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Delay-dependent robust absolute stability criteria for uncertain multiple time-delayed Lur'e systems

Ali Kazemy and Mohammad Farrokhi

Abstract

In this article, the problem of delay-dependent robust absolute stability of uncertain multiple time-delayed Lur'e systems with sector-bounded nonlinearity is investigated. The nonlinearity is assumed to be both time invariant and time varying. Based on the Lyapunov–Krasovskii stability theory and matrix decomposition method, some delay-dependent sufficient conditions for the robust absolute stability of the Lur'e system are derived and expressed in the form of linear matrix inequalities. By solving a convex optimization problem for these linear matrix inequalities, the maximum upper bounds of the allowable delays are obtained. Some numerical examples are given to show that the proposed stability criteria are less conservative than those in the literature.

Keywords

Time-delayed systems, Lur'e systems, delay-dependent stability

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Introduction

In practice, all physical systems are nonlinear in nature and there are various kinds of nonlinearities. An important class of nonlinear systems is the feedback system in which the forward path contains a linear time-invariant subsystem and the feedback path contains a memoryless nonlinearity. When the nonlinearity satisfies certain sector condition, the nonlinear system is called Lur'e system.[1] Since 1957, when Lur'e introduced the concept of absolute stability of such systems,[2] the problem of stability of Lur'e systems has been intensively studied by many researchers.[3–6]

It is a well-known fact that the stability and convergence properties of feedback systems are strongly affected by time delays, which are often encountered in various engineering systems.[7] Since the existence of time delays is the source of instability and performance degradation, the stability analysis of time-delayed Lur'e systems has attracted considerable attention of researchers.[8–14] In general, two types of absolute stability criteria for Lur'e systems with time delays are considered: (1) delay dependent and (2) delay independent.[7,15] Since delay-dependent criteria make use of information on the length of the delay, they are less conservative than the delay-independent criteria, especially when the size of the delay is small.

Recently, practical considerations such as model uncertainties are considered for stability analysis of Lur'e systems.[16–20] However, most of these papers consider Lur'e systems with single time delay. Lur'e systems with multiple time delays are considered by Cao and Zhong,[10] Cao et al.[11] and Tian et al.[12] but without uncertainties. Moreover, in their numerical examples, only single–time-delay systems are considered.

A challenging problem in this field is to find the maximum allowable time delay that can guarantee the absolute stability of the uncertain time-delay dynamic systems. Different methods have been developed by researchers in this regard.[20–25] The problem of delay-dependent stability of neutral Lur'e systems with interval time-varying state delay is investigated by Ramakrishnan and Ray.[13,14] In the study of Xu and Feng,[20] a new Lyapunov–Krasovskii functional is introduced to reduce the conservativeness. In the study of

Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran

Corresponding author:
Mohammad Farrokhi, Department of Electrical Engineering, Iran University of Science and Technology, Farjam Street, Narmak, Tehran 16846-13114, Iran.
Email: farrokhi@iust.ac.ir

Wu et al.,[21] a free-weighting matrix method has been employed to improve the delay-dependent stability of systems with a time-varying delay. The idea of N-segmentation of delay length in Lyapunov–Krasovskii functional is proposed by Wu et al.[24] to acquire less conservative results. This idea is improved by Kazemy and Farrokhi[23] by introducing adaptive segmentation of delay interval.

In this article, the problem of delay-dependent robust absolute stability of uncertain multiple time-delayed Lur'e systems with sector-bounded nonlinearity is investigated. The nonlinearity is assumed to be both time invariant and time varying. Based on the Lyapunov–Krasovskii stability theory and matrix decomposition methods, some delay-dependent sufficient conditions for the robust absolute stability of the Lur'e system are derived and expressed in the form of linear matrix inequalities (LMIs). In addition, multiple time delays for uncertain Lur'e systems will also be considered. In order to obtain higher values for the time delays, a matrix decomposition approach is employed in this article. Some examples are given to show that the proposed stability criteria are less conservative than those reported in well-established literatures.

The main contribution of this article is to obtain less conservative results as compared to the recently proposed methods in well-established literatures. Moreover, Lur'e systems with multiple time delays and parametric uncertainty are considered.

This article is organized as follows. In the section "Problem statement and preliminaries," the Lur'e system with uncertainties is defined. The main results are proposed in the section "Main results." In the section "Illustrative examples," some numerical examples are given and the results are compared with recent methods, reported in literatures. The final section concludes the article.

Notations. Throughout this article, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices. $\mathbf{P} > 0$ means that $\mathbf{P}$ is a real, positive-definite, and symmetric matrix. $\mathbb{C}[-h, 0]$ denotes the space of continuous functions defined on $[-h, 0]$, $\mathbf{I}$ is the identity matrix with appropriate dimensions, $\mathrm{diag}\{\mathbf{W}_1, \ldots, \mathbf{W}_m\}$ refers to a real matrix with diagonal elements $\mathbf{W}_1, \ldots, \mathbf{W}_m$, and $\mathbf{A}^T$ denotes the transpose of the real matrix $\mathbf{A}$. Symmetric terms in a symmetric matrix are denoted by $*$.

Problem statement and preliminaries

Consider the following uncertain Lur'e system with multiple time delays[20]

$$\begin{cases} \dot{\mathbf{x}}(t) = \bar{\mathbf{A}}_o\mathbf{x}(t) + \sum_{i=1}^{m} \bar{\mathbf{B}}_i\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\boldsymbol{\omega}(t), \\ \mathbf{z}(t) = \mathbf{M}\mathbf{x}(t) + \sum_{i=1}^{m} \mathbf{N}_i\mathbf{x}(t-h_i), \\ \boldsymbol{\omega}(t) = -\boldsymbol{\varphi}(t, \mathbf{z}(t)), \\ \mathbf{x}(t) = \boldsymbol{\phi}(t), \quad \forall t \in \left[-\max_{1\leqslant i\leqslant m}\{h_i\}, 0\right] \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ denotes the state vector, $\boldsymbol{\omega}(t) \in \mathbb{R}^p$ is the input, $\mathbf{z}(t) \in \mathbb{R}^p$ is the output, $\boldsymbol{\phi}(t) \in \mathbb{C}([h, 0], \mathbb{R}^n)$ is a continuous vector-valued initial function, m is the number of time delays, and $\boldsymbol{\varphi}(t, \mathbf{z}(t)) \in \mathbb{R}^p$ is a nonlinear function which will be defined in Assumption 1. Moreover, $h_i \geqslant 0$ $(i = 1, \ldots, m)$ are time delays, and $\mathbf{M}$ and $\mathbf{N}_i$ $(i = 1, \ldots, m) \in \mathbb{R}^{p\times n}$ are known real constant matrices. In addition, $\bar{\mathbf{A}}_o, \bar{\mathbf{B}}_i\,(i = 1, \ldots, m) \in \mathbb{R}^{n\times n}$, and $\bar{\mathbf{D}} \in \mathbb{R}^{n\times p}$ are time-varying matrices with the following structures

$$\begin{aligned} \bar{\mathbf{A}}_o &= \mathbf{A}_o + \Delta\mathbf{A}_o(t), \quad \bar{\mathbf{D}} = \mathbf{D} + \Delta\mathbf{D}(t), \\ \bar{\mathbf{B}}_i &= \mathbf{B}_i + \Delta\mathbf{B}_i(t) \quad (i = 1, \ldots, m) \end{aligned} \tag{2}$$

where $\mathbf{A}_o$, $\mathbf{B}_i$ $(i = 1, \ldots, m)$, and $\mathbf{D}$ are known real constant matrices. $\Delta\mathbf{A}_o(t)$, $\Delta\mathbf{B}_i(t)$ $(i = 1, \ldots, m)$, and $\Delta\mathbf{D}(t)$ are norm-bounded parameter uncertainties and are assumed to be of the form

$$\begin{aligned} &[\Delta\mathbf{A}_o(t), \Delta\mathbf{B}_1(t), \ldots, \Delta\mathbf{B}_m(t), \Delta\mathbf{D}(t)] \\ &\quad = \mathbf{L}\mathbf{F}(t)[\mathbf{E}, \mathbf{E}_1, \ldots, \mathbf{E}_m, \mathbf{H}] \end{aligned} \tag{3}$$

where $\mathbf{L}, \mathbf{E}, \mathbf{E}_i$ $(i = 1, \ldots, m)$, and $\mathbf{H}$ are known real constant matrices with appropriate dimensions and $\mathbf{F}(t) \in \mathbb{R}^{q\times k}$ is the unknown time-varying real matrix satisfying

$$\mathbf{F}^T(t)\mathbf{F}(t) \leqslant \mathbf{I} \tag{4}$$

Remark 1. In the uncertainty structure, given by equation (3), matrices $\mathbf{L}, \mathbf{E}, \mathbf{E}_i (i = 1, \ldots, m)$, and $\mathbf{H}$ characterize how uncertain parameters in $\mathbf{F}(t)$ affect matrices $\bar{\mathbf{A}}_o$, $\bar{\mathbf{B}}_i(i = 1, \ldots, m)$, and $\bar{\mathbf{D}}$. Matrix $\mathbf{F}(t)$ can often be restricted as equation (4) by appropriate selections of $\mathbf{L}, \mathbf{E}, \mathbf{E}_i(i = 1, \ldots, m)$, and $\mathbf{H}$.

