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Design of state-dependent impulsive observer for non-linear time-delay systems

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Nasrin Kalamian[1], Hamid Khaloozadeh[1] ✉, Moosa Ayati[2]
[1]Department of Systems and Control Engineering, K.N. Toosi University of Technology, Tehran, Iran
[2]School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran
✉ *E-mail: h_khaloozadeh@kntu.ac.ir*

Abstract: In this study, a new state-dependent impulsive observer (SDIO) is proposed for a class of non-linear time-delay systems. The proposed observer is based on the extended pseudo-linearisation technique that parameterises the non-linear time-delay system to a pseudo-linear structure with time delay and state-dependent coefficients. Applying this technique, the presented observer is utilised for non-linear systems with multiple, time-varying and distributed delays. Furthermore, the extended pseudo-linearisation technique simplifies the procedure of impulsive observer design for non-linear time-delay systems. The proposed SDIO is capable of continuously estimating system states using discrete samples of the system output that are available at discrete impulse times. The stability and convergence of the proposed observer are proven via a theorem utilising time-varying and delay-independent Lyapunov function and the comparison system theory of impulsive systems. It is guaranteed that the estimation error asymptotically converges to zero under well-defined and less-conservative sufficient conditions that are presented in terms of feasible linear matrix inequalities. In addition, the stability theorem specifies an upper bound on the time intervals between consecutive impulses. Results are simulated on Congo Ebola disease model which is an epidemic non-linear time-delay system. Simulation results confirm the effectiveness and performance of the proposed SDIO.

1 Introduction

During recent decades, the impulsive systems theory has attracted the attention of many researchers in control theory field. Impulsive systems are a subclass of the hybrid systems and consist of both continuous and discontinuous parts. The continuous dynamical behaviour of impulsive system is presented by continuous differential equations defined between impulse intervals. The discontinuous dynamical behaviour is described by difference equations defined at impulse times when, the system states suddenly change.

Recently, based on impulsive system theory and its hybrid characteristics, the impulsive observers were developed. Classical continuous-time observers update the system states estimation continuously and thus they require continuous output measurement. On the other hand, classical discrete-time observers only update the states estimations at specific times using discrete output measurement. Interestingly, impulsive observers update the states estimation continuously according to system dynamics while discrete output measurement is used. In many practical applications of the control systems, only discrete-time measurements are feasible [1]. For instance, in the chemical and economic processes, biological applications such as distribution of the drug in the human body and impulsive vaccination, chaos communication systems, renewable resources management, biological neural networks, population ecology, rhythmic models of pathology, the modulated frequency signal processing systems, flying objects and so on, output measurements are available at discrete time instants and the time intervals between instances are not necessarily constant, equal or even known [2]. Some advantages of impulsive observer are as follows [3, 4],

- Unlike the continuous observers, the impulsive observers utilise discrete output measurement for state estimation update. Thus, these observers estimate the states of the system even if data transmission cannot continually go on.
- Impulsive observers reduce transmitted data from the system to the observer and therefore, the communication channel bandwidth is reserved.
- Reduction of transmitted data between system and observer, the cost of data transmission is decreased and channel capacity is increased.

As it is mentioned in [5], the impulsive observers is realisable if the observer is implemented on a microcontroller chip since in that case the full observer state is easily set to any value at any impulse time. The design of the impulsive functional observer for linear systems is proposed in [3]. The stability analysis is presented by considering a new piecewise differentiable Lyapunov function. The sufficient conditions of exponential stability of the proposed observer are derived in terms of linear matrix inequalities (LMIs). Then, the authors proposed the impulsive observer deign for uncertain linear systems in [6], and a time-varying Lyapunov function is introduced for the stability analysis and sufficient conditions are derived in terms of LMIs. In [7], the continuous-discrete time interval observer is proposed for linear systems with the exsistence of additive disturbances. The asymptotic stability of the observer is investigated and the upper and lower bounds of solutions are given.

In the impulsive observer design for non-linear systems, a restrictive assumption is considered in almost all articles which is, the considered non-linear system should have two separable and additive linear and non-linear parts. This limits the class of under study non-linear systems. For non-linear time-delay systems, the assumption is more restrictive that, time-delay part should be just in linear part or separable from the other parts. Thus, the proposed observers are applicable to some special classes of non-linear time-delay systems.

The observers with impulsive dynamical behaviour for linear and Lipschitz non-linear continuous-time systems are presented in [8]. However, it is assumed that the measured output continuously is available at all times. A time-varying Lyapunov function is used for analysis of the stability theorem. The impulsive observer design for non-linear systems is investigated in [9]. The presented observer shares the structure as same as a Luenberger observer with a rule for updating the observer gain. The proposed observer is simulated on a flexible joint robotic arm. In [10], the impulsive observer with timevarying gain is presented for non-linear systems. The stability of the proposed observer is investigated utilising small gain arguments. The design of continuous-discrete observer is presented for continuous-time non-linear time-varying systems with discrete measurements [11]. It is shown that the solution of the proposed observer using the notion of cooperative systems converges to the solution of the original system under sufficient conditions on the non-linear terms and the maximum sampling interval. In [12], the authors presented an impulsive continuous-discrete time observer for a class of uncertain non-linear systems. The output measurements are available in constant sampling intervals. The performance of the proposed observer is shown by some simulations on biochemical reactors. The impulsive observer design is investigated in [13] for a class of non-linear Lipschitz system. The event-triggered technique is used in order to reduce the network usage. It is considered that the measured output is available for every constant sampling interval but, the data is transferred by an event-triggered mechanism. The proposed observer is tested on a flexible joint robot.

Adaptive impulsive observer (AIO) design is proposed for chaotic system synchronisation problem [4, 5, 14]. The proposed AIO is able to estimate both the states and unknown parameters of the uncertain system. The comparison system theorem is used to analysis the stability of the AIO that leads to less-conservative sufficient conditions [5]. The proposed observer is improved [4, 14] and the stability conditions are presented in terms of LMIs. Also, the stochastic AIO is developed in [15]. It is shown in [16] that the stability sufficient conditions of the proposed AIO cannot be satisfied. In [17], AIO stability analysis is revised applying an improved time-varying Lyapunov function, in conjunction with the application of a generalised version of Barbalat's Lemma. Also, the sufficient conditions are formulated in terms of LMIs. In [18], an adaptive observer is designed for a class of uniformly observable non-linear systems with sampled output. The proposed impulsive observer is combined of a continuous-time observer coupled and an intersample output predictor.

