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Research article

Adaptive state-dependent impulsive observer design for nonlinear deterministic and stochastic dynamics with time-delays

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ABSTRACT

The present research has introduced a novel adaptive state-dependent impulsive observer (ASDIO) used to control diverse nonlinear systems with time-varying delay. This designed ASDIO is in conformity with the approach of extended pseudo-linearization for the purpose of parametrizing a nonlinear system with time-delay to a pseudo-linear time-delay structure having state-dependent coefficients. This technique makes the ASDIO applicable to nonlinear systems with distributed, multiple, and time-varying delays. The time-varying and delay-independent Lyapunov functional approach, coupled with the comparison method for impulsive systems, was used to confirm the stability of ASDIO. This new theorem affirmed the state and parameter estimation error to approach zero asymptotically through distinct and less-conservative adequate conditions with respect to practical linear matrix inequalities. Furthermore, the maximum impulse time was specified via the presented stability theorem. The ASDIO has also been offered for a special set of stochastic nonlinear systems with time-delay. An investigation of the asymptotic stability for the intended ASDIO was performed via a new theorem employing the comparison principle for stochastic impulsive systems. Accordingly, this observer was simulated on an epidemic system with time-delay nonlinear features to affirm its performance.

1. Introduction

Impulsive systems containing continuous and discontinuous portions are known as a subclass of hybrid systems. The continuous component of an impulsive system can be described using continuous differential dynamics defined on different impulsive intervals. On the other hand, a discontinuous dynamical system is illustrated by difference equations defined for impulses at fixed times in case of abrupt changes in system states [1]. Recently, impulsive observers are developed based on impulsive system theory and its hybrid characteristics. Classical continuous-time observers update the system states estimation continuously and thus require continuous output measurement. In comparison, classical discrete-time observers only update the state estimations at specific times using discrete output measurement.

In [2], the functional observer is designed for linear impulsive systems. Compared to the other available impulsive observers for linear dynamics, this observer has a parameter introducing a degree of freedom that can be employed to minimize conservativeness. In [3], the impulsive observer for linear systems is designed according to the event-triggered feedback controller, and the closed-loop stability analysis is based on the separation principle for linear systems. In [4], the impulsive observer layout is propounded for uncertain linear systems. The minimum and maximum impulse intervals are determined before the design procedure that can be a practical limitation. The continuous- and discrete-time interval observers are proposed for linear dynamical networks considering external disturbances [5]. The asymptotically stable observer is investigated giving the upper and lower bounds of solutions. A periodically time-varying Lyapunov function is used for the first time in [6]. As an achievement, a condition presents the feasibility of LMI corresponding to the observability condition for discrete-time systems.

Like the same articles in this area, only a specific class of nonlinear system that should have a linear part is considered in [7]. A robust sampled data observer is proposed for electro-hydraulic actuators as a nonlinear system with the presence of output disturbances and measurement noises in [8]. [9] represents a maximum allowable sampling period provided by the proposed observer. Nevertheless, a linear portion is necessary for the discussed nonlinear dynamics which is a constraint condition for utilizing the presented observer. In [10], sliding-mode and impulsive observers are designed for a linear impulsive system with the aim of detecting faults and approximations. [11] has employed the event-triggered control for impulsive observers in order to reduce network usage. It should be emphasized that the

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impulse interval is considered as a known and fixed parameter. Furthermore, a specific class of nonlinear systems has been addressed. The impulsive observer layout is highlighted in [12] for continuous-time dynamic on nonlinear time-varying systems utilizing the cooperative dynamical concept. In [13], the stability and existence conditions of full- and reduced-order impulsive observer are examined using average dwell time switching concept and Riccati equation. An impulsive high-gain observer is applied to uncertain nonlinear systems in [14]. The specific uncertain nonlinear class should have separate linear and uncertain parts.

As discussed in the stability theory for impulsive dynamical systems, it is not obligatory to have a non-positive time-derivative of the Lyapunov candidate function. This is formulated as a comparison principle of an impulsive differential equation and several corollaries [1]. Therefore, considering the comparison principle, sufficient conditions to investigate the system's stability have less conservatism. The stability analysis of a fractional-order impulsive control system is investigated in [15]. The stability conditions are presented by comparing the system theory and the fractional-order system concepts. The numerical results confirm the performance of the intended controller for fractional-order impulsive linear and nonlinear systems. Compared to the classic approach, the comparison system theory provides less-conservative sufficient condition for the controller design. The chaotic system synchronization is addressed by an adaptive impulsive observer (AIO) plan in [16,17]. Accordingly, the stability analysis of the AIO is performed using the comparison system theorem which provides less-conservative sufficient conditions [16]. In addition, the adaptive impulsive observer for stochastic dynamics is extended in [18]. As shown in [19], the stability of sufficient conditions of the proposed AIO in [17] cannot be satisfied. In [20], AIO stability analysis is revised by adopting an improved time-varying Lyapunov function candidate coupled with applying a generalized issue of Barbalat's Lemma. The AIO problem proposed in [17] is solved by the use of a time-varying Lyapunov function in [20]. As a constraint on the proposed observer, the minimum and maximum impulse intervals are regarded as known parameters, which can lead to a practical limitation.

The observer design proposed in [21] involves a special group of nonlinear systems, and the minimum and maximum impulse intervals are determined before the design procedure that can make a practical limitation. In [22], a set of time-delay impulsive switched control systems is studied through the finite-time controller. The time-delays are considered as time-varying and unknown delays in the states and state derivatives. However, the proposed controller has the upper bound constraint on the time-delay value and its time-derivative. The adaptive state-feedback controller is outlined to control a set of nonlinear uncertain time-delay impulsive systems [23]. The time-delays appear time-varying while bounded. The present article investigates a specific group of uncertain time-delay systems containing four separate portions of linear and nonlinear components along with linear and nonlinear parts with delays. Moreover, the uncertainty is placed in linear parts. Thus, there are some limitations to utilize the proposed method in a vast variety of nonlinear systems with uncertainty.

[24] presents the impulsive observer layout in two cases: constant and piece-wise constant time-delays. Therefore, a special type of delay (constant and piece-wise constant) placed in the output signal is considered. Lipschitz nonlinear time-delay systems are examined using an impulsive observer with the variable impulsive interval [25]. Time-delays, in the first case, are considered time-varying but bounded, while in the second case, the derivative of the time-delay is also bounded. Accordingly, the proposed impulsive observer has some limitations on delay types, and their time-derivatives, as well as further design parameters, are considered to make some problems in the design procedure of an observer. [26] investigates the observer layout for a specific set of discrete-time switching nonlinear dynamics with delay and impulses. This intended system involves three separate portions including linear, and nonlinear portions along with linear part with delay. Various nonlinear dynamics with time-delay are investigated in [27,28] using the state-dependent impulsive observer (SDIO) connected with the extended pseudo-linearization technique. The stability analysis is also performed on the proposed SDIO using the comparison system theory. Considering a deterministic Lipschitz nonlinear system having time-delay with known parameters, the stability analysis is investigated under the eigenvalues principle that does not generate a specified solution to achieve the proposed conditions.

The current study has proposed the adaptive impulsive observer with state-dependent parameters for a wide range of nonlinear systems including time-delay in accordance with the approach of extended pseudo-linearization. As can be observed, studies on the impulsive observer layout have expounded just for confined and particular groups of nonlinear systems. In fact, to simplify the calculation of sufficient conditions and routine of stability analysis, the equation of the system is restricted to involve a linear section. These restrictions are more limiting to nonlinear time-delay dynamics. The present article has considerably resolved this problem through the extended pseudo-linearization technique which concerns a nonlinear time-delay system to be factorized into a pseudo-linear structure. By employing the extended pseudo-linearization approach, the meant observer can be broadened into a number of nonlinear systems having different time-delays comprising multiple discrete delays, distributed delay, and time-varying delay. Thus, the suggested state-dependent impulsive observer attributed to extended pseudo-linearization can be applied to various nonlinear time-delay systems. Regardless of the delay value, another benefit of the extended approach of pseudo-linearization is asymptotic stability of the suggested observer. Accordingly, in the present paper, the delay value is not bounded, unlike the previous methods. Furthermore, since the delay-dependent Lyapunov function is not required when employing this technique, the design process of an impulsive observer is simplified for the use in nonlinear time-delay systems. Moreover, there are enormous extended pseudo-linearization factorizations for multivariable systems which cause more degrees of freedom for the process of designing an observer resulting in increased performance for the observer or avoidance of the observability reduction.

It has been demonstrated that a Lyapunov function's non-positive time-derivative is unnecessary for the stability theory on impulsive systems. This is formulated as the comparison system theory for impulsive differential equations and its corollaries [1,29–33]. Thus, this theory reveals less-conservative adequate conditions for stability analysis. The present article carries out the SDIO stability analysis through comparison system theory besides time-varying delay-independent Lyapunov function. As demonstrated, under a number of less-conservative and explicit adequate conditions presented in terms of available LMIs, the asymptotic estimation error approaches zero. Additionally, this suggested stability theorem determines the maximum interval between impulses. The similar ASDIO structure is employed for stochastic nonlinear systems with time-delay to evaluate unknown parameters and states. The sufficient conditions of ASDIO are derived from the presented theorem on the basis of the comparison principle of stochastic impulsive dynamics. Overall, the properties and advantages of the proposed ASDIO are summarized as follows:

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- The ASDIO would assess the states and parameters continuously.
- The ASDIO is applicable in a vast group of nonlinear systems with different time-delay types considering the extended pseudo-linearization approach.
- The time-delay value does not require the upper bound.
- There are enormous extended pseudo-linearization factorizations for multivariable systems leading to more degrees of freedom in the process of designing an observer which are applied to avoid the observability reduction or to increase the observer performance.
- The asymptotic convergence of the estimation error for both state and parameter to zero is guaranteed under explicit and less-conservative adequate conditions that are presented concerning feasible LMIs attributed to the comparison system theory.
- Using the comparison system theory, the ASDIO design procedure is easier than conventional impulsive observers.
- There is no requirement to employ a delay-dependent Lyapunov function; thus making the designing process of an impulsive observer more simplified in delayed nonlinear systems.
- The stability theorem determines an upper bound of time intervals between consecutive impulses, unlike existing methods where the minimum and maximum impulse intervals are regarded as known parameters.
- The stability theorem contains the special case under Lipschitz condition and the general case of the sector bounded condition. Thus, less-conservative sufficient condition and LMIs with larger feasibility are generated.
- The ASDIO can evaluate unknown parameters and states for a vast group of stochastic nonlinear time-delay systems under less-conservative adequate conditions stated regarding available LMIs.

The recent paper outline is provided in the following order: Section 2 explains the system equation of a nonlinear time-delay dynamics as well as extended pseudo-linearization approach. Also, conditions for the existence of extended pseudo-linearization are reviewed in this section. The stability analysis is described using the comparison system theorem and its corollary in Section 3. Section 4 introduces the adaptive state-dependent impulsive observer and the asymptotic stability of the intended ASDIO under sufficient conditions. Moreover, the updating intervals' upper bounds are represented in this section. In Section 5, the suggested ASDIO is simulated on an uncertain nonlinear time-delay system. At last, Section 6 highlights the concluding remarks of this paper.

1.1. Notation

The notations used through this paper are described hereafter: R^n signifies the real space of n-dimensions and $R^+ = [0, +\infty)$. I stands for the identity matrix, besides A^T as the transpose of a matrix A. $\|x\|$ denotes the Euclidean norm of the vector x. For any $\rho \in R^+$, $S_\rho = \{x \in R^n \big| \|x\| < \rho\}$ and for a random vector y, $\mathcal{S}_\rho = \{y \in R^n \big| E\left(\|y\|\right) < \rho\}$. C denotes a set of all continuous functions. C^i is the collection of all i-times continuously differentiable functions defined on x for all continuous functions of $a(x)$. $C^{i,j}$ is the collection of all i-times continuously differentiable functions defined on t, as well as the collection of all j-times continuously differentiable functions defined on x, for all continuous functions of $a(t,x)$. A continuous function $a(t,x)$, $a \in C[R^+, R^+]$, belongs to class κ ($a \in C$ and $a : R^+ \to R^+$), thus, a is strictly increasing in x and $a(t,0) = 0$.