Assumption 1. The nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t)) \in \mathbb{R}^p$ in equation (1) is piecewise continuous in t, globally Lipschitz in $\mathbf{z}(t)$, $\boldsymbol{\varphi}(t, 0) = 0$, and satisfies the following sector condition for any $t \geqslant 0$ and $\mathbf{z}(t) \in \mathbb{R}^p$

$$[\boldsymbol{\varphi}(t, \mathbf{z}(t)) - \mathbf{K}_1\mathbf{z}(t)]^T[\boldsymbol{\varphi}(t, \mathbf{z}(t)) - \mathbf{K}_2\mathbf{z}(t)] \leqslant 0 \tag{5}$$

Such a nonlinear function is said to belong to the sector $[\mathbf{K}_1, \mathbf{K}_2]$.[1]

Definition 1. The nonlinear delay system (1) is said to be robustly absolutely stable in the sector $[\mathbf{K}_1, \mathbf{K}_2]$ if it is globally uniformly asymptotically stable for any nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$ satisfying Assumption 1 and for all admissible uncertainties.[20]

The following section will show the main results proposed in this article.

Main results

In order to improve the delay bounds h_i, let us decompose matrix $\bar{\mathbf{B}}_i$ as $\bar{\mathbf{B}}_i = \mathbf{B}_{i1} + \mathbf{B}_{i2} + \Delta\mathbf{B}_i(t) = \mathbf{B}_{i1} + \bar{\mathbf{B}}_{i2}$. Then, using the Newton–Leibniz equation

$$\mathbf{x}(t - h_i) = \mathbf{x}(t) - \int_{t-h_i}^{t} \dot{\mathbf{x}}(s)ds \tag{6}$$

the original system can be transformed into the following form

$$\dot{\mathbf{x}}(t) = \bar{\mathbf{A}}_o\mathbf{x}(t) + \sum_{i=1}^{m}\mathbf{B}_{i1}\left(\mathbf{x}(t) - \int_{t-h_i}^{t}\dot{\mathbf{x}}(s)ds\right) + \sum_{i=1}^{m}\bar{\mathbf{B}}_{i2}\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\boldsymbol{\omega}(t) \quad (7)$$

or equivalently

$$\frac{d}{dt}\left(\mathbf{x}(t) + \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right) = \bar{\mathbf{A}}\mathbf{x}(t) + \sum_{i=1}^{m}\bar{\mathbf{B}}_{i2}\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\boldsymbol{\omega}(t) \quad (8)$$

where $\bar{\mathbf{A}} = \bar{\mathbf{A}}_o + \sum_{i=1}^{m}\mathbf{B}_{i1}$, with the initial condition given in equation (1). Let $\boldsymbol{\Gamma}(x_t)$ be a new operator defined as

$$\boldsymbol{\Gamma}(x_t) := \mathbf{x}(t) + \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds \quad (9)$$

To guarantee that the difference operator $\boldsymbol{\Gamma}(x_t) \in \mathbb{C}([h,0],\mathbb{R}^n)$ is stable, the following assumption should hold.

Assumption 2. Assume $\sum_{i=1}^{m} h_i\|\mathbf{B}_{i1}\| < 1$, where $\|\cdot\|$ is any matrix norm.[9]

By applying the loop transformation suggested by Khalil,[1] equation (8) with the nonlinear function in equation (5) can be transformed into the following form

$$\frac{d}{dt}\left(\mathbf{x}(t) + \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right) = \left(\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{M}\right)\mathbf{x}(t) + \sum_{i=1}^{m}(\bar{\mathbf{B}}_{i2} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{N}_i)\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\,\underset{\sim}{\boldsymbol{\omega}}(t) \quad (10)$$

where $\underset{\sim}{\boldsymbol{\omega}}(t) = -\boldsymbol{\varphi}(t,\mathbf{z}(t))$. The nonlinear function $\underset{\sim}{\boldsymbol{\varphi}}(t,\mathbf{z}(t))$ satisfies the following inequality

$$\underset{\sim}{\boldsymbol{\varphi}}^T(t,\mathbf{z}(t))\left[\underset{\sim}{\boldsymbol{\varphi}}(t,\mathbf{z}(t)) - \underset{\sim}{\mathbf{K}}\,\mathbf{z}(t)\right] \leqslant 0 \quad \forall t > 0 \quad (11)$$

where $\underset{\sim}{\mathbf{K}} = \mathbf{K}_2 - \mathbf{K}_1$. Considering equations (2) and (3), it gives

$$\begin{aligned}\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{M} &= \mathbf{A}_o + \sum_{i=1}^{m}\mathbf{B}_{i1} + \mathbf{LF}(t)\mathbf{E} - (\mathbf{D} + \mathbf{LF}(t)\mathbf{H})\mathbf{K}_1\mathbf{M}\\ &= \mathbf{A}_o - \mathbf{DK}_1\mathbf{M} + \sum_{i=1}^{m}\mathbf{B}_{i1} + \mathbf{LF}(t)(\mathbf{E} - \mathbf{HK}_1\mathbf{M})\\ &= \underset{\sim}{\mathbf{A}}_o + \sum_{i=1}^{m}\mathbf{B}_{i1} + \mathbf{LF}(t)\,\underset{\sim}{\mathbf{E}}\\ &= \underset{\sim}{\mathbf{A}} + \mathbf{LF}(t)\,\underset{\sim}{\mathbf{E}} = \underset{\sim}{\bar{\mathbf{A}}}\end{aligned} \quad (12)$$

and

$$\begin{aligned}\bar{\mathbf{B}}_{i2} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{N}_i &= \mathbf{B}_{i2} + \mathbf{LF}(t)\mathbf{E}_i - (\mathbf{D} + \mathbf{LF}(t)\mathbf{H})\mathbf{K}_1\mathbf{N}_i\\ &= (\mathbf{B}_{i2} - \mathbf{DK}_1\mathbf{N}_i) + \mathbf{LF}(t)(\mathbf{E}_i - \mathbf{HK}_1\mathbf{N}_i)\\ &= \underset{\sim}{\mathbf{B}}_{i2} + \mathbf{LF}(t)\underset{\sim}{\mathbf{E}}_i \quad (i = 1,\ldots,m).\\ &= \underset{\sim}{\bar{\mathbf{B}}}_{i2}\end{aligned} \quad (13)$$

Substituting equations (12) and (13) into equation (10) yields

$$\frac{d}{dt}\left(\mathbf{x}(t) + \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right) = \underset{\sim}{\bar{\mathbf{A}}}\,\mathbf{x}(t) + \sum_{i=1}^{m}\underset{\sim}{\bar{\mathbf{B}}}_{i2}\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\,\underset{\sim}{\boldsymbol{\omega}}(t) \quad (14)$$

Hence, the absolute stability of system (14) is the same as the absolute stability of system (1).[11] The following theorem shows the absolute stability of system (1).