In [19], an impulsive observer with variable update interval is proposed for Lipschitz non-linear time-delay systems. A novel discontinuous and delay-dependent Lyapunov function and Razumikhin-type technique are applied to analyse the stability, and sufficient conditions are presented using LMIs. The results are presented in two cases: (i) the delay is bounded, (ii) the delay and its time-derivative are bounded. The proposed stability conditions depend on both the lower and the upper bounds of the update intervals. Thus, the proposed conditions are less-conservative than the delay-free conditions in [6]. In [20], the observer design for discrete-time non-linear impulsive switched systems with time-varying delay is investigated. A delay-dependent Lyapunov-Krasovskii function is considered to analyse the stability, and sufficient conditions are established using the average dwell time approach and LMIs. The preliminary model of the state-dependent impulsive observer (SDIO) is presented in [21]. The comparison system theory and eigenvalue approach are used for stability analysis of the proposed SDIO.

As it is seen, in researches of the impulsive observer design for non-linear systems, only special and confined classes of these systems are discussed. Actually, for simplifying of routine of stability analysis and calculation of sufficient conditions, the equation of the system is limited to have a linear part. These conditions for non-linear time-delay systems are more restrictive. In this paper, this problem has been largely resolved using extended pseudo-linearisation technique. This technique is the factorisation procedure of non-linear time-delay system into a pseudo-linear structure. Applying the extended pseudo-linearisation approach, the presented observer can be extended for several kinds of non-linear time-delay systems such as systems with multiple discrete delays, time-varying delay and distributed delay. So, the proposed SDIO based on extended pseudo-linearisation can be applied for a wider class of non-linear time-delay systems. Another advantage of the extended pseudo-linearisation approach is that, irrespective of the delay value, the proposed observer is asymptotically stable. Thus, in this paper, unlike the previous methods, the delay value is not bounded. In addition, using this technique, there is no need to employ delay-dependent Lyapunov function thus, the routine of impulsive observer design for non-linear time-delay systems is simplified. Also, for multivariable systems, there are an infinite number of the extended pseudo-linearisation factorisation. This matter causes additional degrees of freedom in the procedure of the observer design, which can be utilised to avoid the observability reduction or increase the observer performance.

In the stability theory of impulsive systems, it has been shown that it is not necessary to have a non-positive time-derivative of Lyapunov function. This is formulated as the comparison system theory of impulsive differential equation systems and its corollaries [1, 22, 23]. Therefore, considering this theory, the sufficient conditions for the stability analysis are less-conservative. In this paper, the stability analysis of the SDIO is investigated using time-varying delay-independent Lyapunov function and the comparison system theory. It is shown that under some well-defined and less-conservative sufficient conditions that are presented in terms of feasible LMIs, the estimation error converges to zero, exponentially.

In previous works such as [17], the minimum and maximum impulse intervals are regarded as known parameters. However, in SDIO, the proposed stability theorem specifies the maximum interval between impulses. The minimum impulse interval is free, and the maximum is calculated by the third condition of the comparison system theory. Furthermore, in the third condition, the maximum impulse interval is related to three design parameters making a trade-off between increasing the maximum impulse intervals and the conservatism of sufficient conditions. It is worth mentioning that the stability problem of the AIO design is solved using the proposed approach.

In [19, 20], impulsive observer design is investigated for non-linear time-delay systems in a special class with four separated linear without delay, linear with delay, non-linear without delay and non-linear with delay parts. But, there are some non-linear time-delay systems that are not formulated in this form. Thus, neither of designed impuslive observers is not utilisable for them. As a novelity and solution, the proposed SDIO is presented for these systems. To prove this claim, the proposed SDIO is simulated on the SIR epidemic non-linear time-delay model for Congo Ebola disease.

The paper is organised as follows. In Section 2, the basic concepts of impulsive systems are presented. Also, the stability theorem of the comparison system and its corollary are expressed in this section. The non-linear time-delay system equation and extended pseudo-linearisation technique are explained in Section 3. Moreover, the condition of existence of extended pseudo-linearisation is reviewed. In Section 4, the SDIO and the sufficient conditions of the stability are proposed. Furthermore, the upper bound of the updating intervals is presented in this section. The simulation results of Congo Ebola as a non-linear time-delay system are shown in Section 5. At the end, in Section 6, the conclusion of this paper is presented.

1.1 Notation

The following notations are used throughout this paper. R^n denotes the n-dimensional real space and $R^+ = [0, +\infty)$. I is the identity matrix and A^{T} is the transpose of matrix A. For any $\rho \in R^+$, $S_\rho = x \in R^n \mid \| x \| < \rho$, where $\| \ . \ \|$ denotes the Euclidean norm. $\| x \|_P^2 = x^{\mathrm{T}} P x$ is the norm induced by a symmetric positive definite matrix P. C denotes the set of all continuous functions. C^i is the class of all continuous functions $a(x)$, where they are i times continuously differentiable with respect to x. Also, $C^{i,j}$ is the class of all continuous functions $a(t, x)$, where they are i and j times continuously differentiable with respect to t and x, respectively. $a(t, x)$ belongs to class κ if, $a \in C[R^+, R^+]$ ($a \in C$ and $a: R^+ \to R^+$), $a(t, 0) = 0$ and a is strictly increasing in x.

2 Basic concepts of impulsive systems

The impulsive system is described by following the impulsive differential equation as

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & t \neq t_k \\ \Delta x = f_I(x(t)), & t = t_k \end{cases} \tag{1}$$

where $x \in R^n$ is the state vector, t is the time variable and $f\colon R^+ \times R^n \to R^n$, $f_I\colon R^n \to R^n$ are non-linear functions with compatible dimensions. $t_k, k = 1, 2, \ldots$ are the impulse times that $t_k > t_{k-1} > 0$. The state jump vector at the impulse time is $\Delta x(t_k) = x(t_k^+) - x(t_k)$. It is assumed that $x(t_k) = x(t_k^-)$.

In the stability theory of impulsive systems, semi-negative definiteness of Lyapunov function time-derivative is not necessary. Thus, sometimes the Lyapunov function may increase in some time intervals but, the stability of system is still reserved. The following definitions and a theorem are presented [1].

Definition 1: $V\colon R^+ \times R^n \to R^+$ belongs to class V_0 if

1. V is continuous in $(t_{k-1}, t_k] \times R^n$ and for each $x \in R^n$, k=1,2,…, $\lim_{(t, y) \to (t_k^+, x)} V(t, y) = V(t_k^+, x)$ exists;
2. V is locally Lipschitz in x.

Definition 2: For $(t, x) \in (t_{k-1}, t_k] \times R^n$, for all k, Dini's derivatives $D^+V(t, x)$, $D^-V(t, x)$ are defined as

$$\begin{aligned} &D^+V(t, x) \\ &\quad = \lim_{h \to 0^+} \sup \frac{1}{h}\Big(V(t+h, x+hf(t, x)) - V(t, x)\Big) \\ &D^-V(t, x) \\ &\quad = \lim_{h \to 0^-} \sup \frac{1}{h}\Big(V(t+h, x+hf(t, x)) - V(t, x)\Big) \end{aligned} \tag{2}$$

It is worth mentioning using Dini's derivatives instead of conventional derivatives, the concept of derivative can be extended to certain classes of discontinuous functions that exist in impulsive systems.