2. Extended pseudo-linearization technique

With regards to research papers concerning the impulsive observer scheme to control nonlinear dynamics, the confined and special classes of these systems are merely discussed. A restrictive assumption of almost all of these articles is the necessity of two distinct linear and nonlinear portions within a nonlinear system. Actually, a linear component of the equation of the system limits simplifies the calculation of adequate conditions and the stability analysis. There is a more constraining assumption for nonlinear time-delay systems that time-delay part should be just in linear part or separable from the other parts. In this respect, these observers are applicable for some specific classes of delayed nonlinear dynamics. This paper solved this problem considering the extended pseudo-linearization technique.

Ideally, a nonlinear system (time-delay nonlinear system) is factorized into an almost linear structure that has state-dependent matrices using the pseudo-linearization (extended pseudo-linearization) approach [34–36]. By applying an extended pseudo-linearization approach, the presented observer can be employed for nonlinear systems with different kinds of time-delay, including time-varying delay, multiple discrete delays, and distributed delay. Therefore, the proposed impulsive state-dependent observer can be utilized with regards to extended pseudo-linearization to various nonlinear dynamics containing time-delay. As another gain for this approach, the intended observer is asymptotically stable irrespective of the delay value [37]. Thus, unlike the previous methods, this paper does not require the upper bound of the time-delays. There is enormous extended pseudo-linearization factorization for multivariable systems that causes the flexibility in the observer design [35–37].

The pseudo-linearization technique has been employed for designing an optimal tracking controller with an application of super-tankers in autopilot [38]. [39] proposes the state-dependent Riccati equation regarding pseudo-linearization factorization to control autonomous underwater vehicles (AUVs). Moreover, this technique is employed to design a suboptimal tracker to control blood glucose values in type 1 diabetic patients [40]. In [41,42], nonlinear time-delay systems are developed using the pseudo-linearization technique, so that [41] has used a sub-optimal sliding mode controller, while [42] presents a sub-optimal observer based on this approach.

Suppose a nonlinear dynamic with time-delays:

$$\begin{aligned} &\dot{x}(t) = f\big(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big) \\ &y(t) = C_h x(t) \\ &x(t) = \varphi_0(t), \quad -\max_{\nu, i=1:m}\big(\tau_i(\nu)\big) \le t \le 0 \end{aligned} \tag{1}$$

in which, the continuous state vector and output vector are defined as $x \in R^n$ and $y \in R^p$. $C_h \in R^{p\times n}$ represents the output matrix. The function $f : \overbrace{R^n \times \cdots \times R^n}^{m+1} \to R^n$ is the differentiable continuous function defined on its argument. Time delays $\tau_1(\nu) < \cdots < \tau_m(\nu)$ are positive functions of ν that itself is t, x or both. m stands for the number of delays. $\varphi_0(t) : [-\max_{\nu,i=1:m}\big(\tau_i(\nu)\big), 0] \to R^n$ is a continuous function of the system for initiatory conditions. As an assumption, $f\big(x, x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big)|_{x=0} = 0$ is satisfied by the state's augmentation. The extended pseudo-linearization structure of the delayed nonlinear system (1) is:

$$\dot{x}(t) = A\big(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big)x(t) \tag{2}$$

where $A : \overbrace{R^n \times \cdots \times R^n}^{m+1} \to R^{n\times n}$ is the state-dependent system matrix [37]. In this connection, the matrix $A\big(x(t), x(t-$

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$\tau_1(\nu)), \ldots, x(t-\tau_m(\nu)))$ factorizes all delayed terms. Unlike the Jacobian method, the extended pseudo-linearization technique needs no approximations, which is recognized as a principal privilege, thus maintaining the nonlinear characteristics of the system. Furthermore, all delayed parts of this method are placed in the state-dependent matrix. So, the linear methods of designing an observer are likely to be developed to a nonlinear system with time-delay [37].

As mentioned earlier, a multi-state system contains innumerous extended pseudo-linearization parameterizations. This additional degree of freedom in the observer layout is indicated in the following theorem. The subsequent theorems represent the existence condition and non-uniqueness of the matrix A.

Theorem 1. *Assume Ω is an open and bounded subset of R^n Euclidean space containing the origin such that $0 \in \Omega \subseteq R^n$. Remark $f\big(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big) \in C^m$, $m \geq 1$. Then, there is at least one extended pseudo-linearization as following [37]:*

$$A\big(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big) = \int_0^1 \frac{\partial f\big(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big)}{\partial x(t)}\Big|_{x(t)=\lambda x(t)} d\lambda \quad (3)$$

where λ is a dummy variable defined for the integration. Conductive to simplification, the abbreviation (x_t) is expressed as $\big(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))\big)$.

Theorem 2. *Assume two extended pseudo-linearization forms for $f(x_t)$ as $f(x_t) = A_1(x_t)x(t)$ and $f(x_t) = A_2(x_t)x(t)$. Therefore, there is an extended pseudo-linearization form as $A(x_t, \alpha) = \alpha A_1(x_t) + (1-\alpha)A_2(x_t)$ and for any $\alpha \in R$, $A(x_t, \alpha)$, represents enormous extended pseudo-linearization forms of $f(x_t)$ [37].*

Theorem 3. *Assume the matrix-valued function $A(x_t)$ is continuous with respect to x. The following statements are equivalent [37]:*

(a) The extended pseudo-linearization (2) is a point-wise observable parameterization of nonlinear time-delay system (1) in Ω.
(b) The pair $\{A(x_t), C_h\}$ is observable in the linear sense for all $x_t \in \Omega$.
(c) The state-dependent observability matrix that is defined in (4) is full rank (rank $(\Phi_o(x_t)) = n$) for all $x_t \in \Omega$.

$$\Phi_o(x_t) = \Big[C_h \Big| C_h A(x_t) \Big| \ldots \Big| C_h A(x_t)^{n-1}\Big]^T \quad (4)$$

3. Comparison system theory

The design and stability analysis of impulsive systems are performed through the comparison system theorem [1]. One of the advantages of comparison system theory is to limit the overall investigation of the system to an ordinary scalar impulsive dynamic system. So, the stability problem of the comparison system is easier to be examined with one impulsive differential equation instead of the main system [29,30]. This proposed theory provides the stability analysis with adequate conditions that are less-conservative and simpler than an existing impulsive observer. Some research papers report the use of the comparison principle and its corollaries for the stability analysis of impulsive systems. For instance, the stability analysis of the impulsive switching nonlinear systems having time-delay is investigated in [31] with regards to the corollaries of the comparison system theory. This theorem is employed for impulsive controller design in nonlinear discrete time-delay systems [32]. Furthermore, a feedback controller and observer are designed for linear impulsive systems based on a separation principle under the comparison system theory [33]. In the following, the impulsive system equation and then comparison system theory are described. The following impulsive differential equation is used to describe an impulsive system:

$$\begin{aligned} \dot{x}(t) &= f_c\big(t, x(t)\big), \quad t \neq t_k \\ x(t_k^+) &= x(t_k) + f_i\big(x(t_k)\big), \quad t = t_k \end{aligned} \quad (5)$$

where $x \in R^n$ is the state vector with the time variable t. The functions $f_c : R^+ \times R^n \to R^n$ and $f_i : R^n \to R^n$ are nonlinear having compatible dimensions. $t_k, k = 1, 2, \ldots$ is the impulse time so that $t_k > t_{k-1} > 0$. $x(t_k^+)$ represents the state vector after the kth jump. The jump vector of each state at the impulse time is $\Delta x(t_k) = x(t_k^+) - x(t_k)$.

Typically, semi-negative definiteness of Lyapunov function time-derivative is unnecessary for the analysis of the impulsive systems stability. Thus, the Lyapunov function may occasionally be increased but the system stability is still reserved. Some definitions and a new theorem are described in the following to express this characteristic mathematically [1].

Definition 1 (*[1]*). $W : R^+ \times R^n \to R^+$ is member of class $\overline{W}$ if:

(a) W is continuous in $(t_{k-1}, t_k] \times R^n$ and for each $x \in R^n$, $\lim_{(t,y)\to(t_k^+,x)} W(t, y) = W(t_k^+, x)$ exists;
(b) W is locally Lipschitz in x.

Definition 2 (*[1]*). Dini's derivatives for $(t, x) \in (t_{k-1}, t_k] \times R^n$ and for all k are specified as:

$$\begin{aligned} D^+W(t,x) &= \lim_{T\to 0^+} \sup \frac{1}{T}\Big(W\big(t+T, x+Tf_c(t,x)\big) - W(t,x)\Big) \\ D^-W(t,x) &= \lim_{T\to 0^-} \sup \frac{1}{T}\Big(W\big(t+T, x+Tf_c(t,x)\big) - W(t,x)\Big) \end{aligned} \quad (6)$$

If $W \in C^1[R^+ \times R^n, R^+]$, then

$$D^+W(t,x) = D^-W(t,x) = \frac{\partial W(t,x)}{\partial t} + \frac{\partial W(t,x)}{\partial x} f_c(t,x) \quad (7)$$

It is worth mentioning that by utilizing Dini's derivatives instead of conventional ones, the derivative's concept is generalized to the certain classes of discontinuous functions that exist in impulsive systems.

Definition 3 (*[1]*). The comparison system of (5) is presented as:

$$\begin{aligned} \dot{\nu}(t) &= g_c\big(t, \nu(t)\big), \ t \neq t_k \\ \nu(t_k^+) &= g_i\big(\nu(t_k)\big), \ t = t_k \end{aligned} \quad (8)$$

where continuous $g_c : R^+ \times R^+ \to R$ complies with Definition 1.a and $g_i : R^+ \to R^+$ has no decreasing trend in accordance with the subsequent assumption for $W \in \overline{W}$:

$$\begin{aligned} D^+W\big(t, x(t)\big) &\leq g_c\Big(t, W\big(t, x(t)\big)\Big), \ t \neq t_k \\ W\big(t_k, x(t_k^+)\big) &\leq g_i\Big(W\big(t_k, x(t_k)\big)\Big), \quad t = t_k \end{aligned} \quad (9)$$

Theorem 4. *Supposing $g_c(t, 0) = 0$, $f_i(0) = 0$ and $f_c(t, 0) = 0$, there are equal trivial solutions for the main and comparison systems of (5) and (8) in $t_{k-1} < t \leq t_k$. The subsequent conditions must be satisfied as assumptions [1]:*

(a) $W \in \overline{W}$, $W : R^+ \times S_\rho \to R^+$ and $\rho > 0$ in $t_{k-1} < t \leq t_k$: $D^+W\big(t, x(t)\big) \leq g_c\Big(t, W\big(t, x(t)\big)\Big)$.
(b) There exists a $\overline{\rho} > 0$ such that $x \in S_{\overline{\rho}}$ implies $x(t_k^+) \in S_{\overline{\rho}}$ for all k and $W\big(t_k, x(t_k^+)\big) \leq g_i\Big(W\big(t_k, x(t_k)\big)\Big)$.

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(c) $\bar{b}(\|x\|) \le W(t, x(t)) \le \bar{a}(\|x\|)$ *on* $R^+ \times S_\rho$ *where* $\bar{a}, \bar{b} \in \kappa$.

As a result, the stability features of the trivial solution of system (5) are correspondingly extended by that of the comparison system (8).

Corollary 1 ([1]). *Let* $g_c(t, W(t, x(t))) = \dot{\lambda}(t) W(t, x(t))$ *where* $\lambda \in C^1[R^+, R^+]$, $g_i(W(t_k, x(t_k^+))) = \eta_k W(t_k, x(t_k))$. *The asymptotically stable origin of (5) is gained as long as the observance of these conditions for* $\gamma > 1$ *and* $k = 1, 2, \ldots$*: The origin of (5) is asymptotically stable if the following conditions are satisfied:*

$$\dot{\lambda}(t) \ge 0$$
$$\eta_k \ge 0$$
$$\ln(\gamma \eta_k) + \lambda(t_k) - \lambda(t_{k-1}) + \le 0 \tag{10}$$

4. Adaptive state-dependent impulsive observer design

4.1. System description

A time-delay nonlinear system having unknown parameters is described as:

$$\begin{aligned}\dot{x}(t) &= f_1(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu))) \\ &\quad + Bf_2(x(t), x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu)), \theta) \\ y(t) &= C_h x(t)\end{aligned} \tag{11}$$

in which x, y and θ stand respectively for the state vector, output vector, and unknown parameter vector. $f_1 : \overbrace{R^n \times \cdots \times R^n}^{m+1} \to R^n$ and $f_2 : \overbrace{R^n \times \cdots \times R^n}^{m+1} \times R^s \to R^n$ are continuous functions, where the value of s is the number of unknown parameters and B is a $n \times n$ constant matrix. The extended pseudo-linearization form is then:

$$\dot{x}(t) = A(x_t)x(t) + Bf(x_t)\theta(t) \tag{12}$$

the state-dependent matrix $f : \overbrace{R^n \times \cdots \times R^n}^{m+1} \to R^{n\times s}$ and it is assumed $f_1(x, x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu)))|_{x=0} = 0$ and $f_2(x, x(t-\tau_1(\nu)), \ldots, x(t-\tau_m(\nu), \theta))|_{x=0,\theta=0} = 0$ thus, a trivial solution of the system (11) is $x = 0$. For designing the ASDIO, the following assumptions and lemmas are considered.