Theorem 1. The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(t,\mathbf{z}(t))$ satisfying equation (5) and $\boldsymbol{\varphi}(t,0) = 0$ is robustly absolutely stable if there exist scalars $\gamma > 0$ and $\delta > 0$, symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$, $\mathbf{C}_i > 0$, $\mathbf{W}_i > 0$, and $\mathbf{G}_i > 0$, and arbitrary matrices $\mathbf{R}_i$ $(i = 1,\ldots,m)$ such that the following LMIs hold

$$\begin{bmatrix} \boldsymbol{\eta} & \prod_1 & \prod_2 & \cdots & \prod_m & \coprod_1 & \coprod_2 & \cdots & \coprod_m & \boldsymbol{\Psi} & \underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} & \mathbf{PL}\\ * & \boldsymbol{\Sigma}_1 & \delta\,\underset{\sim}{\mathbf{E}}_1^T\underset{\sim}{\mathbf{E}}_2 & \cdots & \delta\,\underset{\sim}{\mathbf{E}}_1^T\underset{\sim}{\mathbf{E}}_m & \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1m} & \boldsymbol{\Psi}_{11} & \underset{\sim}{\mathbf{B}}_1^T\boldsymbol{\Phi} & 0\\ * & * & \boldsymbol{\Sigma}_2 & \cdots & \delta\,\underset{\sim}{\mathbf{E}}_2^T\underset{\sim}{\mathbf{E}}_m & \Omega_{21} & \Omega_{22} & \cdots & \Omega_{2m} & \boldsymbol{\Psi}_{12} & \underset{\sim}{\mathbf{B}}_2^T\boldsymbol{\Phi} & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & * & \boldsymbol{\Sigma}_m & \Omega_{m1} & \Omega_{m2} & \cdots & \Omega_{mm} & \boldsymbol{\Psi}_{1m} & \underset{\sim}{\mathbf{B}}_m^T\boldsymbol{\Phi} & 0\\ * & * & * & * & * & \boldsymbol{\theta}_{11} & \boldsymbol{\theta}_{12} & \cdots & \boldsymbol{\theta}_{1m} & \boldsymbol{\Psi}_{21} & -\mathbf{B}_{11}^T\,\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} & \mathbf{R}_1^T\mathbf{L}\\ * & * & * & * & * & * & \boldsymbol{\theta}_{22} & \cdots & \boldsymbol{\theta}_{2m} & \boldsymbol{\Psi}_{22} & -\mathbf{B}_{21}^T\,\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} & \mathbf{R}_2^T\mathbf{L}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & * & * & * & * & * & \boldsymbol{\theta}_{mm} & \boldsymbol{\Psi}_{2m} & -\mathbf{B}_{m1}^T\,\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} & \mathbf{R}_m^T\mathbf{L}\\ * & * & * & * & * & * & * & * & * & \mho & \mathbf{D}^T\boldsymbol{\Phi} & 0\\ * & * & * & * & * & * & * & * & * & * & -\boldsymbol{\Phi} & \boldsymbol{\Phi}\mathbf{L}\\ * & * & * & * & * & * & * & * & * & * & * & -\delta\mathbf{I} \end{bmatrix} < 0 \quad (15)$$

$$\begin{bmatrix} \mathbf{P}/(mh_i) & \mathbf{R}_i \\ \mathbf{R}_i^T & \mathbf{Q}_i \end{bmatrix} > 0 \quad (i = 1, \ldots, m) \tag{16}$$

where

$$\boldsymbol{\eta} = \underset{\sim}{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\underset{\sim}{\mathbf{A}} + \sum\nolimits_{i=1}^{m}\left(\mathbf{R}_i + \mathbf{R}_i^T + \mathbf{Q}_i + h_i\mathbf{C}_i - \mathbf{G}_i\right) + \delta\underset{\sim}{\mathbf{E}}^T\underset{\sim}{\mathbf{E}},\ \prod\nolimits_i = \mathbf{P}\underset{\sim}{\mathbf{B}}_{i2} - \mathbf{R}_i + \mathbf{G}_i + \delta\underset{\sim}{\mathbf{E}}^T\underset{\sim}{\mathbf{E}}_i$$

$$\prod\nolimits_i = -\mathbf{P}\underset{\sim}{\mathbf{A}}\underset{\sim}{\mathbf{B}}_{i1} + \underset{\sim}{\mathbf{A}}^T\mathbf{R}_i + \boldsymbol{\mu}\mathbf{B}_{i1} + \mathbf{W}_i - \delta\underset{\sim}{\mathbf{E}}^T\underset{\sim}{\mathbf{E}}\mathbf{B}_{i1},\ \boldsymbol{\Psi} = \mathbf{P}\mathbf{D} - \gamma\mathbf{M}^T\underset{\sim}{\mathbf{K}}^T + \delta\underset{\sim}{\mathbf{E}}^T\mathbf{H},$$

$$\boldsymbol{\Sigma}_i = -\mathbf{Q}_i - \mathbf{G}_i + \delta\,\underset{\sim}{\mathbf{E}}_i^T\underset{\sim}{\mathbf{E}}_i,\ \Omega_{ii} = \underset{\sim}{\mathbf{B}}_{i2}^T\mathbf{R}_i - \mathbf{W}_i - \mathbf{G}_i\mathbf{B}_{i1} - \delta\,\underset{\sim}{\mathbf{E}}_i^T\underset{\sim}{\mathbf{E}}\mathbf{B}_{i1},\ \boldsymbol{\Psi}_{1i} = -\gamma\mathbf{N}_i^T\underset{\sim}{\mathbf{K}}^T + \delta\,\underset{\sim}{\mathbf{E}}_i^T\mathbf{H},$$

$$\Omega_{ij} = \underset{\sim}{\mathbf{B}}_{i2}^T\mathbf{R}_j - \mathbf{G}_i\mathbf{B}_{j1} - \delta\,\underset{\sim}{\mathbf{E}}_i^T\underset{\sim}{\mathbf{E}}\mathbf{B}_{j1},\ \boldsymbol{\Psi}_{2i} = \mathbf{R}_i^T\mathbf{D} + \gamma\mathbf{B}_{i1}^T\mathbf{M}^T\underset{\sim}{\mathbf{K}}^T - \delta\mathbf{B}_{i1}^T\underset{\sim}{\mathbf{E}}^T\mathbf{H},$$

$$\mho = -2\gamma + \delta\mathbf{H}^T\mathbf{H},\ \boldsymbol{\theta}_{ij} = \mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\mathbf{W}_j - \mathbf{W}_i\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\underset{\sim}{\mathbf{A}}^T\mathbf{R}_j - \mathbf{R}_i^T\underset{\sim}{\mathbf{A}}\mathbf{B}_{j1} + \delta\mathbf{B}_{i1}^T\underset{\sim}{\mathbf{E}}^T\underset{\sim}{\mathbf{E}}\mathbf{B}_{j1},$$

$$\boldsymbol{\theta}_{ii} = 2\mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{i1} - \mathbf{C}_i/h_i - \mathbf{B}_{i1}^T\mathbf{W}_i - \mathbf{W}_i\mathbf{B}_{i1} - \mathbf{B}_{i1}^T\underset{\sim}{\mathbf{A}}^T\mathbf{R}_i - \mathbf{R}_i^T\underset{\sim}{\mathbf{A}}\mathbf{B}_{i1} + \delta\mathbf{B}_{i1}^T\underset{\sim}{\mathbf{E}}^T\underset{\sim}{\mathbf{E}}\mathbf{B}_{i1},$$

$$\boldsymbol{\Phi} = \sum\nolimits_{i=1}^{m} h_i^2\mathbf{G}_i,\ \boldsymbol{\Sigma} = \sum\nolimits_{j=1}^{m}\left(\mathbf{Q}_j + h_j\mathbf{C}_j - \mathbf{G}_j\right),\ \boldsymbol{\mu} = \sum\nolimits_{j=1}^{m}\left(-\mathbf{R}_j - \mathbf{Q}_j - h_j\mathbf{C}_j + \mathbf{G}_j\right),$$

$$\underset{\sim}{\mathbf{K}} = \mathbf{K}_2 - \mathbf{K}_1,\ \underset{\sim}{\mathbf{A}} = \mathbf{A} - \mathbf{D}\mathbf{K}_1\mathbf{M},\ \underset{\sim}{\mathbf{E}} = \mathbf{E} - \mathbf{H}\mathbf{K}_1\mathbf{M},\ \underset{\sim}{\mathbf{B}}_{i2} = \mathbf{B}_{i2} - \mathbf{D}\mathbf{K}_1\mathbf{N}_i,\ \underset{\sim}{\mathbf{B}}_i = \mathbf{B}_i - \mathbf{D}\mathbf{K}_1\mathbf{N}_i,$$

$$\underset{\sim}{\mathbf{E}}_i = \mathbf{E}_i - \mathbf{H}\mathbf{K}_1\mathbf{N}_i,\ \mathbf{A} = \mathbf{A}_o + \sum\nolimits_{i=1}^{m}\mathbf{B}_{i1},\ \underset{\sim}{\mathbf{A}}_o = \mathbf{A}_o - \mathbf{D}\mathbf{K}_1\mathbf{M},\ (i, j = 1, \ldots, m\ ;\ i \neq j)$$