Note 1: If $V \in C^1[R^+ \times R^n, R^+]$, then

$$D^+V(t, x) = D^-V(t, x) = \frac{\partial V(t, x)}{\partial t} + \frac{\partial V(t, x)}{\partial x} f(t, x) \tag{3}$$

Definition 3: The comparison system of (1) is presented as [1]

$$\begin{cases} \dot{w}(t) = g(t, w(t)), & t \neq t_k \\ w(t_k^+) = \psi_k(w(t_k)), & t = t_k \end{cases} \tag{4}$$

where $g\colon R^+ \times R^+ \to R$ is continuous and satisfies definition 1, $\psi_k\colon R^+ \to R^+$ is non-decreasing with considering the following assumption for $V \in V_0$:

$$\begin{cases} D^+V(t, x) \leq g(t, V(t, x)), & t \neq t_k \\ V(t, x + \Delta x) \leq \psi_k(V(t, x)), & t = t_k \end{cases} \tag{5}$$

One of the advantages of comparison system theory is to reduce study of the main system to the study of a simple scalar impulsive system. So, it is easier to investigate the stability problem of the comparison system (4) with one impulsive differential equation instead of the main system [1, 22, 23].

Theorem 1: Suppose that $f(t, 0) = 0$, $f_I(0) = 0$ and $g(t, 0) = 0$ for all k and $t > 0$ so, that we have the trivial solution of the main system (1). Assume that the following conditions are satisfied [1]:

1. $V \in V_0$, $\rho > 0$, $V\colon R^+ \times S_\rho \to R^+$ and in $(t_{k-1}, t_k]$: $D^+V(t, x) \leq g(t, V(t, x))$.
2. There exists a $\rho_0 > 0$ such that $x \in S_{\rho_0}$ implies $x + \Delta x \in S_{\rho_0}$ for all k and $V(t_k, x + \Delta x) \leq \psi_k(V(t_k, x))$.
3. $b(\| x \|) \leq V(t, x) \leq a(\| x \|)$ on $R^+ \times S_\rho$, where $a, b \in \kappa$.

Then, the stability properties of the trivial solution of the comparison system (4) imply the corresponding stability properties of the trivial solution of system (1).

Corollary 1: Let $g(t, V(t, x)) = \dot{\xi}(t)V(t, x)$, where $\xi \in C^1[R^+, R^+]$, $\psi_k(V(t_k, x)) = d_k V(t_k, x)$. The origin of (1) is asymptotically stable if the following conditions are satisfied [1]:

$$\begin{cases} \dot{\xi}(t) \geq 0 \\ d_k \geq 0, \quad k = 1, 2, \ldots \\ \xi(t_k) - \xi(t_{k-1}) + \ln(\gamma d_k) \leq 0, \quad \gamma > 1 \end{cases} \tag{6}$$

It is worth noting, in the corollary of the comparison system theory even if the equalities are established, the asymptotically stability will be guaranteed [1]. Refer to [1], for proof of Theorem 1 and its corollary. The comparison system theory and its corollaries are used for the stability analysis of the impulsive systems in some researches. For example, in [24], the stability analysis of the non-linear impulsive and switching systems with time-delay is investigated based on the corollaries of the comparison system theory. Furthermore, this theorem is employed to design an impulsive controller for discrete non-linear time-delay systems [25].

3 Extended pseudo-linearisation technique

The pseudo-linearisation approach factorises a non-linear system into a linear-like structure with the state-dependent matrices [26, 27]. In [28], the optimal feedback controller is designed for general non-linear systems based on pseudo-linearisation technique. This approach is used to design an optimal tracking controller for super-tankers in autopilot [29]. Moreover, in [30], The state-dependent Riccati equation based on pseudo-linearisation factorisation is proposed to control autonomous underwater vehicles. The pseudo-linearisation technique is extended for non-linear time-delay systems [31–33]. In [31], suboptimal sliding mode controller is designed for non-linear time-delay systems based on extended pseudo-linearisation. Also, based on this approach, suboptimal observer [32] and controller [33] are presented for non-linear time-delay systems. Applying the extended pseudo-linearisation approach, the presented observer can be utilised for several kinds of non-linear time-delay systems such as systems with multiple discrete delays, time-varying delay and distributed delay. Therefore, the proposed SDIO based on extended pseudo-linearisation can be applied for a wider class of non-linear time-delay systems. Another advantage of the extended pseudo-linearisation approach is that, irrespective of the delay value, the proposed observer is asymptotically stable [31–33]. Thus, in this paper, unlike the previous methods, the upper bound of the time-delays is not required. For multivariable systems, there are an infinite number of extended pseudo-linearisation factorisation that cause the flexibility in the observer design [32].

The non-linear time-delay system equation is considered as

$$\begin{cases} \dot{x}(t) = f(x(t), x(t - \tau_1(\theta)), \ldots, x(t - \tau_m(\theta))) \\ y(t) = C_h x(t) \\ x(t) = \varphi_0(t), \; -\max\limits_{\theta, i = 1:m} (\tau_i(\theta)) \leq t \leq 0 \end{cases} \tag{7}$$

where $x \in R^n$ is a continuous state vector, $y \in R^p$ is the output vector. $C_h \in R^{p \times n}$ is the output matrix and $f\colon R^n \times \ldots \times R^n \to R^n$ is a differentiable continuous functions with respect to its argument. The time-delays $\tau_1(\theta) < \ldots < \tau_m(\theta)$ are positive functions with argument θ that is t,x or both. m is the number of delays. $\varphi_0(t)\colon [< dol > < dol > \; - \; \max_{\theta, i = 1:m} (\tau_i(\theta)) < dol > < dol > ,0] \to R^n$ is a continuous function for initial conditions of the system. It is assumed $f(0, x(t - \tau_1(\theta)), \ldots, x(t - \tau_m(\theta))) = 0$ that is satisfied by the states augmentation. For non-linear time-delay system (7), the extended pseudo-linearisation form is presented as

$$\dot{x}(t) = A\Big(x(t), x(t - \tau_1(\theta)), \ldots, x(t - \tau_m(\theta))\Big)x(t) \tag{8}$$

where $A: R^n \times \ldots \times R^n \to R^{n \times n}$ is the state-dependent system matrix [32]. It should be noted that all delayed terms are factorised in the matrix A. For example, consider the non-linear time-delay system with one state as $\dot{x}(t) = x(t)x(t-\tau)$. For the mentioned system, there are two pseudo-linearisation forms as $L_1: A_1(x(t-\tau))x(t)$, that $A_1 = x(t-\tau)$ and $L_2: A_2(x(t))x(t-\tau)$ that $A_2 = x(t)$. But, only L_1 is extended pseudo-linearisation parameterisation. The main advantage of the extended pseudo-linearisation technique is that unlike Jacobian method, does not use any approximations then, the non-linear characteristics of the system are maintained. Moreover, in this method all delayed parts are placed in the state-dependent system matrix. Thus, it is possible to extend the linear methods of observer design to non-linear time-delay systems [33].

