper, an optimal state dependent Riccati equation controller based on SDIO is designed for controlling a manipulator. The ability to design this controller for a wider range of nonlinear systems and less conservative conditions in preserving stability are two main advantages of the proposed method. In addition, the proposed observer can present a continuous estimation of states even when the output is not accessible at each sampling time with time-varying rates. Simulation results indicate estimation and tracking with proper accuracy despite knowing the output at all 5 samples.

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