Proof. Consider the following Lyapunov–Krasovskii functional

$$V(\mathbf{x}_t) = V_1(\mathbf{x}_t) + V_2(\mathbf{x}_t) + V_3(\mathbf{x}_t) + V_4(\mathbf{x}_t) \tag{17}$$

where

$$V_1(\mathbf{x}_t) = \sum_{j=1}^{m}\int_{t-h_j}^{t}\left(\begin{bmatrix}\mathbf{x}^T(t) & \mathbf{x}^T(s)\end{bmatrix}\begin{bmatrix}\mathbf{P}/(mh_j) & \mathbf{R}_j \\ \mathbf{R}_j^T & \mathbf{Q}_j\end{bmatrix}\begin{bmatrix}\mathbf{x}(t) \\ \mathbf{x}(s)\end{bmatrix}\right)$$

$$V_2(\mathbf{x}_t) = \sum_{j=1}^{m}\int_{-h_j}^{0}\int_{t+\beta}^{t}\mathbf{x}^T(s)\mathbf{C}_j\mathbf{x}(s)\,ds\,d\beta$$

$$V_3(\mathbf{x}_t) = \sum_{j=1}^{m}\left(\int_{t-h_j}^{t}\mathbf{x}^T(s)ds\right)\mathbf{W}_j\left(\int_{t-h_j}^{t}\mathbf{x}(s)ds\right)$$

$$V_4(\mathbf{x}_t) = \sum_{j=1}^{m}h_j\int_{-h_j}^{0}\int_{t+\beta}^{t}\dot{\mathbf{x}}^T(s)\mathbf{G}_j\dot{\mathbf{x}}(s)\,ds\,d\beta$$

where $\mathbf{x}_t$ is defined as $\mathbf{x}_t = \mathbf{x}(t+\theta)$, $\theta \in [-\max_{1\leqslant i\leqslant m}\{h_i\}, 0]$.

Taking the derivative of $V(\mathbf{x}_t)$ with respect to t yields

$$\begin{aligned}\dot{V}_1(\mathbf{x}) = {} & 2\boldsymbol{\Gamma}^T(t)\mathbf{P}\dot{\boldsymbol{\Gamma}}(t) + 2\boldsymbol{\Gamma}^T(t)\sum_{j=1}^{m}\mathbf{R}_j\left(\mathbf{x}(t) - \mathbf{x}(t-h_j)\right) \\ & + 2\dot{\boldsymbol{\Gamma}}^T(t)\sum_{j=1}^{m}\mathbf{R}_j\int_{t-h_j}^{t}\mathbf{x}(s)ds + \mathbf{x}^T(t)\left(\sum_{j=1}^{m}\mathbf{Q}_j\right)\mathbf{x}(t) \\ & - \sum_{j=1}^{m}\mathbf{x}^T(t-h_j)\mathbf{Q}_j\mathbf{x}^T(t-h_j)\end{aligned} \tag{18}$$

where $\mathbf{x}(t)$ and $\dot{\boldsymbol{\Gamma}}(t)$ are

$$\mathbf{x}(t) = \boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds \tag{19}$$

$$\begin{aligned}\dot{\boldsymbol{\Gamma}}(t) = {} & \bar{\underset{\sim}{\mathbf{A}}}\boldsymbol{\Gamma}(t) - \bar{\underset{\sim}{\mathbf{A}}}\sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds \\ & + \sum_{i=1}^{m}\bar{\underset{\sim}{\mathbf{B}}}_{i2}\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\underset{\sim}{\boldsymbol{\omega}}(t)\end{aligned} \tag{20}$$

The second term of equation (17) becomes

$$\dot{V}_2(\mathbf{x}_t) = \mathbf{x}^T(t)\left(\sum_{j=1}^{m}h_j\mathbf{C}_j\right)\mathbf{x}(t) - \sum_{j=1}^{m}\int_{t-h_j}^{t}\mathbf{x}^T(s)\mathbf{C}_j\mathbf{x}(s)ds$$

Using Jensen's inequality[26] and equation (19) gives

$$\begin{aligned}\dot{V}_2(\mathbf{x}_t) \leqslant {} & \left(\boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right)^T\left(\sum_{j=1}^{m}h_j\mathbf{C}_j\right) \\ & \left(\boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right) \\ & - \sum_{j=1}^{m}\left(\int_{t-h_j}^{t}\mathbf{x}^T(s)ds\right)(1/h_j)\mathbf{C}_j\left(\int_{t-h_j}^{t}\mathbf{x}(s)ds\right)\end{aligned} \tag{21}$$

The third and fourth terms of equation (17) become

$$\dot{V}_3(\mathbf{x}_t) = 2\left(\boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right)^T\sum_{j=1}^{m}W_j\left(\int_{t-h_j}^{t}\mathbf{x}(s)ds\right) - 2\sum_{j=1}^{m}\mathbf{x}^T(t-h_j)W_j\left(\int_{t-h_j}^{t}\mathbf{x}(s)ds\right) \tag{22}$$

$$\dot{V}_4(\mathbf{x}_t) = \dot{\mathbf{x}}^T(t)\left(\sum_{j=1}^{m}h_j^2\mathbf{G}_j\right)\dot{\mathbf{x}}(t) - \sum_{j=1}^{m}\int_{t-h_j}^{t}\dot{\mathbf{x}}^T(s)h_j\mathbf{G}_j\dot{\mathbf{x}}(s)ds \leqslant \dot{\mathbf{x}}^T(t)\left(\sum_{j=1}^{m}h_j^2\mathbf{G}_j\right)\dot{\mathbf{x}}(t) - \sum_{j=1}^{m}\left(\int_{t-h_j}^{t}\dot{\mathbf{x}}^T(s)ds\right)\mathbf{G}_j\left(\int_{t-h_j}^{t}\dot{\mathbf{x}}(s)ds\right) \tag{23}$$

where

$$\dot{\mathbf{x}}(t) = \underset{\sim}{\bar{\mathbf{A}}}_o\boldsymbol{\Gamma}(t) - \underset{\sim}{\bar{\mathbf{A}}}_o\sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds + \sum_{i=1}^{m}\underset{\sim}{\bar{\mathbf{B}}}_i\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\underset{\sim}{\boldsymbol{\omega}}(t)$$

Using equation (6), equation (23) can be expressed as

$$\begin{aligned}\dot{V}_4(\mathbf{x}_t) = {} & \left(\underset{\sim}{\bar{\mathbf{A}}}_o\boldsymbol{\Gamma}(t) - \underset{\sim}{\bar{\mathbf{A}}}_o\sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds + \sum_{i=1}^{m}\underset{\sim}{\bar{\mathbf{B}}}_i\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\underset{\sim}{\boldsymbol{\omega}}(t)\right)^T\left(\sum_{j=1}^{m}h_j^2\mathbf{G}_j\right)\\ & \left(\underset{\sim}{\bar{\mathbf{A}}}_o\boldsymbol{\Gamma}(t) - \underset{\sim}{\bar{\mathbf{A}}}_o\sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds + \sum_{i=1}^{m}\underset{\sim}{\bar{\mathbf{B}}}_i\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\underset{\sim}{\boldsymbol{\omega}}(t)\right) - \left(\boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right)^T\left(\sum_{j=1}^{m}\mathbf{G}_j\right)\\ & \left(\boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right) + 2\left(\boldsymbol{\Gamma}(t) - \sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds\right)^T\sum_{j=1}^{m}\mathbf{G}_j\mathbf{x}(t-h_j) - \sum_{j=1}^{m}\mathbf{x}^T(t-h_j)\mathbf{G}_j\mathbf{x}^T(t-h_j)\end{aligned} \tag{24}$$

The sector condition (11) can be written as

$$-\underset{\sim}{\boldsymbol{\omega}}^T(t)\left[\underset{\sim}{\boldsymbol{\omega}}(t) + \underset{\sim}{\mathbf{K}}\mathbf{z}(t)\right] \geqslant 0 \tag{25}$$

where $\mathbf{z}(t)$ is the same as in equation (1).