Theorem 2: Assumed that Ω is a bounded open subset of R^n Euclidean space that contains the origin. Let $f: \Omega \to R^n$ such that $f(0, x(t-\tau_1(\theta)), \ldots, x(t-\tau_m(\theta))) = 0$ and $f(x(t), x(t-\tau_1(\theta)), \ldots, x(t-\tau_m(\theta))) \in C^m$, $m \ge 1$. Then, for all $x_t \in \Omega$, an extended pseudo-linearisation form (8) of $f(.)$ always exists for some $A: \Omega \to R^{n \times n}$. Under the above conditions, one extended pseudo-linearisation parameterisation is guaranteed to exist as follows [27, 32]:

$$\begin{aligned} &A\big(x(t), x(t-\tau_1(\theta)), \ldots, x(t-\tau_m(\theta))\big) \\ &= \int_0^1 \frac{\partial f(x(t), x(t-\tau_1(\theta)), \ldots, x(t-\tau_m(\theta)))}{\partial x(t)} \Big|_{x(t) = \lambda x(t)} \mathrm{d}\lambda \end{aligned} \tag{9}$$

where λ is a dummy variable introduced in the integration. For simplifying, the abbreviation of (x_t) is defined as $(x(t), x(t-\tau_1(\theta)), \ldots, x(t-\tau_m(\theta)))$.

As it is mentioned, if the system has more than one state, there are infinite number of the extended pseudo-linearisation parameterisation. This additional degree of freedom in the observer design is presented in the following theorem.

Theorem 3: Assume that there are two extended pseudo-linearisation forms for $f(x_t)$ as $f(x_t) = A_1(x_t)x(t)$ and $f(x_t) = A_2(x_t)x(t)$. Therefore, there is an extended pseudo-linearisation form as $A(x_t, \alpha) = \alpha A_1(x_t) + (1-\alpha)A_2(x_t)$ and for any $\alpha \in R$, $A(x_t, \alpha)$ represents infinite number of the extended pseudo-linearisation forms of $f(x_t)$ [27, 32].

Theorem 4: The extended pseudo-linearisation (8) is a point-wise observable parameterisation of non-linear time-delay system (7) in Ω (the pair $\{A(x_t), C_h\}$ is observable in the linear sense for all $x_t \in \Omega$) if the state-dependent observability matrix that is defined as (10) is full rank ($\mathrm{rank}(\Phi_o(x_t)) = n$) for all $x_t \in \Omega$ [27, 32]

$$\Phi_o(x_t) = \left[C_h \middle| C_h A(x_t) \middle| \ldots \middle| C_h A(x_t)^{n-1}\right]^{\mathrm{T}} \tag{10}$$

4 State-dependent impulsive observer design

The SDIO for non-linear time-delay system (7) is proposed as

$$\begin{cases} \dot{\hat{x}}(t) = A(\hat{x}_t)\hat{x}(t), & t \ne t_k \\ \hat{y}(t) = C_h \hat{x}(t) & \\ \Delta\hat{x}(t) = F(\hat{x}_t)(y(t) - \hat{y}(t)), & t = t_k \end{cases} \tag{11}$$

where $\hat{x}, \hat{y}$ are the estimated state and output vectors, respectively, and $F(\hat{x}_t) \in R^{n \times p}$ is the states impulses gain matrix. For designing of the SDIO, the following assumptions are considered.

Assumption 1: $f(x_t)$ satisfies the following Lipschitz condition:

$$\| f(x_t) - f(\hat{x}_t) \| \le K_f \| x(t) - \hat{x}(t) \| \tag{12}$$

where $K_f \in R^+$ is Lipschitz constant.

Assumption 2: The matrix-valued function $A(x_t)$ is continuous with respect to x.

Assumption 3: The pair $\{A(x_t), C_h\}$ is observable in the linear sense for all $x_t \in \Omega$.

The following lemmas have been used during the proof of the proposed SDIO stability theorem.

Lemma 1: For any matrix $P, D \in R^{n \times n}$ and n-dimensional vectors x, w, the following inequality is satisfied for any positive constant $\varepsilon > 0$ [34]:

$$x^{\mathrm{T}}P^{\mathrm{T}}Dw + w^{\mathrm{T}}D^{\mathrm{T}}Px \le \varepsilon w^{\mathrm{T}}w + \frac{1}{\varepsilon}x^{\mathrm{T}}P^{\mathrm{T}}DD^{\mathrm{T}}Px \tag{13}$$

Theorem 5: Suppose that the Assumptions 1 to 3 are established. The state estimation error $e = x - \hat{x}$ of the presented SDIO by (11) asymptotically converges to zero if the following conditions are satisfied:

$$\begin{bmatrix} \overline{\Sigma}_i & P_i \\ P_i & -\varepsilon I \end{bmatrix} \le 0, \quad i = 1, 2 \tag{14}$$

$$\begin{bmatrix} -\sigma P_1 & P_2 - \overline{F(\hat{x}_t)C_h}^{\mathrm{T}} \\ P_2 - \overline{F(\hat{x}_t)C_h} & -P_2 \end{bmatrix} \le 0 \tag{15}$$

$$\alpha\Delta_k + \ln(\gamma\sigma) \le 0 \tag{16}$$

where $\varepsilon > 0$, $\alpha \ge 0$, $\gamma > 1$ and $\sigma \ge 0$ are constants that satisfy $\gamma\sigma \le 1$. The matrix $P(t) > 0$ is presented as (22) and $\rho(t)$ is defined as [8]

$$\rho(t) = \frac{t_k - t}{\Delta_k}, \quad t \in (t_{k-1}, t_k] \tag{17}$$

where $\Delta_k = t_k - t_{k-1}$ is the kth impulse interval. P_1, P_2 are symmetric positive definite matrices and $\overline{\Sigma}_i$ is defined as

$$\begin{aligned} \overline{\Sigma}_i = {} & A^{\mathrm{T}}(\hat{x}_t)P_i + P_i A(\hat{x}_t) + (P_1 - P_2)/\Delta_k \\ & + 2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) - \alpha P_i \end{aligned} \tag{18}$$

Proof: Using (8) and (11), the dynamic and jump of the state estimation error are

$$\begin{cases} \dot{e}(t) = A(\hat{x}_t)e(t) + (A(x_t) - A(\hat{x}_t))x(t), & t \ne t_k \\ \Delta e(t) = -F(\hat{x}_t)(y(t) - \hat{y}(t)) = -F(\hat{x}_t)C_h e(t), & t = t_k \end{cases} \tag{19}$$