Assumption 1. The matrix-valued function $A(x_t)$ and $f(x_t)$ are continuous with respect to x.

Assumption 2. The system equation (12) is pointwise observable in the linear sense for all $x_t \in \Omega$.

Assumption 3. Two following conditions are considered:

$$\|f_1(x_t) - f_1(\hat{x}_t)\| \le K_{f_1}\|x(t) - \hat{x}(t)\| \tag{13}$$

$$\|f_2(x_t, \theta) - f_2(\hat{x}_t, \hat{\theta})\| \le K_{f_2}\|x(t) - \hat{x}(t)\| \tag{14}$$

where $K_{f_1}, K_{f_2} \in R^+$ are Lipschitz constants.

Lemma 1 ([43]). *For any matrix* $D_1, D_2 \in R^{n\times n}$ *and n-dimensional vectors* z_1, z_2*, the following inequality is satisfied for any positive constant* ε

$$z_1^T D_1^T D_2 z_2 + z_2^T D_2^T D_1 z_1 \le \varepsilon z_2^T z_2 + \frac{1}{\varepsilon} z_1^T D_1^T D_2 D_2^T D_1 z_1 \tag{15}$$

Lemma 2 ([43]). *Suppose matrices of* S_1, S_2, S_3 *and* S_4 *with compatible dimensions and* $S = \begin{bmatrix} S_1 & S_2 \\ S_4 & S_3 \end{bmatrix}$, *so:*

$$\begin{aligned} S < 0 &\iff S_1 < 0,\ S_3 - S_4 S_1^{-1} S_2 < 0 \\ S < 0 &\iff S_3 < 0,\ S_1 - S_2 S_3^{-1} S_4 < 0 \end{aligned} \tag{16}$$

here, S_1 *and* S_3 *are recognized as invertible matrices with Schur complements.*

4.2. Main results

The adaptive state-dependent impulsive observer used for nonlinear dynamics containing time-delay has been brought up on the system (11) as:

$$\begin{aligned}\dot{\hat{x}}(t) &= A(\hat{x}_t)\hat{x}(t) + Bf(\hat{x}_t)\hat{\theta}(t), \quad t \ne t_k \\ \hat{y}(t) &= C_h \hat{x}(t), \\ \hat{x}(t_k^+) &= \hat{x}(t_k) + F_1(\hat{x}_t)(y(t_k) - \hat{y}(t_k)), \quad t = t_k\end{aligned} \tag{17}$$

$$\begin{aligned}\dot{\hat{\theta}}(t) &= \varphi^{-1}(t) f^T(\hat{x}_t) H(t) C_h e(t), \quad t \ne t_k \\ \hat{\theta}(t_k^+) &= \hat{\theta}(t_k) + F_2(\hat{x}_t)(y(t_k) - \hat{y}(t_k)), \quad t = t_k\end{aligned} \tag{18}$$

where $\hat{x}$ and $\hat{y}$ are the estimated state and estimated output vectors, and $\hat{\theta}$ is the estimated parameter vector. $e(t) = x(t) - \hat{x}(t)$ and $\tilde{\theta}(t) = \theta(t) - \hat{\theta}(t)$ are state and parameter estimation errors, respectively. $F_1(\hat{x}_t) \in R^{n\times p}$ and $F_2(\hat{x}_t) \in R^{s\times p}$ are impulse gain matrices for the state and parameter vectors, and design parameters φ, H are calculated by Theorem 5.

Theorem 5. *The ASDIO's estimation errors for the state and parameter,* $e(t)$ *and* $\tilde{\theta}(t)$*, are presented by (17) and (18) which asymptotically converge to zero as the satisfaction of the subsequent conditions:*

$$\begin{aligned} &\begin{bmatrix} \Xi_i & P_i\left(\frac{1}{\varepsilon_1}I + \frac{1}{\varepsilon_2}BB^T\right) \\ \left(\frac{1}{\varepsilon_1}I + \frac{1}{\varepsilon_2}BB^T\right)P_i & -\left(\frac{1}{\varepsilon_1}I + \frac{1}{\varepsilon_2}BB^T\right) \end{bmatrix} \le 0 \\ &\begin{bmatrix} \frac{\varphi_1-\varphi_2}{\Delta k} - \alpha\varphi_i & f^T(\hat{x}_t) \\ f(\hat{x}_t) & -\frac{1}{2\varepsilon_2}I \end{bmatrix} \le 0 \end{aligned} \tag{19}$$

$$\begin{bmatrix} -\sigma M_1 & M_2 - \overline{FC}_h^T(\hat{x}_t) \\ M_2 - \overline{FC}_h(\hat{x}_t) & -M_2 \end{bmatrix} \le 0 \tag{20}$$

$$\alpha\Delta_k + \ln(\gamma\sigma) \le 0 \tag{21}$$

where

$$\begin{aligned}\Xi_i &= A^T(\hat{x}_t)P_i + P_iA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) \\ &\quad + 2(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2)I + \frac{P_1 - P_2}{\Delta_k} - \alpha P_i\end{aligned} \tag{22}$$

and $i = 1, 2$. *Also,* $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\alpha \ge 0$, $\sigma \ge 0$ *and* $\gamma > 1$ *are constants that satisfy* $\gamma\sigma \le 1$. *The matrix* $P(t) > 0$ *is presented as (24) and* $\mu(t)$ *is specified as:*

$$\mu(t) = \frac{t_k - t}{\Delta_k}, \quad t_{k-1} < t \le t_k \tag{23}$$

$\Delta_k = t_k - t_{k-1}$ *is the kth impulse interval,* H_1 *and* H_2 *are arbitrary and* $P_i, \varphi_j; i, j = 1, 2$ *are symmetric and positive definite periodic matrices as:*

$$\begin{aligned} P(t) &= P(t + \Delta_k), \quad t_{k-1} < t \le t_k \\ P(t) &= P_1 + \mu(t)(P_2 - P_1) \end{aligned} \tag{24}$$

$$\begin{aligned} \varphi(t) &= \varphi(t + \Delta_k), \quad t_{k-1} < t \le t_k \\ \varphi(t) &= \varphi_1 + \mu(t)(\varphi_2 - \varphi_1) \end{aligned} \tag{25}$$

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$$H(t) = H(t+\Delta_k), \qquad t_{k-1} < t \le t_k$$
$$H(t) = H_1 + \mu(t)(H_2 - H_1) \tag{26}$$

$$M(t) = \begin{bmatrix} P(t) & 0 \\ 0 & \varphi(t) \end{bmatrix}, \quad M_1 = \begin{bmatrix} P_1 & 0 \\ 0 & \varphi_1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} P_2 & 0 \\ 0 & \varphi_2 \end{bmatrix} \tag{27}$$

where $k = 1, 2, \ldots$ *and*

$$\begin{matrix} P_1 B = {C_h}^T {H_1}^T \\ P_2 B = {C_h}^T {H_2}^T \end{matrix} \rightarrow PB = {C_h}^T H^T \tag{28}$$

Proof. Using (12) and (17), the dynamic state estimation error, as well as the state estimation error of jump are:

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) = A(x_t)x(t) + Bf(x_t)\theta(t) - A(\hat{x}_t)\hat{x}(t) \\ &\quad - Bf(\hat{x}_t)\hat{\theta}(t), \quad t \ne t_k \\ e(t_k^+) &= x(t_k^+) - \hat{x}(t_k^+) \\ &= x(t_k) - \Big(\hat{x}(t_k) + F_1(\hat{x}_t)\big(y(t_k) - \hat{y}(t_k)\big)\Big), \quad t = t_k \end{aligned} \tag{29}$$

Now, addition and subtraction of the two terms $A(\hat{x}_t)x(t)$ and $Bf(\hat{x}_t)\theta(t)$ to the right sentence of $\dot{e}(t)$ and $e(t_k^+)$ is sorted based on $e(t_k)$:

$$\begin{aligned} \dot{e}(t) &= A(x_t)x(t) + Bf(x_t)\theta(t) - A(\hat{x}_t)\hat{x}(t) - Bf(\hat{x}_t)\hat{\theta}(t) \\ &\quad + A(\hat{x}_t)x(t) - A(\hat{x}_t)x(t) + Bf(\hat{x}_t)\theta(t) - Bf(\hat{x}_t)\theta(t) \\ &= A(\hat{x}_t)\big(x(t) - \hat{x}(t)\big) + \big(A(x_t) - A(\hat{x}_t)\big)x(t) \\ &\quad + Bf(\hat{x}_t)\big(\theta(t) - \hat{\theta}(t)\big) + B\big(f(x_t) - f(\hat{x}_t)\big)\theta(t) \\ &= A(\hat{x}_t)e(t) + \tilde{A}x(t) + Bf(\hat{x}_t)\tilde{\theta}(t) + B\tilde{f}\theta(t), \quad t \ne t_k \\ e(t_k^+) &= e(t_k) - F_1(\hat{x}_t)C_h e(t_k) = \big(I - F_1(\hat{x}_t)C_h\big)e(t_k), \quad t = t_k \end{aligned} \tag{30}$$

where for simplifying, two abbreviations are defined as $\tilde{A} = A(x_t) - A(\hat{x}_t), \tilde{f} = f(x_t) - f(\hat{x}_t)$. Using (18), the parameter estimation error for dynamic and jumping behaviors are:

$$\begin{aligned} \dot{\tilde{\theta}}(t) &= \dot{\theta}(t) - \dot{\hat{\theta}}(t) = -\varphi^{-1}(t)f^T(\hat{x}_t)H(t)C_h e(t), \quad t \ne t_k \\ \tilde{\theta}(t_k^+) &= \theta(t_k^+) - \hat{\theta}(t_k^+) \\ &= \theta(t_k) - \hat{\theta}(t_k) - F_2(\hat{x}_t)\big(y(t_k) - \hat{y}(t_k)\big) \\ &= \tilde{\theta}(t_k) - F_2(\hat{x}_t)C_h e(t_k), \quad t = t_k \end{aligned} \tag{31}$$

The time-varying Lyapunov function candidate is remarked as:

$$V(t, e, \tilde{\theta}) = x_{aug}^T(t)M(t)x_{aug}(t) \tag{32}$$

where $x_{aug} = \left[e^T, \tilde{\theta}^T\right]^T$. So, the right Dini's derivative of Lyapunov function for $t_{k-1} < t \le t_k$ is:

$$\begin{aligned} D^+V(t, e, \tilde{\theta}) &= \dot{x}_{aug}^T(t)M(t)x_{aug}(t) + x_{aug}^T(t)M(t)\dot{x}_{aug}(t) \\ &\quad + x_{aug}^T(t)\dot{M}(t)x_{aug}(t) \\ &= \dot{e}^T(t)P(t)e(t) + e^T(t)P(t)\dot{e}(t) + \dot{\tilde{\theta}}^T(t)\varphi(t)\tilde{\theta}(t) \\ &\quad + \tilde{\theta}^T(t)\varphi(t)\dot{\tilde{\theta}}(t) + e^T(t)\dot{P}(t)e(t) \\ &\quad + \tilde{\theta}^T(t)\dot{\varphi}(t)\tilde{\theta}(t) \end{aligned} \tag{33}$$