Hence, $\dot{V}(\mathbf{x}_t)$ can be written as

$$\dot{V}(\mathbf{x}_t) \leqslant \dot{V}_1(\mathbf{x}_t) + \dot{V}_2(\mathbf{x}_t) + \dot{V}_3(\mathbf{x}_t) + \dot{V}_4(\mathbf{x}_t) - 2\gamma\underset{\sim}{\boldsymbol{\omega}}^T(t)\left[\underset{\sim}{\boldsymbol{\omega}}(t) + \underset{\sim}{\mathbf{K}}\mathbf{z}(t)\right] \tag{26}$$

where $\gamma > 0$. Considering equations (18) to (26), it is straightforward to show that

$$\dot{V}(\mathbf{x}_t) \leqslant \boldsymbol{\xi}^T(t)\boldsymbol{\Xi}\boldsymbol{\xi}(t) \tag{27}$$

where

$$\boldsymbol{\Xi} = \begin{bmatrix} \hat{\boldsymbol{\eta}} & \hat{\prod}_1 & \hat{\prod}_2 & \cdots & \hat{\prod}_m & \hat{\coprod}_1 & \hat{\coprod}_2 & \cdots & \hat{\coprod}_m & \hat{\boldsymbol{\Psi}} \\ * & \hat{\boldsymbol{\Sigma}}_1 & 0 & \cdots & 0 & \hat{\Omega}_{11} & \hat{\Omega}_{12} & \cdots & \hat{\Omega}_{1m} & \hat{\boldsymbol{\Psi}}_{11} \\ * & * & \hat{\boldsymbol{\Sigma}}_2 & \cdots & 0 & \hat{\Omega}_{21} & \hat{\Omega}_{22} & \cdots & \hat{\Omega}_{2m} & \hat{\boldsymbol{\Psi}}_{12} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \hat{\boldsymbol{\Sigma}}_m & \hat{\Omega}_{m1} & \hat{\Omega}_{m2} & \cdots & \hat{\Omega}_{mm} & \hat{\boldsymbol{\Psi}}_{1m} \\ * & * & * & * & * & \hat{\boldsymbol{\theta}}_{11} & \hat{\boldsymbol{\theta}}_{12} & \cdots & \hat{\boldsymbol{\theta}}_{1m} & \hat{\boldsymbol{\Psi}}_{21} \\ * & * & * & * & * & * & \hat{\boldsymbol{\theta}}_{22} & \cdots & \hat{\boldsymbol{\theta}}_{2m} & \hat{\boldsymbol{\Psi}}_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & \hat{\boldsymbol{\theta}}_{mm} & \hat{\boldsymbol{\Psi}}_{2m} \\ * & * & * & * & * & * & * & * & * & \hat{\mho} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{\bar{\mathbf{A}}}_o^T \\ \underset{\sim}{\bar{\mathbf{B}}}_1^T \\ \underset{\sim}{\bar{\mathbf{B}}}_2^T \\ \vdots \\ \underset{\sim}{\bar{\mathbf{B}}}_m^T \\ -\mathbf{B}_{11}^T\underset{\sim}{\bar{\mathbf{A}}}_o^T \\ -\mathbf{B}_{21}^T\underset{\sim}{\bar{\mathbf{A}}}_o^T \\ \vdots \\ -\mathbf{B}_{m1}^T\underset{\sim}{\bar{\mathbf{A}}}_o^T \\ \bar{\mathbf{D}}^T \end{bmatrix} \boldsymbol{\Phi} \begin{bmatrix} \underset{\sim}{\bar{\mathbf{A}}}_o^T \\ \underset{\sim}{\bar{\mathbf{B}}}_1^T \\ \underset{\sim}{\bar{\mathbf{B}}}_2^T \\ \vdots \\ \underset{\sim}{\bar{\mathbf{B}}}_m^T \\ -\mathbf{B}_{11}^T\underset{\sim}{\bar{\mathbf{A}}}_o^T \\ -\mathbf{B}_{21}^T\underset{\sim}{\bar{\mathbf{A}}}_o^T \\ \vdots \\ -\mathbf{B}_{m1}^T\underset{\sim}{\bar{\mathbf{A}}}_o^T \\ \bar{\mathbf{D}}^T \end{bmatrix}^T \tag{28}$$

$$\boldsymbol{\xi}(t) = \begin{bmatrix}\boldsymbol{\Gamma}^T(t) & \boldsymbol{\xi}_1^T(t) & \boldsymbol{\xi}_2^T(t) & \underset{\sim}{\boldsymbol{\omega}}^T(t)\end{bmatrix}^T$$

in which

$$\boldsymbol{\xi}_1(t) = \left[\mathbf{x}^T(t-h_1) \quad \cdots \quad \mathbf{x}^T(t-h_m)\right]^T,\ \boldsymbol{\xi}_2(t) = \left[\int_{t-h_1}^{t}\mathbf{x}^T(s)ds \quad \cdots \quad \int_{t-h_m}^{t}\mathbf{x}^T(s)ds\right]^T,$$

$$\hat{\boldsymbol{\eta}} = \underset{\sim}{\bar{\mathbf{A}}}^T\mathbf{P} + \mathbf{P}\,\underset{\sim}{\bar{\mathbf{A}}} + \sum_{i=1}^{m}\left(\mathbf{R}_i + \mathbf{R}_i^T + \mathbf{Q}_i + h_i\mathbf{C}_i - \mathbf{G}_i\right),\ \hat{\prod}_i = \mathbf{P}\underset{\sim}{\bar{\mathbf{B}}}_{i2} - \mathbf{R}_i + \mathbf{G}_i,$$

$$\hat{\Psi} = \mathbf{P}\bar{\mathbf{D}} - \gamma\mathbf{M}^T\underset{\sim}{\mathbf{K}}^T,\ \hat{\boldsymbol{\theta}}_{ii} = 2\mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{i1} - \mathbf{C}_i/h_i - \mathbf{B}_{i1}^T\mathbf{W}_i - \mathbf{W}_i\mathbf{B}_{i1} - \mathbf{B}_{i1}^T\underset{\sim}{\bar{\mathbf{A}}}^T\mathbf{R}_i - \mathbf{R}_i^T\,\underset{\sim}{\bar{\mathbf{A}}}\,\mathbf{B}_{i1},$$

$$\hat{\coprod}_i = -\mathbf{P}\,\underset{\sim}{\bar{\mathbf{A}}}\,\mathbf{B}_{i1} + \underset{\sim}{\bar{\mathbf{A}}}^T\mathbf{R}_i + \boldsymbol{\mu}\mathbf{B}_{i1} + \mathbf{W}_i,\ \hat{\boldsymbol{\Sigma}}_i = -\mathbf{Q}_i - \mathbf{G}_i,\ \hat{\Omega}_{ii} = \underset{\sim}{\bar{\mathbf{B}}}_{i2}^T\mathbf{R}_i - \mathbf{W}_i - \mathbf{G}_i\mathbf{B}_{i1}$$

$$\hat{\Omega}_{ij} = \underset{\sim}{\bar{\mathbf{B}}}_{i2}^T\mathbf{R}_j - \mathbf{G}_i\mathbf{B}_{j1},\ \hat{\boldsymbol{\Psi}}_{1i} = -\gamma\mathbf{N}_i^T\underset{\sim}{\mathbf{K}}^T,\ \hat{\boldsymbol{\Psi}}_{2i} = \mathbf{R}_i^T\bar{\mathbf{D}} + \gamma\mathbf{B}_{i1}^T\mathbf{M}^T\underset{\sim}{\mathbf{K}}^T,\ \hat{\mho} = -2\gamma,$$

$$\hat{\boldsymbol{\theta}}_{ij} = \mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\mathbf{W}_j - \mathbf{W}_i\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\underset{\sim}{\bar{\mathbf{A}}}^T\mathbf{R}_j - \mathbf{R}_i^T\,\underset{\sim}{\bar{\mathbf{A}}}\,\mathbf{B}_{j1},\ (i,j = 1,\ldots,m\ ;\ i \neq j)$$

The other parameters are defined in equation (15).