For simplifying, the abbreviation $\tilde{A}$ is defined as $A(x_t) - A(\hat{x}_t)$. The dynamic of the discrete part at impulse times is calculated as

$$\begin{aligned} \Delta e(t_k) &= e(t_k^+) - e(t_k) = -F(\hat{x}_t)C_h e(t_k) \\ &\to e(t_k^+) = (I - F(\hat{x}_t)C_h)e(t_k) \end{aligned} \tag{20}$$

The time-varying Lyapunov function candidate is considered as

$$V(t, e) = \| e(t) \|_P^2 = e^{\mathrm{T}}(t)P(t)e(t) \tag{21}$$

where $P(t)$ is a time-varying periodic and symmetric positive definite matrix as [6]

$$\begin{aligned} P(t) = P(t + \Delta_k), t \in (t_{k-1}, t_k], k = 1, 2, \ldots \\ P(t) = (1 - \rho(t))P_1 + \rho(t)P_2 \end{aligned} \tag{22}$$

Considering this definition at $t = t_k$

$$\rho(t_k) = 0 \to P(t_k) = P_1 \tag{23}$$

Step 1: Consider (16), (37) *and the convergence rate of the states, the design parameters* (α, γ *and* σ) *are chosen.*
Step 2: Calculate the matrix-value $A(\hat{x}_t)$ *and then solving LMI* (14) *to find the value of* $P(t)$.
Step 3: Solve LMI (15) *to find the value of* F.

Three steps of Algorithm 1 should be implemented every sample time. The YALMIP as a MATLAB toolbox is very efficient in calculating the LMIs.

It is worth noting considering the time-varying Lyapunov function as (21) and time-varying periodic matrix $P(t)$ as (22) the problem of the stability theorem for the AIO design [4] is solved. The asymptotic and exponential stability of the SDIO is presented as remarks 2 and 3, respectively, without the comparison system theory and with the classical approach.

Fig. 1 *Algorithm 1: The following procedure is presented to design the proposed SDIO*

Moreover, at $t = t_k^+$

$$\begin{aligned} P(t_k^+) &= P(t_{k-1}^+ + \Delta k) = P(t_{k-1}^+) \\ \rho(t_{k-1}^+) &= 1 \rightarrow P(t_k^+) = P_2 \end{aligned} \tag{24}$$

Thus, the time-derivative of Lyapunov function at $t \in (t_{k-1}, t_k]$ is

$$\begin{aligned} D^+V(t,e) = {} & e^{\mathrm{T}}(A^{\mathrm{T}}(\hat{x}_t)P(t) + P(t)A(\hat{x}_t))e \\ & + e^{\mathrm{T}}((P_1 - P_2)/\Delta_k)e \\ & + x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}P(t)e + e^{\mathrm{T}}P(t)\tilde{A}x \end{aligned} \tag{25}$$

Utilising Lemma 1, it is concluded

$$x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}P(t)e + e^{\mathrm{T}}P(t)\tilde{A}x \le \varepsilon x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}\tilde{A}x + \frac{1}{\varepsilon}e^{\mathrm{T}}P(t)^2 e \tag{26}$$

With considering Assumption 1, the following result is obtained:

$$\begin{aligned} x^{\mathrm{T}}\tilde{A}^{\mathrm{T}}\tilde{A}x &= \|\tilde{A}x\|^2 = \|(A(x_t) - A(\hat{x}_t))x \pm A(\hat{x}_t)\hat{x}\|^2 \\ &\le 2\|f(x_t) - f(\hat{x}_t)\|^2 + 2\|A(\hat{x}_t)(x - \hat{x})\|^2 \\ &\le 2K_f^2 e^{\mathrm{T}}e + 2e^{\mathrm{T}}A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t)e \end{aligned} \tag{27}$$

Thus, (25) is rewritten as

$$\begin{aligned} D^+V(t,e) \le {} & e^{\mathrm{T}}(A^{\mathrm{T}}(\hat{x}_t)P(t) + P(t)A(\hat{x}_t))e \\ & + e^{\mathrm{T}}((P_1 - P_2)/\Delta_k)e \\ & + e^{\mathrm{T}}(2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) + \frac{1}{\varepsilon}P(t)^2)e \end{aligned} \tag{28}$$

Now the right side of (28) is added to $\pm\alpha V(t,e)$

$$D^+V(t,e) \le e^{\mathrm{T}}\Sigma e + \alpha V(t,e) \tag{29}$$

where

$$\begin{aligned} \Sigma = {} & A^{\mathrm{T}}(\hat{x}_t)P(t) + P(t)A(\hat{x}_t) + (P_1 - P_2)/\Delta_k \\ & + 2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) + \frac{1}{\varepsilon}P(t)^2 - \alpha P(t) \end{aligned} \tag{30}$$

With regard to the definition of Matrices P as (22), Σ is written as

$$\Sigma = \Sigma_1 + \rho\Sigma_2 + \rho^2\Sigma_3 \tag{31}$$

where

$$\begin{aligned} \Sigma_1 = {} & A^{\mathrm{T}}(\hat{x}_t)P_1 + P_1A(\hat{x}_t) + \frac{P_1 - P_2}{\Delta_k} + 2\varepsilon K_f^2 I \\ & + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) + \frac{1}{\varepsilon_1}P_1^2 - \alpha P_1 \\ \Sigma_2 = {} & A^{\mathrm{T}}(\hat{x}_t)(P_2 - P_1) + (P_2 - P_1)A(\hat{x}_t) \\ & + \frac{2}{\varepsilon}P_1(P_2 - P_1) - \alpha(P_2 - P_1) \\ \Sigma_3 = {} & \frac{1}{\varepsilon}(P_2 - P_1)^2 \end{aligned}$$

With regard to $\Sigma_1 \le 0$, $\Sigma_3 \ge 0$ and $0 \le \rho \le 1$, thus, if $\Sigma_1 + \Sigma_2 + \Sigma_3 \le 0$, then, $\Sigma \le 0$. These inequalities are driven to LMIs (14) based on Schur complement lemma [4]. Satisfying of two LMIs of (14), $e^{\mathrm{T}}\Sigma e \le 0$ is obtained, thus

$$D^+V(t,e) \le \alpha V(t,e) \tag{32}$$

where it is considered $\dot{\xi}(t) = \alpha \ge 0$ and thus, the first condition of corollary 1 (6) is satisfied. Due to (24), the Lyapunov function at $t = t_k^+$ is

$$V(t_k^+, e) = e^{\mathrm{T}}(t_k)(I - F(\hat{x}_t)C_h)^{\mathrm{T}}P_2(I - F(\hat{x}_t)C_h)e(t_k) \tag{33}$$

With considering the following condition:

$$(I - F(\hat{x}_t)C_h)^{\mathrm{T}}P_2(I - F(\hat{x}_t)C_h) \le \sigma P_1 \tag{34}$$