Suppose that $\dot{\mu}(t) = -\frac{1}{\Delta_k}$, so:

$$\begin{aligned} \dot{P}(t) &= -\dot{\mu}(t)\big(P_1 - P_2\big) = \frac{P_1 - P_2}{\Delta_k} \\ \dot{\varphi}(t) &= -\dot{\mu}(t)\big(\varphi_1 - \varphi_2\big) = \frac{\varphi_1 - \varphi_2}{\Delta_k} \end{aligned} \tag{34}$$

Replacing (30), (31) and (34) to (33):

$$\begin{aligned} D^+V(t, e, \tilde{\theta}) &= \Big(A(\hat{x}_t)e + \tilde{A}x + Bf(\hat{x}_t)\tilde{\theta} + B\tilde{f}\theta\Big)^T Pe \\ &\quad + e^T P\Big(A(\hat{x}_t)e + \tilde{A}x + Bf(\hat{x}_t)\tilde{\theta} + B\tilde{f}\theta\Big) \\ &\quad - \Big(\varphi^{-1}f^T(\hat{x}_t)H\Big)^T \varphi\tilde{\theta} - \tilde{\theta}^T\varphi\Big(\varphi^{-1}f^T(\hat{x}_t)H\Big) \\ &\quad + e^T\frac{P_1 - P_2}{\Delta_k}e + \tilde{\theta}^T\frac{\varphi_1 - \varphi_2}{\Delta_k}\tilde{\theta} \\ &= e^T\Big(A^T(\hat{x}_t)P + PA(\hat{x}_t)\Big)e \\ &\quad + e^T\Big(PBf(\hat{x}_t) - C_h^T H^T f(\hat{x}_t)\varphi^{-1}\varphi\Big)\tilde{\theta} + \tilde{\theta}^T\Big(f^T(\hat{x}_t)B^T P \\ &\quad - \varphi\varphi^{-1}f^T(\hat{x}_t)HC_h\Big)e + x^T\tilde{A}^T Pe + e^T P\tilde{A}x \\ &\quad + \theta^T\tilde{f}^T B^T Pe + e^T PB\tilde{f}\theta \\ &\quad + e^T\frac{P_1 - P_2}{\Delta_k}e + \tilde{\theta}^T\frac{\varphi_1 - \varphi_2}{\Delta_k}\tilde{\theta} \end{aligned} \tag{35}$$

Considering condition (28), it is concluded that $PB - C_h^T H^T = 0$, so:

$$\begin{aligned} D^+V(t, e, \tilde{\theta}) &= e^T\Big\{A^T(\hat{x}_t)P + PA(\hat{x}_t)\Big\}e + x^T\tilde{A}^T Pe + e^T P\tilde{A}x \\ &\quad + \theta^T\tilde{f}^T B^T Pe + e^T PB\tilde{f}\theta + e^T\frac{P_1 - P_2}{\Delta_k}e \\ &\quad + \tilde{\theta}^T\frac{\varphi_1 - \varphi_2}{\Delta_k}\tilde{\theta} \end{aligned} \tag{36}$$

Using Lemma 1, it is achieved:

$$x^T\tilde{A}^T Pe + e^T P\tilde{A}x \le \varepsilon_1 x^T\tilde{A}^T\tilde{A}x + \frac{1}{\varepsilon_1}e^T P^2 e \tag{37}$$

$$\theta^T\tilde{f}^T B^T Pe + e^T PB\tilde{f}\theta \le \varepsilon_2\theta^T\tilde{f}^T\tilde{f}\theta + \frac{1}{\varepsilon_2}e^T PBB^T Pe \tag{38}$$

The term of $x^T\tilde{A}^T\tilde{A}x$ is defined as:

$$x^T\tilde{A}^T\tilde{A}x = \left\|\tilde{A}x\right\|^2 \tag{39}$$

Now, the term $A(\hat{x_t})\hat{x}$ is added and subtracted to right sentence:

$$\begin{aligned} x^T\tilde{A}^T\tilde{A}x &= \left\|A(x_t)x - A(\hat{x}_t)x + A(\hat{x}_t)\hat{x} - A(\hat{x}_t)\hat{x}\right\|^2 \\ &= \left\|\Big(A(x_t)x - A(\hat{x}_t)\hat{x}\Big) - \Big(A(\hat{x}_t)x - A(\hat{x}_t)\hat{x}\Big)\right\|^2 \end{aligned} \tag{40}$$

Considering the extended pseudo-linearization (11) and (12) $f_1(x_t) = A(x_t)x$, so:

$$x^T\tilde{A}^T\tilde{A}x = \left\|\Big(f_1(x_t) - f_1(\hat{x}_t)\Big) - A(\hat{x}_t)\big(x - \hat{x}\big)\right\|^2 \tag{41}$$

Utilizing Appendix:

$$x^T\tilde{A}^T\tilde{A}x \le 2\left\|f_1(x_t) - f_1(\hat{x}_t)\right\|^2 + 2\left\|A(\hat{x}_t)(x - \hat{x})\right\|^2 \tag{42}$$

Considering assumptions (13), the following result is obtained:

$$x^T\tilde{A}^T\tilde{A}x \le 2K_{f_1}^2 e^T e + 2e^T A^T(\hat{x}_t)A(\hat{x}_t)e \tag{43}$$

The term $\theta^T\tilde{f}^T\tilde{f}\theta$ is defined as:

$$\theta^T\tilde{f}^T\tilde{f}\theta = \left\|\tilde{f}\theta\right\|^2 \tag{44}$$

Now, the term $f(\hat{x_t})\hat{\theta}$ is added and subtracted to right sentence:

$$\begin{aligned} \theta^T\tilde{f}^T\tilde{f}\theta &= \left\|f(x_t)\theta - f(\hat{x}_t)\theta + f(\hat{x}_t)\hat{\theta} - f(\hat{x}_t)\hat{\theta}\right\|^2 \\ &= \left\|\Big(f(x_t)\theta - f(\hat{x}_t)\hat{\theta}\Big) - \Big(f(\hat{x}_t)\theta - f(\hat{x}_t)\hat{\theta}\Big)\right\|^2 \end{aligned} \tag{45}$$

Considering the extended pseudo-linearization (11) and (12) $f_2(x_t, \theta) = f(x_t)\theta$, so:

$$\theta^T\tilde{f}^T\tilde{f}\theta = \left\|\Big(f_2(x_t, \theta) - f_2(\hat{x}_t, \hat{\theta})\Big) - f(\hat{x}_t)\big(\theta - \hat{\theta}\big)\right\|^2 \tag{46}$$

Utilizing Appendix:

$$\theta^T\tilde{f}^T\tilde{f}\theta \le 2\left\|f_2(x_t, \theta) - f_2(\hat{x}_t, \hat{\theta})\right\|^2 + 2\left\|f(\hat{x}_t)(\theta - \hat{\theta})\right\|^2 \tag{47}$$

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Considering assumptions (14), the subsequent result is obtained:

$$\theta^T\tilde{f}^T\tilde{f}\theta \le 2K_{f_2}^2 e^T e + 2\tilde{\theta}^T f^T(\hat{x}_t)f(\hat{x}_t)\tilde{\theta} \tag{48}$$

Thus, (36) is written as:

$$\begin{aligned} &D^+V(t,e,\tilde{\theta}) \\ &\le e^T\Big\{A^T(\hat{x}_t)P + PA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) \\ &+2\Big(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\Big)I + \frac{1}{\varepsilon_1}P^2 + \frac{1}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k}\Big\}e \\ &+\tilde{\theta}^T\Big\{\frac{\varphi_1-\varphi_2}{\Delta_k} + 2\varepsilon_2 f^T(\hat{x}_t)f(\hat{x}_t)\Big\}\tilde{\theta} \end{aligned} \tag{49}$$

Now, the term $\alpha V(t,e,\tilde{\theta}) = \alpha\big(e^TPe + \tilde{\theta}^T\varphi\tilde{\theta}\big)$ is added and subtracted to the right side of (49):

$$\begin{aligned} D^+V(t,e,\tilde{\theta}) \le e^T\Big\{&A^T(\hat{x}_t)P + PA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) \\ &+2\Big(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\Big)I + \frac{1}{\varepsilon_1}P^2 + \frac{1}{\varepsilon_2}PBB^TP \\ &+\frac{P_1-P_2}{\Delta_k} - \alpha P\Big\}e \\ &+\tilde{\theta}^T\Big\{\frac{\varphi_1-\varphi_2}{\Delta_k} + 2\varepsilon_2 f^T(\hat{x}_t)f(\hat{x}_t) - \alpha\varphi\Big\}\tilde{\theta} \\ &+\alpha V(t,e,\tilde{\theta}) \end{aligned} \tag{50}$$

Thus:

$$D^+V(t,e,\tilde{\theta}) \le x_{aug}^T \Sigma x_{aug} + \alpha V(t,e,\tilde{\theta}) \tag{51}$$

where

$$\begin{aligned} &\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \\ &\Sigma_1 = A^T(\hat{x}_t)P + PA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) \\ &\quad +2\Big(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\Big)I + \frac{1}{\varepsilon_1}P^2 + \frac{1}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k} - \alpha P \\ &\Sigma_2 = \frac{\varphi_1-\varphi_2}{\Delta_k} + 2\varepsilon_2 f^T(\hat{x}_t)f(\hat{x}_t) - \alpha\varphi \end{aligned} \tag{52}$$

With regard to the definition of matrices P and φ as in (24) and (25), Σ_1 and Σ_2 are written as:

$$\begin{aligned} &\Sigma_1 = N_1 + \mu N_2 + \mu^2 N_3 \\ &\Sigma_2 = L_1 + \mu L_2 \end{aligned} \tag{53}$$

where

$$\begin{aligned} &N_1 = A^T(\hat{x}_t)P_1 + P_1A(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) \\ &\quad +2\Big(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\Big)I + \frac{1}{\varepsilon_1}P_1^2 + \frac{1}{\varepsilon_2}P_1BB^TP_1 + \frac{P_1-P_2}{\Delta_k} - \alpha P_1 \\ &N_2 = A^T(\hat{x}_t)(P_2-P_1) + (P_2-P_1)A(\hat{x}_t) + \frac{2}{\varepsilon_1}P_1(P_2-P_1) \\ &\quad +\frac{2}{\varepsilon_2}P_1BB^T(P_2-P_1) - \alpha(P_2-P_1) \\ &N_3 = \frac{1}{\varepsilon_1}(P_2-P_1)^2 + \frac{1}{\varepsilon_2}(P_2-P_1)BB^T(P_2-P_1) \\ &L_1 = \frac{\varphi_1-\varphi_2}{\Delta_k} + 2\varepsilon_2 f^T(\hat{x}_t)f(\hat{x}_t) - \alpha\varphi_1 \\ &L_2 = -\alpha(\varphi_2-\varphi_1) \end{aligned} \tag{54}$$

With regard to $N_1 \le 0$, $N_3 \ge 0$ and $0 \le \mu \le 1$, thus, if $N_1 + N_2 + N_3 \le 0$, then, $\Sigma_1 \le 0$. In the same way, if $L_1 \le 0$ and $L_1 + L_2 \le 0$ then, $\Sigma_2 \le 0$. These inequalities are driven to LMIs (19) based on Schur complement (Lemma 2). Satisfying LMIs of (19), $x_{aug}^T \Sigma x_{aug} \le 0$ is obtained therefore, (51) leads to:

$$D^+V(t,e,\tilde{\theta}) \le \alpha V(t,e,\tilde{\theta}) \tag{55}$$

Utilizing (24), it is concluded that $\mu(t_k) = 0$ and $\mu(t_{k-1}^+) = 1$, then:

$$\begin{aligned} &P(t_k) = P_1 \\ &P(t_k^+) = P(t_{k-1}^+ + \Delta_k) = P(t_{k-1}^+) \to P(t_k^+) = P_2 \\ &\varphi(t_k) = \varphi_1 \\ &\varphi(t_k^+) = \varphi(t_{k-1}^+ + \Delta_k) = \varphi(t_{k-1}^+) \to \varphi(t_k^+) = \varphi_2 \end{aligned} \tag{56}$$

So, $M(t_k) = M_1$, $M(t_k^+) = M_2$ and

$$\begin{aligned} &V(t_k,e,\tilde{\theta}) = x_{aug}^T(t_k)M(t_k)x_{aug}(t_k) = x_{aug}^T(t_k)M_1x_{aug}(t_k) \\ &V(t_k^+,e,\tilde{\theta}) = x_{aug}^T(t_k^+)M(t_k^+)x_{aug}(t_k^+) = x_{aug}^T(t_k^+)M_2x_{aug}(t_k^+) \end{aligned} \tag{57}$$