If we can show that $\boldsymbol{\Xi} < 0$ in equation (28), then $\dot{V}(\mathbf{x}_t) < 0$. Therefore, by Definition 1 and Lyapunov–Krasovskii theorem,[26] the considered nonlinear delayed system is robustly absolutely stable. But the matrix $\boldsymbol{\Xi}$ is not an LMI and should be transformed into one.

Using Schur complement,[27] $\boldsymbol{\Xi}$ can be rewritten as

$$\begin{bmatrix}
\hat{\boldsymbol{\eta}} & \hat{\prod}_1 & \hat{\prod}_2 & \cdots & \hat{\prod}_m & \hat{\coprod}_1 & \hat{\coprod}_2 & \cdots & \hat{\coprod}_m & \hat{\boldsymbol{\Psi}} & \underset{\sim}{\bar{\mathbf{A}}}_o^T\boldsymbol{\Phi} \\
* & \hat{\boldsymbol{\Sigma}}_1 & 0 & \cdots & 0 & \hat{\Omega}_{11} & \hat{\Omega}_{12} & \cdots & \hat{\Omega}_{1m} & \hat{\boldsymbol{\Psi}}_{11} & \underset{\sim}{\bar{\mathbf{B}}}_1^T\boldsymbol{\Phi} \\
* & * & \hat{\boldsymbol{\Sigma}}_2 & \cdots & 0 & \hat{\Omega}_{21} & \hat{\Omega}_{22} & \cdots & \hat{\Omega}_{2m} & \hat{\boldsymbol{\Psi}}_{12} & \underset{\sim}{\bar{\mathbf{B}}}_2^T\boldsymbol{\Phi} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & \hat{\boldsymbol{\Sigma}}_m & \hat{\Omega}_{m1} & \hat{\Omega}_{m2} & \cdots & \hat{\Omega}_{mm} & \hat{\boldsymbol{\Psi}}_{1m} & \underset{\sim}{\bar{\mathbf{B}}}_m^T\boldsymbol{\Phi} \\
* & * & * & * & * & \hat{\boldsymbol{\theta}}_{11} & \hat{\boldsymbol{\theta}}_{12} & \cdots & \hat{\boldsymbol{\theta}}_{1m} & \hat{\boldsymbol{\Psi}}_{21} & -\mathbf{B}_{11}^T\,\underset{\sim}{\bar{\mathbf{A}}}_o^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & \hat{\boldsymbol{\theta}}_{22} & \cdots & \hat{\boldsymbol{\theta}}_{2m} & \hat{\boldsymbol{\Psi}}_{22} & -\mathbf{B}_{21}^T\,\underset{\sim}{\bar{\mathbf{A}}}_o^T\boldsymbol{\Phi} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & \hat{\boldsymbol{\theta}}_{mm} & \hat{\boldsymbol{\Psi}}_{2m} & -\mathbf{B}_{m1}^T\,\underset{\sim}{\bar{\mathbf{A}}}_o^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & * & * & * & \hat{\mho} & \bar{\mathbf{D}}^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & * & * & * & * & -\boldsymbol{\Phi}
\end{bmatrix} < 0 \qquad (29)$$

where the parameters are defined in equation (28).

Considering equations (2) and (3), equation (29) can be written as

$$\begin{bmatrix}
\tilde{\boldsymbol{\eta}} & \tilde{\prod}_1 & \tilde{\prod}_2 & \cdots & \tilde{\prod}_m & \tilde{\coprod}_1 & \tilde{\coprod}_2 & \cdots & \tilde{\coprod}_m & \tilde{\boldsymbol{\Psi}} & \underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
* & \hat{\boldsymbol{\Sigma}}_1 & 0 & \cdots & 0 & \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \cdots & \tilde{\Omega}_{1m} & \hat{\boldsymbol{\Psi}}_{11} & \underset{\sim}{\mathbf{B}}_1^T\boldsymbol{\Phi} \\
* & * & \hat{\boldsymbol{\Sigma}}_2 & \cdots & 0 & \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & \cdots & \tilde{\Omega}_{2m} & \hat{\boldsymbol{\Psi}}_{12} & \underset{\sim}{\mathbf{B}}_2^T\boldsymbol{\Phi} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & \hat{\boldsymbol{\Sigma}}_m & \tilde{\Omega}_{m1} & \tilde{\Omega}_{m2} & \cdots & \tilde{\Omega}_{mm} & \hat{\boldsymbol{\Psi}}_{1m} & \underset{\sim}{\mathbf{B}}_m^T\boldsymbol{\Phi} \\
* & * & * & * & * & \tilde{\boldsymbol{\theta}}_{11} & \tilde{\boldsymbol{\theta}}_{12} & \cdots & \tilde{\boldsymbol{\theta}}_{1m} & \tilde{\boldsymbol{\Psi}}_{21} & -\mathbf{B}_{11}^T\,\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & \tilde{\boldsymbol{\theta}}_{22} & \cdots & \tilde{\boldsymbol{\theta}}_{2m} & \tilde{\boldsymbol{\Psi}}_{22} & -\mathbf{B}_{21}^T\,\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & \tilde{\boldsymbol{\theta}}_{mm} & \tilde{\boldsymbol{\Psi}}_{2m} & -\mathbf{B}_{m1}^T\,\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & * & * & * & \hat{\mho} & \mathbf{D}^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & * & * & * & * & -\boldsymbol{\Phi}
\end{bmatrix} + \mathbf{U}\mathbf{F}(t)\mathbf{V} + \mathbf{V}^T\mathbf{F}^T(t)\mathbf{U}^T < 0 \qquad (30)$$

where

$$\tilde{\boldsymbol{\eta}} = \underset{\sim}{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\underset{\sim}{\mathbf{A}} + \sum\nolimits_{i=1}^{m}\left(\mathbf{R}_i + \mathbf{R}_i^T + \mathbf{Q}_i + h_i\mathbf{C}_i - \mathbf{G}_i\right),\ \tilde{\prod}_i = \mathbf{P}\underset{\sim}{\mathbf{B}}_{i2} - \mathbf{R}_i + \mathbf{G}_i,$$

$$\tilde{\Psi} = \mathbf{PD} - \gamma\mathbf{M}^T\underset{\sim}{\mathbf{K}}^T,\ \tilde{\coprod}_i = -\mathbf{P}\underset{\sim}{\mathbf{A}}\mathbf{B}_{i1} + \underset{\sim}{\mathbf{A}}^T\mathbf{R}_i + \boldsymbol{\mu}\mathbf{B}_{i1} + \mathbf{W}_i,\ \tilde{\Omega}_{ii} = \underset{\sim}{\mathbf{B}}_{i2}^T\mathbf{R}_i - \mathbf{W}_i - \mathbf{G}_i\mathbf{B}_{i1},$$

$$\tilde{\Omega}_{ij} = \underset{\sim}{\mathbf{B}}_{i2}^T\mathbf{R}_j - \mathbf{G}_i\mathbf{B}_{j1},\ \tilde{\boldsymbol{\Psi}}_{2i} = \mathbf{R}_i^T\mathbf{D} + \gamma\mathbf{B}_{i1}^T\mathbf{M}^T\underset{\sim}{\mathbf{K}}^T,$$

$$\tilde{\boldsymbol{\theta}}_{ii} = 2\mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{i1} - \mathbf{C}_i/h_i - \mathbf{B}_{i1}^T\mathbf{W}_i - \mathbf{W}_i\mathbf{B}_{i1} - \mathbf{B}_{i1}^T\underset{\sim}{\mathbf{A}}^T\mathbf{R}_i - \mathbf{R}_i^T\underset{\sim}{\mathbf{A}}\mathbf{B}_{i1},$$