Utilising Schur complement lemma [4], (34) is rewritten as LMI as (15) where $F(\hat{x}_t)C_h = P_2^{-1}\overline{F(\hat{x}_t)C_h}$. The Lyapunov function is

$$V(t_k^+, e) \le e^{\mathrm{T}}(t_k)\sigma P_1 e(t_k) = \sigma V(t_k, e) \tag{35}$$

where $d_k = \sigma \ge 0$ and the second condition of corollary 1 (6) is satisfied. In the end, according to the third condition of corollary 1 (6)

$$\alpha t_k - \alpha t_{k-1} + \ln(\gamma d_k) \le 0 \rightarrow \ln(\gamma d_k) \le -\alpha\Delta_k \tag{36}$$

With regards to $\alpha\Delta_k \ge 0$ and $\gamma > 1$, the argument of ln function should be less than or equal to ;1 hence, it is concluded $\gamma d_k \le 1$, which is satisfied by $\sigma\gamma \le 1 \rightarrow \sigma \le 1$. □

Remark 1: The maximum distance of the impulses is presented as

$$\Delta_k^{\max} = \max_{k = 1,2,\dots}(t_k - t_{k-1}) = \left|\frac{\ln(\gamma\sigma)}{\alpha}\right| \tag{37}$$

Three steps of Algorithm 1 (see Fig. 1) should be implemented every sample time. The YALMIP as a MATLAB toolbox is very efficient in calculating the LMIs.

It is worth noting considering the time-varying Lyapunov function as (21) and time-varying periodic matrix $P(t)$ as (22) the problem of the stability theorem for the AIO design [4] is solved. The asymptotic and exponential stability of the SDIO is presented as Remarks 2 and 3, respectively, without the comparison system theory and with the classical approach.

Remark 2: The state estimation error of the proposed SDIO by (11) asymptotically converges to zero if the following conditions and LMI (15) are satisfied:

$$\begin{bmatrix} \overline{\Sigma}_i & P_i \\ P_i & -\varepsilon I \end{bmatrix} \le 0, \quad i = 1, 2 \tag{38}$$

where $\overline{\Sigma}_i$ is defined as

Table 1 SIR model parameters

Parameter	Value	Parameter	Value
μ	0.014 day^{-1}	b	0.07 day^{-1}
d	0.0123 day^{-1}	β	0.21 day^{-1}
λ	0.0476 day^{-1}	τ	10 day
K	10,900		

$$\begin{aligned}\overline{\Sigma}_i' = A^{\mathrm{T}}(\hat{x}_t)P_i + P_iA(\hat{x}_t) + (P_1 - P_2)/\Delta_k \\ +2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t)\end{aligned} \tag{39}$$

Proof: Equation (28) can be rewritten as

$$D^+V(t,e) \le e^{\mathrm{T}}\Sigma' e \tag{40}$$

where

$$\begin{aligned}\Sigma' = A^{\mathrm{T}}(\hat{x}_t)P(t) + P(t)A(\hat{x}_t) + (P_1 - P_2)/\Delta_k \\ +2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) + \frac{1}{\varepsilon}P(t)^2\end{aligned} \tag{41}$$

Satisfying two LMIs of (38), $e^{\mathrm{T}}\Sigma' e$ is obtained, thus $D^+V(t,e) \le 0$. It is worth noting, with the existence of α in Σ, the LMI condition of (14) is satisfied easier than (38) and the Theorem 5 presents less-conservative sufficient conditions than the classical approach in [8]. Besides, (16) presents the upper bound of the time interval of impulses. This condition shows that the maximum distance of the impulses has the inverse relation with α. Thus, there must be a tradeoff between a bigger upper bound of the time interval of impulses and solving the LMIs (14). □

Remark 3: The state estimation error of the proposed SDIO by (11) exponentially converges to zero if the following conditions and LMI (15) are satisfied:

$$\begin{bmatrix}\overline{\Sigma}_i'' & P_i \\ P_i & -\varepsilon I\end{bmatrix} \le 0, \quad i = 1,2 \tag{42}$$

where $\delta > 0$ and $\overline{\Sigma}_i''$ are defined as

$$\begin{aligned}\Sigma_i'' = A^{\mathrm{T}}(\hat{x}_t)P_i + P_iA(\hat{x}_t) + (P_1 - P_2)/\Delta_k \\ +2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) + \delta P_i\end{aligned} \tag{43}$$

Proof: The right side of (28) is added to $\pm\delta V(t,e)$, so

$$D^+V(t,e) \le e^{\mathrm{T}}\Sigma'' e - \delta V(t,e) \tag{44}$$

where

$$\begin{aligned}\Sigma'' = A^{\mathrm{T}}(\hat{x}_t)P(t) + P(t)A(\hat{x}_t) + (P_1 - P_2)/\Delta_k \\ +2\varepsilon K_f^2 I + 2\varepsilon A^{\mathrm{T}}(\hat{x}_t)A(\hat{x}_t) + \frac{1}{\varepsilon}P^2 + \delta P(t)\end{aligned} \tag{45}$$

Satisfying two LMIs of (42), $e^{\mathrm{T}}\Sigma'' e \le 0$ is obtained and $D^+V(t,e) \le -\delta V(t,e)$. As a result, the Lyapunov function is calculated as

$$V(t,e) \le V(t_{k-1},e)\exp(-\delta(t - t_{k-1})); t_{k-1} < t \le t_k \tag{46}$$

which monotonically decreases between impulses. □

5 Simulation results

In this section, the effectiveness of the proposed SDIO is illustrated by numerical simulations. The susceptible-infected-recovered (SIR) epidemic non-linear time-delay model is considered as [35, 36]

$$\begin{cases}\dot{S}(t) = \left(b - \mu\dfrac{rN(t)}{K}\right)N(t) - \dfrac{\beta S(t)I(t-\tau)}{N(t-\tau)} \\ -\left(d + (1-\mu)\dfrac{rN(t)}{K}\right)S(t) \\ \dot{I}(t) = -\left(d + (1-\mu)\dfrac{rN(t)}{K} + \lambda\right)I(t) \\ +\dfrac{\beta S(t)I(t-\tau)}{N(t-\tau)} \\ \dot{R}(t) = \lambda I(t) - \left(d + (1-\mu)\dfrac{rN(t)}{K}\right)R(t)\end{cases} \tag{47}$$

where S,I,R are susceptible, infected and recovered individuals, respectively. $N(t) = S(t) + I(t) + R(t)$ is the number of total population. $b > 0$, $d > 0$, $\lambda > 0$, $\beta > 0$ are the birth, death, recovery and contact rate, respectively. $r = b - d$ is the intrinsic growth rate, μ is convex combination constant, K is the carrying capacity of the population and τ is a non-negative constant represents a time delay on infected individuals I and total individuals N during the spread of disease. The state R is considered as the measured output, so $C_h = [0, 0, 1]$. The value of the parameters in a particular disease Congo Ebola is presented in Table 1 [36]. The following form is one of the an infinite number of extended pseudo-linearisation parameterisation of (47)

$$A(x_t) = \begin{bmatrix} A_{11} & b - \mu\dfrac{rN(t)}{K} & b - \mu\dfrac{rN(t)}{K} \\ A_{21} & A_{22} & -(1-\mu)\dfrac{rI(t)}{K} \\ 0 & \lambda & A_{33}\end{bmatrix} \tag{48}$$

where

$$A_{11} = b - \mu\tfrac{rN(t)}{K} - \tfrac{\beta I(t-\tau)}{N(t-\tau)} - \left(d + (1-\mu)\tfrac{rN(t)}{K}\right)$$
$$A_{21} = \tfrac{\beta I(t-\tau)}{N(t-\tau)} - (1-\mu)\tfrac{rI(t)}{K}$$
$$A_{22} = -\left(d + (1-\mu)\tfrac{rI(t)}{K} + \lambda\right)$$
$$A_{33} = -\left(d + (1-\mu)\tfrac{rN(t)}{K}\right)$$