Considering (29) and (31), at $t = t_k^+$:

$$x_{aug}(t_k^+) = \begin{bmatrix} I - F_1(\hat{x}_t)C_h & 0 \\ -F_2(\hat{x}_t)C_h & I \end{bmatrix} x_{aug}(t_k) = \Big(I - FC_h(\hat{x}_t)\Big)x_{aug}(t_k) \tag{58}$$

where $FC_h(\hat{x}_t) = \begin{bmatrix} F_1(\hat{x}_t)C_h & 0 \\ F_2(\hat{x}_t)C_h & 0 \end{bmatrix}$ resulting in the achievement of the Lyapunov function:

$$V\big(t_k^+, e(t_k^+), \tilde{\theta}(t_k^+)\big) = x_{aug}^T(t_k)\Big(I - FC_h(\hat{x}_t)\Big)^T M_2\Big(I - FC_h(\hat{x}_t)\Big)x_{aug}(t_k) \tag{59}$$

Considering the following condition:

$$\Big(I - FC_h(\hat{x}_t)\Big)^T M_2\Big(I - FC_h(\hat{x}_t)\Big) \le \sigma M_1 \tag{60}$$

Using Schur complement (Lemma 2), (60) is written as LMI as (20), where $FC_h(\hat{x}_t) = M_2^{-1}\overline{FC}_h(\hat{x}_t)$. Thus, the Lyapunov function would be:

$$V\big(t_k^+, e(t_k^+), \tilde{\theta}(t_k^+)\big) \le x_{aug}^T(t_k)\sigma M_1 x_{aug}(t_k) = \sigma V\big(t_k, e(t_k), \tilde{\theta}(t_k)\big) \tag{61}$$

Remark 1. According to Theorem 4, the stability properties of the trivial solutions of the comparison systems (55) and (61) are correspondingly extended by those of ASDIO (17) and (18). Note that it is considered $\dot{\lambda}(t) = \alpha \ge 0$ and $\eta_k = \sigma \ge 0$ as the first and second conditions of Corollary 1. Thus, the asymptotically stable ASDIO origin is found under the presented conditions of Theorem 5.

Remark 2. Regarding Corollary 1, the third condition implies that:

$$\ln(\gamma d_k) + \alpha(t_k - t_{k-1}) \le 0 \to \ln(\gamma d_k) \le -\alpha\Delta_k \tag{62}$$

In accordance with $\alpha\Delta_k \ge 0$ and $\gamma > 1$, the "Ln" argument of the function must be less than or equal to 1; as a result $\gamma\eta_k \le 1$, which is met by $\sigma\gamma \le 1 \to \sigma \le 1$.

Remark 3. The maximum impulse interval can be described so that:

$$\Delta_k^{\max} = \max_{k=1,2,\ldots}(t_k - t_{k-1}) = \left|\frac{\ln(\gamma\sigma)}{\alpha}\right| \tag{63}$$

Remark 4. In previous works such as [20], the minimum and maximum impulse intervals are regarded as known parameters. However, in ASDIO, the minimum impulse interval is free, and the maximum is calculated by "Ln" condition of the comparison system theory. Furthermore, in "Ln" condition, the maximum impulse interval is related to three design parameters making a trade-off between increasing the maximum impulse intervals and the conservatism of sufficient conditions.

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Remark 5. In comparison to ref [20], the first contribution of our method is using the comparison system theory that makes some differences in the procedure of observer design and less-conservative sufficient conditions and then provides feasible LMIs with an appropriate region of attractions. In [20], a particular class of nonlinear system is regarded and has a limiting condition on varying parameter. Therefore, the second contribution of our observer is using the extended pseudo-linearization technique such that the ASDIO could be used for various nonlinear time-delay systems with different kinds of delays. In addition, there is no limitation on varying-parameter or delay values unlike [25]. In [25], the proposed impulsive observer has districted limits on delays.

The asymptotic and exponential stability of the ASDIO is presented as Remarks 6 and 7, respectively, without the comparison system theory and with the classical approach.

Remark 6. The state estimation error of the proposed ASDIO by (17) and (18) have asymptotic convergence to zero in cases of observing the subsequent conditions and LMI (20):

$$\begin{bmatrix} \Xi_i' & P_i\left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right) \\ \left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right)P_i & -\left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right) \end{bmatrix} \le 0$$
$$\begin{bmatrix} \frac{\varphi_1-\varphi_2}{\Delta k} & f^T(\hat{x}_t) \\ f(\hat{x}_t) & -\frac{1}{2\varepsilon_2}I \end{bmatrix} \le 0 \tag{64}$$

where $i = 1, 2$ and Ξ_i' is defined as:

$$\Xi_i' = A^T(\hat{x}_t)P_i + P_iA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A^T(\hat{x}_t) + 2\left(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\right)I + \frac{P_1-P_2}{\Delta_k} \tag{65}$$

Proof. Eq. (49) is written as:

$$D^+V(t,e,\tilde{\theta}) \le x_{aug}^T \Xi' x_{aug} \tag{66}$$

where

$$\Sigma' = \begin{bmatrix} \Sigma_1' & 0 \\ 0 & \Sigma_2' \end{bmatrix}$$
$$\Sigma_1' = A^T(\hat{x}_t)P + PA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) + 2\left(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\right)I + \frac{1}{\varepsilon_1}P^2 + \frac{1}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k}$$
$$\Sigma_2' = \frac{\varphi_1-\varphi_2}{\Delta_k} + 2\varepsilon_2 f^T(\hat{x}_t)f(\hat{x}_t) \tag{67}$$

Satisfying two LMIs of (64), $x_{aug}^T \Sigma' x_{aug} \le 0$ is obtained so $D^+V(t,e,\tilde{\theta}) \le 0$. It should be emphasized that with the existence of α in Σ_1, Σ_2, the LMI condition of (19) is satisfied easier than (50) and Theorem 4 presents less-conservative sufficient conditions than the classical approach in [6]. Besides, Eq. (21) presents the upper bound of the time interval of impulses. This condition shows that the maximum impulse intervals has the inverse relation with α. Thus, there must be a trade-off between a bigger upper bound of the time interval of impulses and solving the LMIs (19).

Remark 7. The state estimation error of the proposed ASDIO by (17) and (18) has an exponential convergence to zero in cases of satisfaction of the subsequent conditions and LMI (20):

$$\begin{bmatrix} \Xi_i'' & P_i\left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right) \\ \left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right)P_i & -\left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right) \end{bmatrix} \le 0$$
$$\begin{bmatrix} \frac{\varphi_1-\varphi_2}{\Delta k} + \delta\varphi_i & f^T(\hat{x}_t) \\ f(\hat{x}_t) & -\frac{1}{2\varepsilon_2}I \end{bmatrix} \le 0 \tag{68}$$

where $i = 1, 2$ and Ξ_i'' is defined as

$$\Xi_i'' = A^T(\hat{x}_t)P_i + P_iA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) + 2\left(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\right)I + \frac{P_1-P_2}{\Delta_k} + \delta P_i \tag{69}$$

Proof. The term $\delta V(t,e,\tilde{\theta})$ is added and subtracted to the right side of (49), so:

$$D^+V(t,e,\tilde{\theta}) \le x_{aug}^T \Xi'' x_{aug} - \delta V(t,e,\tilde{\theta}) \tag{70}$$

where $\delta > 0$ and:

$$\Sigma'' = \begin{bmatrix} \Sigma_1'' & 0 \\ 0 & \Sigma_2'' \end{bmatrix}$$
$$\Sigma_1'' = A^T(\hat{x}_t)P + PA(\hat{x}_t) + 2\varepsilon_1 A^T(\hat{x}_t)A(\hat{x}_t) + 2\left(\varepsilon_1 K_{f_1}^2 + \varepsilon_2 K_{f_2}^2\right)I + \frac{1}{\varepsilon_1}P^2 + \frac{1}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k} + \delta P$$
$$\Sigma_2'' = \frac{\varphi_1-\varphi_2}{\Delta_k} + 2\varepsilon_2 f^T(\hat{x}_t)f(\hat{x}_t) + \delta\varphi \tag{71}$$

Satisfying two LMIs of (68), $x_{aug}^T \Xi'' x_{aug} \le 0$ is obtained and $D^+V(t,e,\tilde{\theta}) \le -\delta V(t,e,\tilde{\theta})$. As a result, the Lyapunov function is calculated as:

$$V(t,e,\tilde{\theta}) \le V(t_{k-1},e,\tilde{\theta})\exp\left(-\delta(t-t_{k-1})\right), \quad t_{k-1} < t \le t_k \tag{72}$$

which monotonically decreases between impulses.

Remark 8. Lipschitz condition is a special case of the sector bounded condition. Assumption 3 is relaxed by considering the sector condition thus, less-conservative sufficient condition and LMIs with more likely feasibility are generated [44–46]. The vector-value functions $f_1(x_t)$ and $f_2(x_t,\theta)$ belong to sectors $[R_1, R_2]$ and $[Q_1, Q_2]$ if the following assumptions are satisfied:

$$\left(f_1(x_t) - f_1(\hat{x}_t) - R_1(x-\hat{x})\right)^T \left(f_1(x_t) - f_1(\hat{x}_t) - R_2(x-\hat{x})\right) \le 0 \tag{73}$$

$$\left(f_2(x_t,\theta) - f_2(\hat{x}_t,\hat{\theta}) - Q_1(\theta-\hat{\theta})\right)^T \left(f_2(x_t,\theta) - f_2(\hat{x}_t,\hat{\theta}) - Q_2(\theta-\hat{\theta})\right) \le 0 \tag{74}$$

where $R_1, R_2 \in R^{n\times n}$ and $Q_1, Q_2 \in R^{n\times s}$ are known real constant matrices. These sector conditions are written as in [45,46]:

$$\begin{bmatrix} x-\hat{x} \\ f_1(x_t)-f_1(\hat{x}_t) \end{bmatrix}^T \begin{bmatrix} \overline{R}_1 & \overline{R}_2 \\ \overline{R}_2^T & I \end{bmatrix} \begin{bmatrix} x-\hat{x} \\ f_1(x_t)-f_1(\hat{x}_t) \end{bmatrix} \le 0 \tag{75}$$

$$\begin{bmatrix} \theta-\hat{\theta} \\ f_2(x_t,\theta)-f_2(\hat{x}_t,\hat{\theta}) \end{bmatrix}^T \begin{bmatrix} \overline{Q}_1 & \overline{Q}_2 \\ \overline{Q}_2^T & I \end{bmatrix} \begin{bmatrix} \theta-\hat{\theta} \\ f_2(x_t,\theta)-f_2(\hat{x}_t,\hat{\theta}) \end{bmatrix} \le 0 \tag{76}$$

where

$$\overline{R}_1 = \frac{R_1^TR_2 + R_2^TR_1}{2}; \ \overline{R}_2 = -\frac{R_1^T + R_2^T}{2} \tag{77}$$

$$\overline{Q}_1 = \frac{Q_1^TQ_2 + Q_2^TQ_1}{2}; \ \overline{Q}_2 = -\frac{Q_1^T + Q_2^T}{2} \tag{78}$$

For choosing the sectors, the following conditions should be satisfied for all $\hat{x} \in \Omega$:

$$A^T(\hat{x}_t)A(\hat{x}_t) \le \overline{R}_1; \ -A^T(\hat{x}_t) \le \overline{R}_2 \tag{79}$$

$$f^T(\hat{x}_t)f(\hat{x}_t) \le \overline{Q}_1; \ -f^T(\hat{x}_t) \le \overline{Q}_2 \tag{80}$$

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Considering the sector conditions (75), (76) and assumptions (79), (80), two LMIs (19) are written as follows:

$$\begin{bmatrix} \Xi_i & P_i\left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right) \\ \left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right)P_i & -\left(\frac{1}{\varepsilon_1}I+\frac{1}{\varepsilon_2}BB^T\right) \end{bmatrix} \le 0$$

$$\frac{\varphi_1-\varphi_2}{\Delta k}+\alpha\varphi_i \le 0 \tag{81}$$

where $i = 1, 2$ and

$$\Xi_i = A^T(\hat{x}_t)P_i + P_iA(\hat{x}_t) + \frac{P_1-P_2}{\Delta_k} - \alpha P_i \tag{82}$$