$$\tilde{\boldsymbol{\theta}}_{ij} = \mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\mathbf{W}_j - \mathbf{W}_i\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\underset{\sim}{\mathbf{A}}^T\mathbf{R}_j - \mathbf{R}_i^T\underset{\sim}{\mathbf{A}}\mathbf{B}_{j1},\ (i,j = 1, \ldots, m\ ;\ i \neq j),$$

$$\mathbf{U} = \begin{bmatrix}\mathbf{L}^T\mathbf{P} & 0 & \cdots & 0 & \mathbf{L}^T\mathbf{R}_1 & \cdots & \mathbf{L}^T\mathbf{R}_m & 0 & \mathbf{L}^T\boldsymbol{\Phi}\end{bmatrix}^T,$$
$$\mathbf{V} = \begin{bmatrix}\underset{\sim}{\mathbf{E}} & \underset{\sim}{\mathbf{E}}_1 & \cdots & \underset{\sim}{\mathbf{E}}_m & -\underset{\sim}{\mathbf{E}}\mathbf{B}_{11} & \cdots & -\underset{\sim}{\mathbf{E}}\mathbf{B}_{m1} & \mathbf{H} & 0\end{bmatrix}$$

By using the lemma that is introduced by Peterson and Hollot[28] and equation (4), equation (30) becomes

$$\begin{bmatrix}
\boldsymbol{\eta} & \prod_1 & \prod_2 & \cdots & \prod_m & \coprod_1 & \coprod_2 & \cdots & \coprod_m & \boldsymbol{\Psi} & \underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
* & \boldsymbol{\Sigma}_1 & \delta\underset{\sim}{\mathbf{E}}_1^T\underset{\sim}{\mathbf{E}}_2 & \cdots & \delta\underset{\sim}{\mathbf{E}}_1^T\underset{\sim}{\mathbf{E}}_m & \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1m} & \boldsymbol{\Psi}_{11} & \underset{\sim}{\mathbf{B}}_1^T\boldsymbol{\Phi} \\
* & * & \boldsymbol{\Sigma}_2 & \cdots & \delta\underset{\sim}{\mathbf{E}}_2^T\underset{\sim}{\mathbf{E}}_m & \Omega_{21} & \Omega_{22} & \cdots & \Omega_{2m} & \boldsymbol{\Psi}_{12} & \underset{\sim}{\mathbf{B}}_2^T\boldsymbol{\Phi} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & \boldsymbol{\Sigma}_m & \Omega_{m1} & \Omega_{m2} & \cdots & \Omega_{mm} & \boldsymbol{\Psi}_{1m} & \underset{\sim}{\mathbf{B}}_m^T\boldsymbol{\Phi} \\
* & * & * & * & * & \boldsymbol{\theta}_{11} & \boldsymbol{\theta}_{12} & \cdots & \boldsymbol{\theta}_{1m} & \boldsymbol{\Psi}_{21} & -\mathbf{B}_{11}^T\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & \boldsymbol{\theta}_{22} & \cdots & \boldsymbol{\theta}_{2m} & \boldsymbol{\Psi}_{22} & -\mathbf{B}_{21}^T\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & \boldsymbol{\theta}_{mm} & \boldsymbol{\Psi}_{2m} & -\mathbf{B}_{m1}^T\underset{\sim}{\mathbf{A}}_o^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & * & * & * & \mho & \mathbf{D}^T\boldsymbol{\Phi} \\
* & * & * & * & * & * & * & * & * & * & -\boldsymbol{\Phi}
\end{bmatrix} + \delta^{-1}\mathbf{U}\mathbf{U}^T < 0 \tag{31}$$

where all elements are the same as in equation (15).

Again, by using Schur complement, equation (31) can be transformed to the form of equation (15). This completes the proof.

Remark 2. In the Lyapunov functional (17), the terms related to $\mathbf{P}$ and $\mathbf{Q}_i$ are conventional Lyapunov–Krasovskii functional terms. $V_2(\mathbf{x}_t)$ is introduced to obtain the delay-dependent results. The other terms, especially the cross terms $\mathbf{R}_i$, are added to reduce the conservativeness of the results.

Remark 3. By introducing the matrix decomposition ($\bar{\mathbf{B}}_i = \mathbf{B}_{i1} + \bar{\mathbf{B}}_{i2}$), the structure of the system matrices $\bar{\mathbf{A}}_o$ and $\bar{\mathbf{B}}_i$ are changed to $\bar{\mathbf{A}}_o + \sum_{i=1}^{m}\mathbf{B}_{i1}$ and $\bar{\mathbf{B}}_{i2} = \bar{\mathbf{B}}_i - \mathbf{B}_{i1}$, respectively. Hence, tuning matrices $\mathbf{B}_{i1}$ affect the matrices of the new system and provide larger feasible region to the LMI solver software. In other words, the matrix decomposition expands the searching area of the feasible solutions for the LMI solver. As a consequence, the results can be less conservative. However, it should be noted that proper values for matrices $\mathbf{B}_{i1}$ should be found with some methods (e.g. trial and error).

Remark 4. The matrix inequality (15) is linear in the unknown parameters $\gamma > 0$, $\delta > 0$, $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$, $\mathbf{C}_i > 0$, $\mathbf{W}_i > 0$, $\mathbf{G}_i > 0$, and R_i $(i = 1, \ldots, m)$. Therefore, it can be solved using available LMI solving software such as MATLAB LMI Toolbox.

Remark 5. Since condition (15) is independent of $\mathbf{F}(t)$, the robust absolute stability of system (1) with nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$, satisfying equation (5), is guaranteed for all admissible $\mathbf{F}(t)$ satisfying equation (4).

Next, it is assumed that the nonlinear function in equation (1) is time-invariant decentralized. Hence, $\boldsymbol{\omega}(t)$ in equation (1) will be $\boldsymbol{\omega}(t) = -\boldsymbol{\varphi}(\mathbf{z}(t))$. In this case, the nonlinear function

$$\boldsymbol{\varphi}(\mathbf{z}(t)) = \begin{bmatrix}\varphi_1(\mathbf{z}_1(t)) & \varphi_2(\mathbf{z}_2(t)) & \cdots & \varphi_p(\mathbf{z}_p(t))\end{bmatrix} \tag{32}$$

satisfies

$$\alpha_i z_i^2(t) \leqslant z_i(t)\varphi_i(z_i(t)) \leqslant \beta_i z_i^2(t) \quad \forall t \geqslant 0, \qquad \beta_i \geqslant \alpha_i > 0 \quad (i = 1, \ldots, p) \tag{33}$$

The following theorem shows the delay-dependent robust absolute stability result of this system.

Theorem 2. The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(\mathbf{z}(t))$ satisfying equation (33) and $\boldsymbol{\varphi}(0) = 0$ is robustly absolutely stable if there exist scalar $\delta > 0$, symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$, $\mathbf{C}_i > 0$, $\mathbf{W}_i > 0$, $\mathbf{G}_i > 0$, and $\mathbf{T}_i > 0$, arbitrary matrices $\mathbf{R}_i$ $(i = 1, \ldots, m)$, and two diagonal matrices $\mathbf{Y} = \mathrm{diag}\{y_1, y_2, \ldots, y_p\} > 0$ and $\mathbf{S} = \mathrm{diag}\{s_1, s_2, \ldots, s_p\} > 0$ such that the following LMIs hold