The benefit of the presented extended pseudo-linearisation form (48) is that the state-dependent observability matrix is full rank. The state-dependent observability matrix is calculated as

$$\Phi_o(\hat{x}_t) = \left[C_h \middle| C_hA(\hat{x}_t) \middle| C_hA(\hat{x}_t)^2\right]^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & \lambda\left(\dfrac{\beta\hat{I}(t-\tau)}{\hat{N}(t-\tau)} - (1-\mu)\dfrac{r\hat{I}(t)}{K}\right) \\ 0 & \lambda & * \\ 1 & * & *\end{bmatrix}^{\mathrm{T}} \tag{49}$$

where * specifies uncalculated elements. The determinant of $\Phi_o(\hat{x}_t)$ is obtained as (50). Due to the parameters values in Table 1, this determinant is always non-zero

$$|\Phi_o(\hat{x}_t)| = -\lambda^2\left(\frac{\beta\hat{I}(t-\tau)}{\hat{N}(t-\tau)} - (1-\mu)\frac{r\hat{I}(t)}{K}\right) \tag{50}$$

As it is mentioned, the maximum impulse interval is related to three design parameters making a trade-off between increasing the maximum impulse intervals and the conservatism of sufficient conditions. The design parameters are considered as $\alpha = 0.068$, $\sigma = 0.98$, $\gamma = 1.01$ and $\varepsilon = 1$. Due to the Remark 1, the maximum impulse interval is calculated as $\Delta_k^{\max} = 10$. In the present simulation results, $\Delta = 5$ is considered and the effect of bigger and smaller impulse intervals are shown in Table 2. It is obvious that increasing the impulse interval and decreasing the number of measurement output, the impulsive observer has less efficiency in the state estimation. Furthermore, due to Algorithm 1 (see Fig. 1), the matrices P_1, P_2 and F are calculated every sample time. The

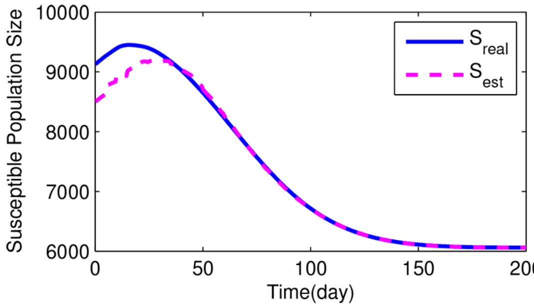

Fig. 2 *Real and estimated susceptible individual by SDIO*

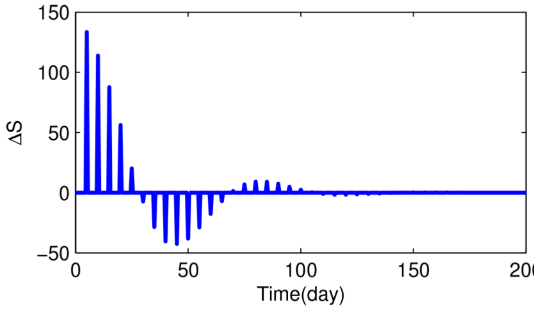

Fig. 3 *Jump of estimated susceptible individual in impulse times*

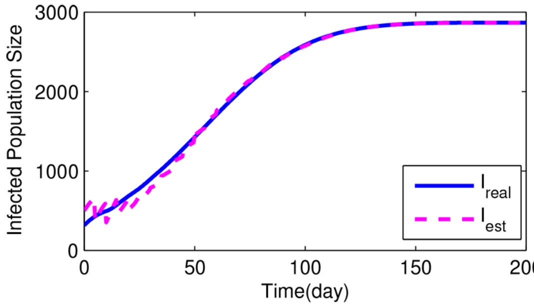

Fig. 4 *Real and estimated infected individual by SDIO*

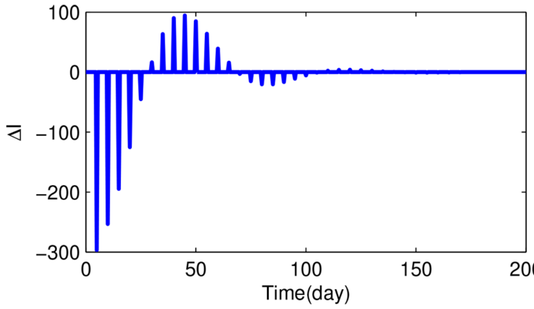

Fig. 5 *Jump of estimated infected individual in impulse times*

initial state is considered as $x(0) = [9126, 315, 59]^{\mathrm{T}}$ and $\hat{x}(0) = [8500, 500, 200]^{\mathrm{T}}$.

In Fig. 2, the real and estimated susceptible individuals using the SDIO are shown. It is obvious that estimated state follows real state even the output is available only in the fifth samples once. Also, the jump values of the estimated susceptible individual are shown in Fig. 3. In impulse times, the output is available and the SDIO updates the states. So, over time, the amplitude of the impulses decreases and the estimated state converges to the real one. Figs. 4 and 5 show the convergence of the estimated infected individual to the real one and its jumps in impulse times. The estimated and real recovered individual are shown in Fig. 6 and its jumps in impulse times are presented in Fig. 7. Fig. 8 shows the real and estimation of the total population and Fig. 9 presents the estimated total population jumps in impulse times.