Proof. Eq. (41) is written as:

$$\begin{aligned} x^T\tilde{A}^T\tilde{A}x &= \left(f_1(x_t)-f_1(\hat{x}_t)-A(\hat{x}_t)e\right)^T\left(f_1(x_t)-f_1(\hat{x}_t)-A(\hat{x}_t)e\right) \\ &= \begin{bmatrix} e \\ f_1(x_t)-f_1(\hat{x}_t)\end{bmatrix}^T \begin{bmatrix} A^T(\hat{x}_t)A(\hat{x}_t) & -A^T(\hat{x}_t) \\ -A(\hat{x}_t) & I \end{bmatrix} \\ &\quad\times \begin{bmatrix} e \\ f_1(x_t)-f_1(\hat{x}_t)\end{bmatrix} \le 0 \end{aligned} \tag{83}$$

According to assumptions (75) and (79), $x^T\tilde{A}^T\tilde{A}x \le 0$ is achieved. In the same way, (46) is written as:

$$\begin{aligned} \theta^T\tilde{f}^T\tilde{f}\theta &= \left(f_2(x_t,\theta)-f_2(\hat{x}_t,\hat{\theta})-f(\hat{x}_t)\tilde{\theta}\right)^T \\ &\quad\times\left(f_2(x_t,\theta)-f_2(\hat{x}_t,\hat{\theta})-f(\hat{x}_t)\tilde{\theta}\right) \\ &= \begin{bmatrix} \tilde{\theta} \\ f_2(x_t,\theta)-f_2(\hat{x}_t,\hat{\theta})\end{bmatrix}^T \begin{bmatrix} f^T(\hat{x}_t)f(\hat{x}_t) & -f^T(\hat{x}_t) \\ -f(\hat{x}_t) & I \end{bmatrix} \\ &\quad\times \begin{bmatrix} \tilde{\theta} \\ f_2(x_t,\theta)-f_2(\hat{x}_t,\hat{\theta})\end{bmatrix} \le 0 \end{aligned} \tag{84}$$

According to assumptions (76) and (80), $\theta^T\tilde{f}^T\tilde{f}\theta \le 0$ is achieved. Continuing the proof of Theorem 5, the LMIs (81) are obtained. As it is mentioned, the sector bounded conditions make less-conservative sufficient conditions in comparison to Lipschitz conditions and the LMIs (81) with more likely feasibility than (19).

Algorithm 1. The following procedure is presented to design the proposed ASDIO.

(a) Choose design parameters $\alpha, \sigma, \gamma, \varepsilon_1$ and ε_2 by considering (21), (63), and the convergence rate of the states, respectively.
(b) Calculate the matrix-values $A(\hat{x}_t), f(\hat{x}_t)$ and then solve LMIs (19) (or (81)) and LMI (20) to find the values of P, φ, F_1 and F_2.
(c) Solve $PB = C_h^TH^T$ to find H.

Three steps should be implemented every sample time. MATLAB LMI toolbox is very efficiency to calculate the proposed LMIs.

In [18,47–51], the different stability methods are presented for linear or nonlinear stochastic impulsive systems and delayed impulsive dynamics are considered in [49–51]. The proposed ASDIO can be applied for the state estimation of stochastic nonlinear time-delay systems. A stochastic impulsive system is defined as:

$$\begin{aligned} dx(t) &= f_c\big(t,x(t)\big)dt + \Gamma g_B\big(t,x(t)\big)dB_B(t), \quad t \ne t_k \\ x(t_k^+) &= x(t_k) + f_i\big(x(t_k)\big), \qquad\qquad t = t_k \end{aligned} \tag{85}$$

in which, $\Gamma \in R^n$ is a constant vector, $g_B : R^+ \times R^n \to R^{1\times l}$, $f_c : R^+ \times R^n \to R^n$ and $f_i : R^n \to R^n$ are nonlinear and continuous functions with $g_B(t,0) = 0$, $f_c(t,0) = 0$ and $f_i(0) = 0$ for all times thus, the trivial solution of the system (85) is $x = 0$. A complete probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, P\right)$ contains $B_B(t) = \left[B_{B_1}(t), B_{B_2}(t), \ldots, B_{B_l}(t)\right]^T$ l-dimensional Brownian motion with a natural filtration $\{\mathcal{F}_t\}_{t\ge0}$ and probability measure P observing ordinary conditions (i.e. right continuousness and all P-null sets for F_0). The stochastic impulsive dynamic comparison system is defined as (8) with considering $\nu(t_0^+) = E\big(V(t_0,x_0)\big)$, where E is the expectation operator [18,47,48].

Definition 4 ([47,48]). The stochastic impulsive dynamics' trivial solution is stable for any $\varepsilon > 0$, regarding $\delta = \delta(t_0,\varepsilon) > 0$ in such a way that $E\big(\|x\|\big) \le \varepsilon$ points $E\big(\|x_0\|\big) \le \delta$ for all $t \ge t_0$.

Definition 5 ([47,48]). The stochastic impulsive dynamics' trivial solution is asymptotically stable in cases of holding Definition 4 and regarding $\delta = \delta(t_0) > 0$ so that $E\big(\|x_0\|\big) \le \delta$ points $\lim_{t\to\infty} E\big(\|x\|\big) = 0$.

Definition 6 ([47,48]). If Definition 1 holds, The diffusion operator $\mathcal{L}$ is illustrated as:

$$\begin{aligned} \mathcal{L}W(t,x) =& W_t(t,x) + W_x(t,x)f_c(t,x) \\ &+ \frac{1}{2}\mathrm{trace}\Big(g_B^T(t,x)\Gamma^T W_{x^2}(t,x)\Gamma g_B(t,x)\Big) \end{aligned} \tag{86}$$

where $W_t(t,x) = \frac{\partial W(t,x)}{\partial t}$, $W_x(t,x) = \left[\frac{\partial W(t,x)}{\partial x_1}, \frac{\partial W(t,x)}{\partial x_2}, \ldots, \frac{\partial W(t,x)}{\partial x_n}\right]$, $W_{x^2}(t,x) = \left[\frac{\partial^2 W(t,x)}{\partial x_i \partial x_j}\right]_{n\times n}$.

Theorem 6 ([48,49]). *Assume that Lyapunov candidate satisfies Definition 1, the continuous $g_c : R^+ \times R^+ \to R$ observes Definition 1.a and $g_i : R^+ \to R^+$ has no decreasing trend thereby to ensure the subsequent terms:*

(a) $E\Big(g_c\big(t, W(t,x)\big)\Big) \le g_c\Big(t, E\big(W(t,x)\big)\Big)$ while $\bar{b}\Big(E\big(\|x\|\big)\Big) \le E\big(W(t,x)\big) \le \bar{a}\Big(E\big(\|x\|\big)\Big)$ where $\bar{a}, \bar{b} \in \kappa$.
(b) $E\Big(\mathcal{L}W(t,x)\Big) \le E\Big(g_c\big(t, W(t,x)\big)\Big)$ for $t \ne t_k$.
(c) There exist $\rho_0 > 0$ and $\rho_1 > 0$ such that $E\Big(\|x(t_k)\|\Big) \in S_{\rho_0}$ implies $E\Big(\|x(t_k^+)\| \in\Big)S_{\rho_1}$ for all k.
(d) $E\Big(W\big(t_k, x(t_k^+)\big)\Big) \le g_i\Big(E\big(W(t_k, x(t_k))\big)\Big)$.

Then, the stability properties of the trivial solution of the comparison system (8) correspondingly conform with those of the stochastic impulsive system (85) regarding $\nu(t_0^+) = E\big(V(t_0,x_0)\big)$.

Corollary 2 ([48,49]). *Assume that the terms of Theorem 6 are ensured then, the asymptotically stable origin of (85) is regarded though observance of the conditions of Corollary 1.*

The stochastic nonlinear time-delay dynamic in extended pseudo-linearization form is illustrated as:

$$dx(t) = \Big(A(x_t)x(t) + Bf(x_t)\theta(t)\Big)dt + \Gamma g_B(t,x)dB_B(t) \tag{87}$$

The adaptive state-dependent impulsive observer (17) and (18) are used for the stochastic nonlinear time-delay system (87) where the states and parameters impulses gain matrices F_1 and F_2 are calculated by Theorem 7.

Assumption 4. The following condition is ensured for all t and x:

$$\left\|g_B^T(t,x)\right\| \le K_{g_B} \tag{88}$$

where $K_{g_B} \in R^+$.

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Theorem 7. *The state estimation error $e(t)$, as well as the parameter estimation error $\tilde{\theta}(t)$, of the presented ASDIO for the stochastic nonlinear time-delay systems converge to zero asymptotically by the observance of the subsequent conditions:*

$$\begin{bmatrix} \Xi_i & 2P_i\left(\frac{1}{\varepsilon_1}I + \frac{1}{\varepsilon_2}BB^T\right) \\ \left(\frac{1}{\varepsilon_1}I + \frac{1}{\varepsilon_2}BB^T\right)2P_i & -2\left(\frac{1}{\varepsilon_1}I + \frac{1}{\varepsilon_2}BB^T\right) \end{bmatrix} \le 0$$
$$\begin{bmatrix} \frac{\varphi_1-\varphi_2}{\Delta k} - \alpha\varphi_i & f^T(\hat{x}_t) \\ f(\hat{x}_t) & -\frac{1}{4\varepsilon_2}I \end{bmatrix} \le 0$$
$$\left(K_{g_B}^2 - \alpha\right)P_i + \frac{P_1 - P_2}{\Delta_k} \le 0 \tag{89}$$

$$P_2 \le \sigma P_1 \tag{90}$$

where

$$\Xi_i = 2\left(A^T(\hat{x}_t)P_i + P_iA(\hat{x}_t)\right) + 4\varepsilon_1A^T(\hat{x}_t)A(\hat{x}_t) + 4\left(\varepsilon_1K_{f_1}^2 + \varepsilon_2K_{f_2}^2\right)I + \frac{P_1 - P_2}{\Delta_k} - \alpha P_i \tag{91}$$

$i = 1, 2$ and the other conditions of Theorem 5 are hold.

Proof. The dynamic of the state estimation error for $t \ne t_k$ is rewritten as:

$$de(t) = \left(A(\hat{x}_t)e(t) + \tilde{A}x(t) + Bf(\hat{x}_t)\tilde{\theta}(t) + B\tilde{f}\theta(t)\right)dt + \Gamma g_B(t,x)dB_B(t) \tag{92}$$

The second term of (30) and the dynamic and jump of the parameter estimation error in (31) are hold. The augmented stochastic system is demonstrated as:

$$dx_{aug}(t) = \begin{bmatrix} A(\hat{x}_t)e(t) + \tilde{A}x(t) + Bf(\hat{x}_t)\tilde{\theta}(t) + B\tilde{f}\theta(t) \\ -\varphi^{-1}(t)f^T(\hat{x}_t)H(t)C_he(t) \end{bmatrix} dt + \begin{bmatrix} \Gamma g_B(t,x) \\ 0 \end{bmatrix} dB_B(t) \tag{93}$$

where $x_{aug} = \left[e^T, \tilde{\theta}^T\right]^T$. The jump equation is similar to (58). The new Lyapunov candidate is considered as:

$$V(t, x_{aug}) = x_{aug}^T(t)M(t)x_{aug}(t) + \Gamma^T P\Gamma \tag{94}$$

The $\mathcal{L}$-operator of Lyapunov candidate is calculated as:

$$\begin{aligned} \mathcal{L}V(t,e,\tilde{\theta}) = {} & e^T\left\{2\left(A^T(\hat{x}_t)P + PA(\hat{x}_t)\right)\right\}e + 2x^T\tilde{A}^TPe + 2e^TP\tilde{A}x \\ & + 2\theta^T\tilde{f}^TB^TPe + 2e^TPB\tilde{f}\theta \\ & + e^T\frac{P_1-P_2}{\Delta_k}e + \tilde{\theta}^T\frac{\varphi_1-\varphi_2}{\Delta_k}\tilde{\theta} + \Gamma^T\frac{P_1-P_2}{\Delta_k}\Gamma \\ & + e^TPBf(\hat{x}_t)\tilde{\theta} + \tilde{\theta}^Tf^T(\hat{x}_t)B^TPe \\ & - e^TC_h^TH^Tf(\hat{x}_t)\tilde{\theta} - \tilde{\theta}^Tf^T(\hat{x}_t)HC_he \\ & + \mathrm{trace}\left(g_B^T(t,x)\Gamma^TP\Gamma g_B(t,x)\right) \end{aligned} \tag{95}$$

Considering condition (28):

$$\begin{aligned} \mathcal{L}V(t,e,\tilde{\theta}) = {} & e^T\left\{2\left(A^T(\hat{x}_t)P + PA(\hat{x}_t)\right)\right\}e + 2x^T\tilde{A}^TPe + 2e^TP\tilde{A}x \\ & + 2\theta^T\tilde{f}^TB^TPe + 2e^TPB\tilde{f}\theta \\ & + e^T\frac{P_1-P_2}{\Delta_k}e + \tilde{\theta}^T\frac{\varphi_1-\varphi_2}{\Delta_k}\tilde{\theta} + \Gamma^T\frac{P_1-P_2}{\Delta_k}\Gamma \\ & + \mathrm{trace}\left(g_B^T(t,x)\Gamma^TP\Gamma g_B(t,x)\right) \end{aligned} \tag{96}$$

According to Lemma 1 and the routine that is done in (39) to (48), (96) is rewritten as:

$$\begin{aligned} \mathcal{L}V(t,e,\tilde{\theta}) \le {} & e^T\left\{2\left(A^T(\hat{x}_t)P + PA(\hat{x}_t)\right) + 4\varepsilon_1A^T(\hat{x}_t)A(\hat{x}_t) \right. \\ & \left. + 4\left(\varepsilon_1K_{f_1}^2 + \varepsilon_2K_{f_2}^2\right)I + \frac{2}{\varepsilon_1}P^2 + \frac{2}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k}\right\}e \\ & \tilde{\theta}^T\left\{\frac{\varphi_1-\varphi_2}{\Delta_k} + 4\varepsilon_2f^T(\hat{x}_t)f(\hat{x}_t)\right\}\tilde{\theta} + \Gamma^T\frac{P_1-P_2}{\Delta_k}\Gamma \\ & + \mathrm{trace}\left(g_B^T(t,x)\Gamma^TP\Gamma g_B(t,x)\right) \end{aligned} \tag{97}$$

For two vectors $x, y \in R^n$, the equality $\mathrm{trace}\left(yx^T\right) = y^Tx$ is ensured. Thus,

$$\begin{aligned} \mathcal{L}V(t,e,\tilde{\theta}) \le {} & e^T\left\{2\left(A^T(\hat{x}_t)P + PA(\hat{x}_t)\right) + 4\varepsilon_1A^T(\hat{x}_t)A(\hat{x}_t) \right. \\ & \left. + 4\left(\varepsilon_1K_{f_1}^2 + \varepsilon_2K_{f_2}^2\right)I + \frac{2}{\varepsilon_1}P^2 + \frac{2}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k}\right\}e \\ & \tilde{\theta}^T\left\{\frac{\varphi_1-\varphi_2}{\Delta_k} + 4\varepsilon_2f^T(\hat{x}_t)f(\hat{x}_t)\right\}\tilde{\theta} + \Gamma^T\frac{P_1-P_2}{\Delta_k}\Gamma \\ & + g_B(t,x)g_B^T(t,x)\Gamma^TP\Gamma \end{aligned} \tag{98}$$

Considering Assumption 4, and adding and subtracting the right side of (98) with $\alpha V(t,e,\tilde{\theta})$, (98) is rewritten as:

$$\begin{aligned} \mathcal{L}V(t,e,\tilde{\theta}) \le {} & e^T\left\{2\left(A^T(\hat{x}_t)P + PA(\hat{x}_t)\right) + 4\varepsilon_1A^T(\hat{x}_t)A(\hat{x}_t) \right. \\ & + 4\left(\varepsilon_1K_{f_1}^2 + \varepsilon_2K_{f_2}^2\right)I + \frac{2}{\varepsilon_1}P^2 + \frac{2}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k} \\ & \left. - \alpha P\right\}e + \tilde{\theta}^T\left\{\frac{\varphi_1-\varphi_2}{\Delta_k} + 4\varepsilon_2f^T(\hat{x}_t)f(\hat{x}_t) - \alpha\varphi\right\}\tilde{\theta} \\ & + \Gamma^T\left\{\frac{P_1-P_2}{\Delta_k} + K_{g_B}^2P - \alpha P\right\}\Gamma + \alpha V(t,e,\tilde{\theta}) \end{aligned} \tag{99}$$

The $\mathcal{L}$-operator of Lyapunov candidate is obtained as:

$$\mathcal{L}V(t,e,\tilde{\theta}) \le e^T\Sigma_1e + \tilde{\theta}^T\Sigma_2\tilde{\theta} + \Gamma^T\Sigma_3\Gamma + \alpha V(t,e,\tilde{\theta}) \tag{100}$$

where

$$\begin{aligned} \Sigma_1 = {} & 2\left(A^T(\hat{x}_t)P + PA(\hat{x}_t)\right) + 4\varepsilon_1A^T(\hat{x}_t)A(\hat{x}_t) \\ & + 4\left(\varepsilon_1K_{f_1}^2 + \varepsilon_2K_{f_2}^2\right)I + \frac{2}{\varepsilon_1}P^2 + \frac{2}{\varepsilon_2}PBB^TP + \frac{P_1-P_2}{\Delta_k} - \alpha P \\ \Sigma_2 = {} & \frac{\varphi_1-\varphi_2}{\Delta_k} + 4\varepsilon_2f^T(\hat{x}_t)f(\hat{x}_t) - \alpha\varphi \\ \Sigma_3 = {} & \frac{P_1-P_2}{\Delta_k} + K_{g_B}^2P - \alpha P \end{aligned} \tag{101}$$

Satisfying the LMIs of (89) with regard to the same procedure of Theorem 5, $\Sigma_i \le 0; i = 1:3$ is concluded thus,

$$\mathcal{L}V(t,e,\tilde{\theta}) \le \alpha V(t,e,\tilde{\theta}) \Rightarrow E\left(\mathcal{L}V(t,e,\tilde{\theta})\right) \le E\left(\alpha V(t,e,\tilde{\theta})\right) \tag{102}$$

Lyapunov candidate in $t = t_k$ is:

$$V\left(t_k^+, e(t_k^+), \tilde{\theta}(t_k^+)\right) = x_{aug}^T(t_k)\left(I - FC_h(\hat{x}_t)\right)^TM_2\left(I - FC_h(\hat{x}_t)\right) x_{aug}(t_k) + \Gamma^TP_2\Gamma \tag{103}$$

Thus, considering LMI (20) and new condition (90) leads to $V\left(t_k^+, e(t_k^+), \tilde{\theta}(t_k^+)\right) \le \sigma V\left(t_k, e(t_k), \tilde{\theta}(t_k)\right)$ then,

$$E\left(V\left(t_k^+, x_{aug}(t_k^+)\right)\right) \le \sigma E\left(V\left(t_k, x_{aug}(t_k)\right)\right) \tag{104}$$

According to Theorem 6 and Corollary 2, the stability properties of the trivial equilibrium of the comparison system correspondingly conform with those of the proposed ASDIO. Thus, the asymptotically stable origin of ASDIO is regarded under the presented conditions through Theorem 7.

5. Simulation results

Prevalence of contagious diseases has always been amongst the most significant challenges in human societies that must

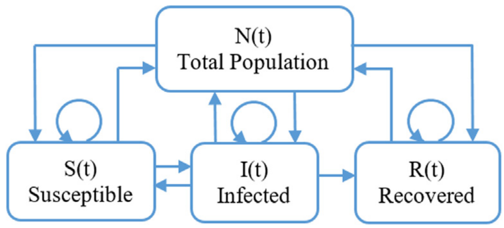

Fig. 1. Transmission diagram of the SIR model.

be prevented. One of the most dangerous infectious diseases is Ebola Virus Disease (EVD) or Congo Ebola. Based on the report presented by the centers for disease control and prevention (CDC), the EVD has spread West Africa rapidly and the result was so deadly. The EVD virus is recognized as the most fatal virus among deadly viruses, with a case fatality rate of between 25 to 90% in the former prevalence [52]. Symptoms of Ebola virus disease comprise sudden onset of fever, sore throat, headache, weakness, diarrhea, and vomiting. If the disease became severe, it may bring internal and external bleeding and cause multiple organ failure. The virus is transmitted from an infected person to healthy humans through close physical striking with blood or other body fluids or indirect touch with contaminated objects and surfaces. EVD may be transmitted even from dead patients as long as their blood contains the virus [52]. To prevent the prevalence of contagious diseases, the disease can be identified and treated through modeling as an important step. In this respect, epidemic diseases can be effectively modeled using the Susceptible–Infected–Recovered (SIR). Several models of SIR have been presented and developed for infectious diseases. In primary models, the total population size is assumed constant and the effect of delay is not considered [53,54]. In [53,54], density-dependent birth and death rates are assumed with the aim of the model investigation on disease dynamics with more demographic implications. In addition, the incubation time is also considered for this practical model as a time needed for the development of the infectious agents within the vector. So, this model is defined as an epidemic SIR model having time-delay. The transmission diagram of the presented model is depicted in Fig. 1.

The epidemic SIR nonlinear model with time-delay is defined as [53,54]:

$$\begin{aligned}\frac{dS(t)}{dt} &= -\left(d+(1-\overline{\mu})\frac{rN(t)}{K}\right)S(t)-\frac{\beta S(t)I(t-\tau)}{N(t-\tau)}\\ &\quad+\left(b-\overline{\mu}\frac{rN(t)}{K}\right)N(t)\\ \frac{dI(t)}{dt} &= -\left(d+(1-\overline{\mu})\frac{rN(t)}{K}+\lambda\right)I(t)+\frac{\beta S(t)I(t-\tau)}{N(t-\tau)}\\ \frac{dR(t)}{dt} &= -\left(d+(1-\overline{\mu})\frac{rN(t)}{K}\right)R(t)+\lambda I(t)\end{aligned} \tag{105}$$

where S, I and R represent susceptible, infected and recovered individuals, respectively. The count of the whole population is $N = S + I + R$. d, λ, b and β are the mortality, recovery, birth and contact rates, and are positive constants, respectively. $\overline{\mu}$ and K stand for convex combination constant and the population's carrying capacity, respectively. $r = b - d$ is the intrinsic growth rate. As well, constant τ is non-negative representing the time-delay on the whole population N and infected persons I during the spread of illness. The state R is the measured output, so $C_h = [0, 0, 1]$. The Congo Ebola also shows some parameters the values of which are summarized in Table 1 [54]. Assume that the birth and recovery rates are unknown, so $\theta = [\lambda, b]^T$ is an unknown parameter vector. For designing the proposed ASDIO, the subsequent form is presented as a member of the enormous extended pseudo-linearization parameterizations of (105):

$$A(x_t)=\begin{bmatrix} s_1 & -\overline{\mu}\frac{rN(t)}{K} & -\overline{\mu}\frac{rN(t)}{K}\\ \frac{\beta I(t-\tau)}{N(t-\tau)}-(1-\overline{\mu})\frac{rI(t)}{K} & -\left(d+(1-\overline{\mu})\frac{rI(t)}{K}\right) & -(1-\overline{\mu})\frac{rI(t)}{K}\\ 0 & 0 & -\left(d+(1-\overline{\mu})\frac{rN(t)}{K}\right)\end{bmatrix}$$

$$B=\begin{bmatrix}0&1&0\\-1&0&0\\1&0&0\end{bmatrix}, f(x_t)=\begin{bmatrix}I(t)&0\\0&N(t)\\0&0\end{bmatrix} \tag{106}$$

where

$$s_1=-\left(d+(1-\overline{\mu})\frac{rN(t)}{K}\right)-\frac{\beta I(t-\tau)}{N(t-\tau)}-\overline{\mu}\frac{rN(t)}{K}$$

The state-dependent observability matrix for the proposed extended pseudo-linearization form would be calculated as:

$$\Phi_o(\hat{x}_t)=\begin{bmatrix}0&0&1\\0&\lambda&*\\ \lambda\left(\frac{\beta\hat{I}(t-\tau)}{\hat{N}(t-\tau)}-(1-\overline{\mu})\frac{r\hat{I}(t)}{K}\right)&*&*\end{bmatrix} \tag{107}$$

$$\left|\Phi_o(\hat{x}_t)\right|=-\lambda^2\left(\frac{\beta\hat{I}(t-\tau)}{\hat{N}(t-\tau)}-(1-\overline{\mu})\frac{r\hat{I}(t)}{K}\right) \tag{108}$$

where * specifies uncalculated elements. This matrix is full rank because its determinant is always non-zero due to the value of the parameter in Table 1.