Table 2 SDIO compared to conventional observer

	Δ (Second)	1	2	5	10
SDIO	NMSE	0.1115	0.1329	0.1906	0.2765
	CC_1	0.9999	0.9999	0.9998	0.9997
	CC_2	0.9982	0.9993	0.9997	0.9996
	CC_3	0.9998	0.9997	0.9996	0.9993
Conventional Ob.	NMSE	0.4722	0.5748	0.7266	0.5261
	CC_1	0.9943	0.9922	0.9814	0.1469
	CC_2	0.9952	0.9910	0.9802	0.5027
	CC_3	0.9981	0.9932	0.9826	−0.0952

Remark 4: As it is mentioned in introduction section, one of the advantages of the proposed method is that the SDIO could be used for a wider class of non-linear time-delay systems with different kinds of delays using the extended pseudo-linearisation technique. Previous researches in the field of impulsive observer design investigated a specific class of non-linear time-delay systems. Mostly, the considered system has four separate parts: linear, linear with delay, non-linear and non-linear with delay [19, 20]. But, there are some systems that they could not be formulated in this form. The proposed SDIO is simulated on the SIR epidemic non-linear time-delay model for Congo Ebola disease. In [19, 20], impulsive observer design is investigated for non-linear time-delay systems. But, Congo Ebola disease cannot be presented in the special class with two separated linear without delay and non-linear with delay parts. In [19], the non-linear time-delay system is considered as follows:

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau(t)) + G_0 f_0(H_0 x(t)) \\ &\quad + G_1 f_1(H_1 x(t-\tau(t)))\end{aligned} \tag{51}$$

where $A_0, A_1, G_0, H_0, G_1, H_1$ are constant matrices with compatible dimensions, f_0, f_1 are non-linear functions and τ is time-delay. The delay is departed in two linear and non-linear sections. But, Congo Ebola disease model that is presented in (47) cannot be formulated as (51). Because, the sentence $(S(t)I(t-\tau))/(N(t-\tau))$ cannot be separated into linear without delay and non-linear with delay or vice versa. The non-linear time-delay system is considered in the same form as in [20]. Therefore, the proposed SDIO can be used for a wider class of non-linear time-delay systems.

In order to illustrate the efficiency of the proposed observer, numerical simulation results of the SDIO and a non-impulsive state-dependent observer are compared. The conventional state-dependent observer is considered as

$$\begin{cases}\dot{\hat{x}}(t) = A(\hat{x}_t)\hat{x}(t) + F(\hat{x}_t)(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_h \hat{x}(t)\end{cases} \tag{52}$$

where $A(\hat{x}_t) - F(\hat{x}_t)C_h$ is point-wise Hurwitz for all $x_t \in \Omega$. The output of system is sampled by constant sampling interval Δ and utilised in both observers. The normalised mean square error (NMSE) and correlation coefficient (CC) are employed to illustrate the performance of estimation of the observers

$$\mathrm{NMSE} = \frac{1}{N}\sum_{k=1}^{N}\sum_{i=1}^{3}\left(\frac{e_i(k)}{\max_k |e_i(k)|}\right)^2 \tag{53}$$

$$CC_i = \frac{\sum_{k=1}^{N} x_i(k)\hat{x}_i(k)}{\sqrt{\left(\sum_{k=1}^{N} x_i^2(k)\right)\left(\sum_{k=1}^{N} \hat{x}_i^2(k)\right)}} \tag{54}$$

where $e_i = x_i - \hat{x}_i, i = 1, 2, 3$. NMSE and CC_i represent the power of the estimation error and the similarity of the real and estimated states. Two signals are completely similar if $CC_i = 1$ and completely different if $CC_i = -1$. As it is shown in Table 2, the state estimated by SDIO is more similar to real state and in $\Delta = 10\,\mathrm{s}$ the conventional observer loses the performance. Increasing the sampling interval, estimation error of both observers

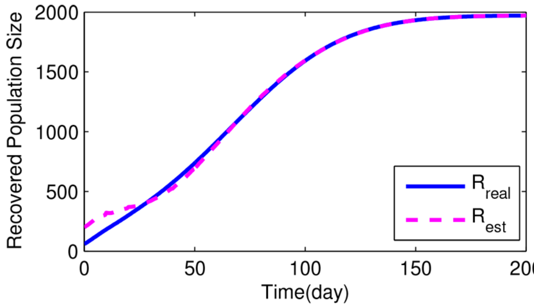

Fig. 6 *Real and estimated recovered individual by SDIO*

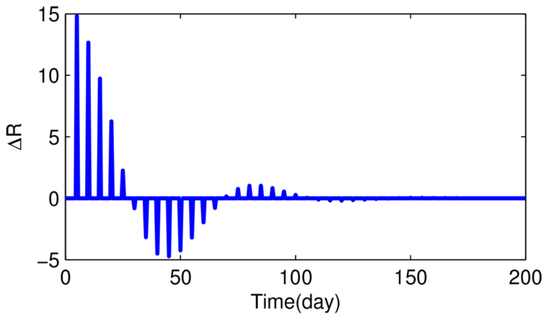

Fig. 7 *Jump of estimated recovered individual in impulse times*

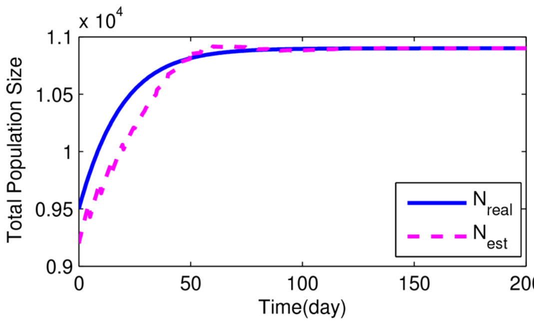

Fig. 8 *Real and estimated total population by SDIO*

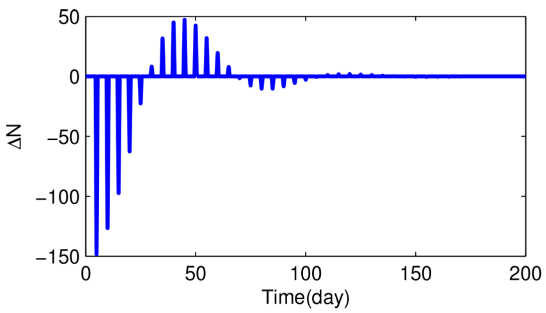

Fig. 9 *Jump of estimated total population in impulse times*

will increase, however, estimation error of SDIO is smaller than the conventional observer especially when the sampling interval has larger values.

6 Conclusion

In this paper, a new SDIO is presented for non-linear time-delay systems based on extended pseudo-linearisation technique. The proposed observer estimated the system states, continuously using system output that was available at discrete and variable impulse instants. The stability analysis of the proposed observer investigated using time-varying and delay-independent Lyapunov function and the comparison system theory of impulsive systems. Two design features of the proposed impulsive observer are worth to be emphasised: first, applicability of the SDIO for a wider class of non-linear time-delay systems with different kinds of delays utilising the extended pseudo-linearisation technique. Thus, there was no requirement to exist a separated linear or delay structure in the original non-linear model. Second, the guaranty of asymptotic convergence of the estimation error to zero under well-defined and less-conservative sufficient conditions that derived in terms of feasible LMIs based on the comparison system theory. The comparison results between the proposed observer and classical one confirmed this matter. In addition, the maximum time interval of impulses is presented. The simulation results of SIR epidemic non-linear time-delay model for Congo Ebola verified the effectiveness of the proposed impulsive observer.

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