Remark 9. As mentioned in the introduction section, one of the advantages of the proposed observer is that the ASDIO could be used for some nonlinear systems with diverse time-delays using the extended pseudo-linearization approach. Previous researches in the field of designing an observer investigated a specific set of delayed nonlinear systems. Mostly, the considered system has four distinct parts including linear and nonlinear sections, as well as linear and nonlinear parts with delays [25,26]. However, there are some systems that are not formulated in this form. To prove this claim, the proposed ASDIO is simulated on the epidemic SIR nonlinear model containing a time-delay for the fatal Congo Ebola disease. [25,26] investigated an observer outline for a group of nonlinear systems with time-delay, but Congo Ebola disease cannot be presented in a special class with two distinct linear and nonlinear portions without any delay. [25] defines a nonlinear time-delay system as follows:

$$\dot{x}(t)=A_0x(t)+A_1x(t-\tau(t))+G_0f_0\left(H_0x(t)\right)+G_1f_1\left(H_1x(t-\tau(t))\right) \tag{109}$$

where the constant matrices of A_0, A_1, G_0, H_0, G_1 and H_1 have compatible dimensions while using nonlinear functions of f_0, f_1 and τ as the time-delay. The delay is departed in two linear and nonlinear sections. But, Congo Ebola disease model that is presented in (105) cannot be formulated as (109). Because the sentence $\frac{S(t)I(t-\tau)}{N(t-\tau)}$ cannot be separated into linear without delay and nonlinear with delay or vice versa. The nonlinear time-delay system is remarked in the same form as in [26]. Therefore, as we claimed, the proposed ASDIO can be used in a vast set of delayed nonlinear systems.

By implementation of Algorithm 1, the intended ASDIO is simulated on Congo Ebola disease. Fig. 2 illustrates the real and estimated susceptible individuals using the ASDIO. The estimated state follows real state, although the output is feasible only once in every five samples. The jump value of the estimated susceptible

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Table 1
Parameters of Congo Ebola disease [54].

Parameter	Value	Parameter	Value
$\overline{\mu}$	0.014 day^{-1}	b	0.07 day^{-1}
d	0.0123 day^{-1}	β	0.21 day^{-1}
λ	0.0476 day^{-1}	τ	10 day
K	10900		

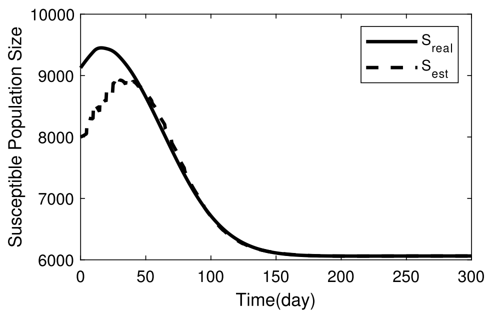

Fig. 2. Real and estimated susceptible individual.

individual is also demonstrated in Fig. 3. In impulse times, the output is available and the ASDIO updates the states. So, over time, the amplitude of the impulses is decreasing and the estimated state converges to the real one. Figs. 4 and 5 show the convergence of the estimated infected individual to the real one and its jumps in impulse times. The estimated and real recovered individual are shown in Fig. 6 and its jumps in impulse times are presented in Fig. 7. Fig. 8 shows the real and estimation of the total population are plotted and Fig. 9 presents the estimated total population jumps in impulse times. In Figs. 10 and 11, real and estimated unknown parameters (recovery and birth rates) are plotted. For the number of almost 100 samples, the estimated parameters would converge to the real value.

6. Conclusion

In the present paper, a novel adaptive state-dependent impulsive observer is introduced to be used in uncertain nonlinear dynamics containing time-delay accompanied by extended pseudo-linearization technique. This intended ASDIO determines both system states as well as unknown parameters, continuously. One of the advantages of the proposed ASDIO is the applicability of the SDIO in a vast group of uncertain nonlinear time-delay dynamical networks with several types of delays using the extended pseudo-linearization technique. Furthermore, by utilizing the extended pseudo-linearization approach, the delay-independent Lyapunov function employed for the stability analysis that made the observer design routine easier. A further advantage of the ASDIO is the estimated state and parameter errors asymptotically converge to zero under well-defined adequate conditions obtained in terms of feasible LMIs. The stability theorem was presented by comparing the system theory and its corollaries, which led to less conservative adequate conditions to preserve the asymptotic stability of the proposed ASDIO. Moreover, the maximum time interval of impulses was presented. Moreover, a new theorem is presented which applies ASDIO for stochastic nonlinear time-delay dynamics. The comparison principle for stochastic impulsive systems is utilized for the stability analysis. Therefore, the efficiency and performance of the proposed ASDIO have been confirmed based on the results of simulating an epidemic SIR uncertain nonlinear time-delay model for Congo Ebola.

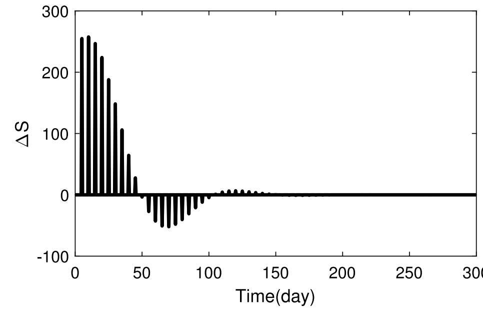

Fig. 3. Jump of estimated susceptible individual in impulse times.

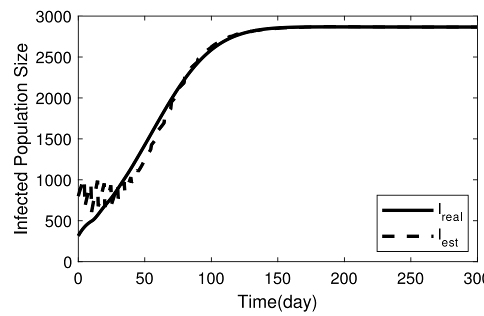

Fig. 4. Real and estimated infected individual.

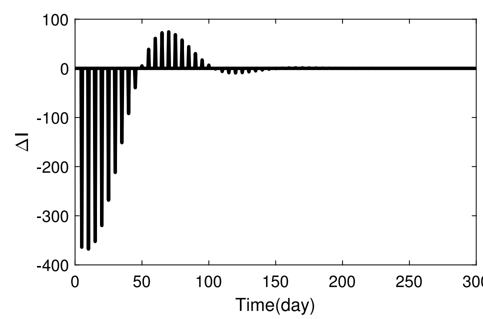

Fig. 5. Jump of estimated infected individual in impulse times.

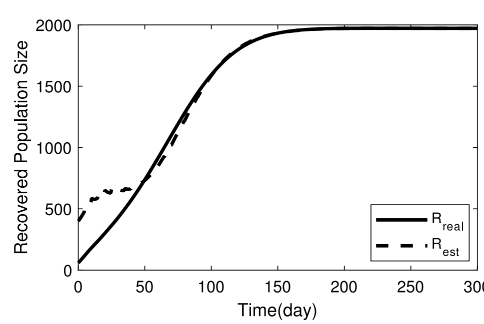

Fig. 6. Real and estimated recovered individual.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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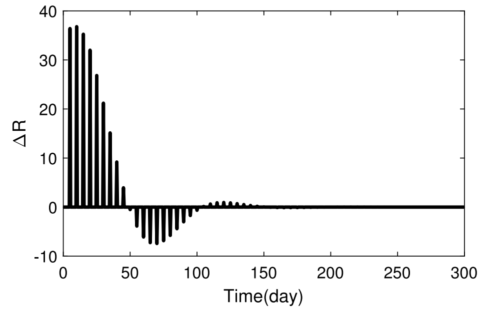

Fig. 7. Jump of estimated recovered individual in impulse times.

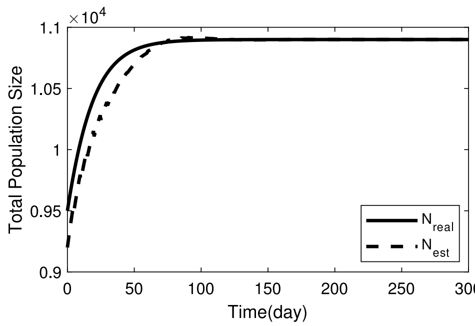

Fig. 8. Real and estimated total population.

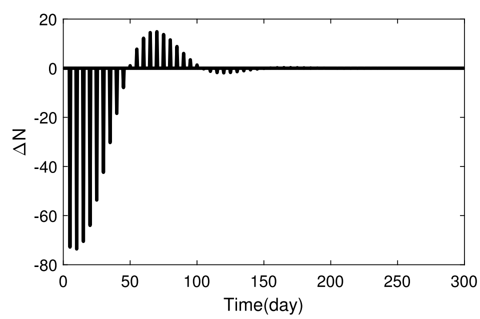

Fig. 9. Jump of estimated total population in impulse times.

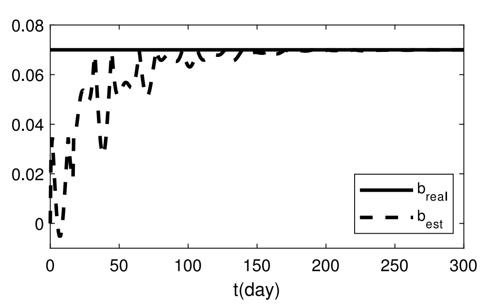

Fig. 10. Real and estimated recovery rate.

Appendix

For any vector $z \in R^n$, the Euclidean norm is defined as $\|z\| = \sqrt{z_1^2 + z_2^2 + \cdots + z_n^2}$ and $\|z\| \geq 0$ so:

$$\begin{aligned}\|z_1 - z_2\|^2 &= (z_{11} - z_{21})^2 + (z_{12} - z_{22})^2 + \cdots + (z_{1n} - z_{2n})^2 \\ &= z_{11}^2 - 2z_{11}z_{21} + z_{21}^2 + \cdots + z_{1n}^2 - 2z_{1n}z_{2n} + z_{2n}^2 \\ &= \left(z_{11}^2 + \cdots + z_{1n}^2\right) - 2\left(z_{11}z_{21} + \cdots + z_{1n}z_{2n}\right) \\ &\quad + \left(z_{21}^2 + \cdots + z_{2n}^2\right) \\ &= \|z_1\|^2 + \|z_2\|^2 - 2z_1^T z_2\end{aligned} \tag{110}$$

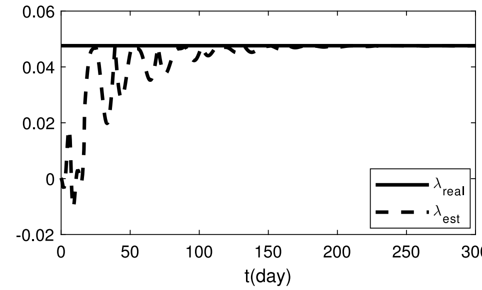

Fig. 11. Real and estimated birth rate.

In the same way following result is achieved:

$$\|z_1 + z_2\|^2 = \|z_1\|^2 + \|z_2\|^2 + 2z_1^T z_2 \tag{111}$$

Adding (110) and (111):

$$\begin{aligned}\|z_1 + z_2\|^2 + \|z_1 - z_2\|^2 &= \|z_1\|^2 + \|z_2\|^2 + 2z_1^T z_2 + \|z_1\|^2 \\ &+ \|z_2\|^2 - 2z_1^T z_2 = 2\|z_1\|^2 + 2\|z_2\|^2\end{aligned} \tag{112}$$

Considering $\|z_1 + z_2\| \geq 0$ so it is concluded:

$$\|z_1 - z_2\|^2 \leq 2\|z_1\|^2 + 2\|z_2\|^2 \tag{113}$$

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