Proof. Let us select a Lyapunov–Krasovskii functional candidate as

$$\hat{V}(\mathbf{x}_t) = V(\mathbf{x}_t) + \sum_{j=1}^{m} \int_{t-h_j}^{t} \dot{\mathbf{x}}^T(s)\mathbf{T}_j\dot{\mathbf{x}}(s)ds + 2\sum_{j=1}^{p} s_j \int_0^{z_j(t)} \varphi_j(s)ds \tag{36}$$

where $V(\mathbf{x}_t)$ is defined in equation (17). Taking the derivative of $\hat{V}(\mathbf{x}_t)$

$$\dot{\hat{V}}(\mathbf{x}_t) = \dot{V}(\mathbf{x}_t) + \dot{\mathbf{x}}^T(t)\left(\sum_{j=1}^{m}\mathbf{T}_j\right)\dot{\mathbf{x}}(t) - \sum_{j=1}^{m}\dot{\mathbf{x}}^T(t-h_j)\mathbf{T}_j\dot{\mathbf{x}}(t-h_j) + 2\boldsymbol{\varphi}^T(\mathbf{z}(t))\mathbf{S}\dot{\mathbf{z}}(t) \tag{37}$$

$$\begin{bmatrix}
\boldsymbol{\eta} & \prod_1 & \prod_2 & \cdots & \prod_m & \coprod_1 & \coprod_2 & \cdots & \coprod_m & \boldsymbol{\Psi} & 0 & \mathbf{A}_o^T\boldsymbol{\Phi} & \mathbf{PL} \\
* & \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \cdots & \boldsymbol{\Sigma}_{1m} & \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1m} & \boldsymbol{\Psi}_{11} & 0 & \mathbf{B}_1^T\boldsymbol{\Phi} & 0 \\
* & * & \boldsymbol{\Sigma}_{22} & \cdots & \boldsymbol{\Sigma}_{2m} & \Omega_{21} & \Omega_{22} & \cdots & \Omega_{2m} & \boldsymbol{\Psi}_{12} & 0 & \mathbf{B}_2^T\boldsymbol{\Phi} & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & \boldsymbol{\Sigma}_{mm} & \Omega_{m1} & \Omega_{m2} & \cdots & \Omega_{mm} & \boldsymbol{\Psi}_{1m} & 0 & \mathbf{B}_m^T\boldsymbol{\Phi} & 0 \\
* & * & * & * & * & \boldsymbol{\theta}_{11} & \boldsymbol{\theta}_{12} & \cdots & \boldsymbol{\theta}_{1m} & \boldsymbol{\Psi}_{21} & 0 & -\mathbf{B}_{11}^T\mathbf{A}_o^T\boldsymbol{\Phi} & \mathbf{R}_1^T\mathbf{L} \\
* & * & * & * & * & * & \boldsymbol{\theta}_{22} & \cdots & \boldsymbol{\theta}_{2m} & \boldsymbol{\Psi}_{22} & 0 & -\mathbf{B}_{21}^T\mathbf{A}_o^T\boldsymbol{\Phi} & \mathbf{R}_2^T\mathbf{L} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & \boldsymbol{\theta}_{mm} & \boldsymbol{\Psi}_{2m} & 0 & -\mathbf{B}_{m1}^T\mathbf{A}_o^T\boldsymbol{\Phi} & \mathbf{R}_m^T\mathbf{L} \\
* & * & * & * & * & * & * & * & * & \mho & -\mathbf{S}\tilde{\mathbf{N}} & \mathbf{D}^T\boldsymbol{\Phi} & -\mathbf{SML} \\
* & * & * & * & * & * & * & * & * & * & -\tilde{\mathbf{T}} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\boldsymbol{\Phi} & \boldsymbol{\Phi}\mathbf{L} \\
* & * & * & * & * & * & * & * & * & * & * & * & -\delta\mathbf{I}
\end{bmatrix} < 0$$

$$\begin{bmatrix} \mathbf{P}/(mh_i) & \mathbf{R}_i \\ \mathbf{R}_i^T & \mathbf{Q}_i \end{bmatrix} > 0 \, (i = 1, \ldots, m) \tag{35}$$

where

$$\boldsymbol{\eta} = \mathbf{A}^T\mathbf{P} + \mathbf{PA} + \sum\nolimits_{i=1}^{m}\left(\mathbf{R}_i + \mathbf{R}_i^T + \mathbf{Q}_i + h_i\mathbf{C}_i - \mathbf{G}_i\right) - 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YM} + \delta\mathbf{E}^T\mathbf{E},$$

$$\prod\nolimits_i = \mathbf{PB}_{i2} - \mathbf{R}_i + \mathbf{G}_i - 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YN}_1 + \delta\mathbf{E}^T\mathbf{E}_i, \ \boldsymbol{\Sigma}_{ii} = -\mathbf{Q}_i - \mathbf{G}_i - 2\mathbf{N}_i^T\boldsymbol{\alpha\beta}\mathbf{YN}_i + \delta\mathbf{E}_i^T\mathbf{E}_i,$$

$$\coprod\nolimits_i = -\mathbf{PAB}_{i1} + \mathbf{A}^T\mathbf{R}_i + \boldsymbol{\mu}\mathbf{B}_{i1} + \mathbf{W}_i - \delta\mathbf{E}^T\mathbf{EB}_{i1}, \ \Psi = \mathbf{PD} - \mathbf{A}_o^T\mathbf{M}^T\mathbf{S} - \mathbf{M}^T(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y} + \delta\mathbf{E}^T\mathbf{H},$$

$$\boldsymbol{\Sigma}_{ij} = -\mathbf{N}_i^T\boldsymbol{\alpha\beta}\mathbf{YN}_j + \delta\mathbf{E}_i^T\mathbf{E}_j, \ \Omega_{ii} = \mathbf{B}_{i2}^T\mathbf{R}_i - \mathbf{W}_i - \mathbf{G}_i\mathbf{B}_{i1} + 2\mathbf{N}_i^T\boldsymbol{\alpha\beta}\mathbf{YMB}_{i1} - \delta\mathbf{E}_i^T\mathbf{EB}_{i1},$$

$$\Omega_{ij} = \mathbf{B}_{i2}^T\mathbf{R}_j - \mathbf{G}_i\mathbf{B}_{j1} + 2\mathbf{N}_i^T\boldsymbol{\alpha\beta}\mathbf{YMB}_{j1} - \delta\mathbf{E}_i^T\mathbf{EB}_{j1}, \ \boldsymbol{\Psi}_{1i} = -\mathbf{B}_i^T\mathbf{M}^T\mathbf{S} - \mathbf{N}_i^T(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y} + \delta\mathbf{E}_i^T\mathbf{H},$$

$$\boldsymbol{\Psi}_{2i} = \mathbf{R}_i^T\mathbf{D} + \mathbf{B}_{i1}^T\mathbf{A}_o^T\mathbf{M}^T\mathbf{S} + \mathbf{B}_{i1}^T\mathbf{M}^T(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y} - \delta\mathbf{B}_{i1}^T\mathbf{E}^T\mathbf{H}, \ \boldsymbol{\Phi} = \sum\nolimits_{i=1}^{m}\left(h_i^2\mathbf{G}_i + \mathbf{T}_i\right),$$

$$\mho = -2\mathbf{Y} - \mathbf{SMD} - \mathbf{D}^T\mathbf{M}^T\mathbf{S} + \delta\mathbf{H}^T\mathbf{H}, \ \boldsymbol{\Sigma} = \sum\nolimits_{j=1}^{m}\left(\mathbf{Q}_j + h_j\mathbf{C}_j - \mathbf{G}_j\right), \ \tilde{\mathbf{T}} = \mathrm{diag}\{\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_m\},$$

$$\boldsymbol{\theta}_{ii} = 2\mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{i1} - \mathbf{C}_i/h_i - \mathbf{B}_{i1}^T\mathbf{W}_i - \mathbf{W}_i\mathbf{B}_{i1} - \mathbf{B}_{i1}^T\mathbf{A}^T\mathbf{R}_i - \mathbf{R}_i^T\mathbf{AB}_{i1} + 2\mathbf{B}_{i1}^T\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YMB}_{i1} + \delta\mathbf{B}_{i1}^T\mathbf{E}^T\mathbf{EB}_{i1},$$

$$\boldsymbol{\theta}_{ij} = \mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\mathbf{W}_j - \mathbf{W}_i\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\mathbf{A}^T\mathbf{R}_j - \mathbf{R}_i^T\mathbf{AB}_{j1} + 2\mathbf{B}_{i1}^T\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YMB}_{j1} + \delta\mathbf{B}_{i1}^T\mathbf{E}^T\mathbf{EB}_{j1},$$

$$\boldsymbol{\mu} = \sum\nolimits_{j=1}^{m}\left(-\mathbf{R}_j - \mathbf{Q}_j - h_j\mathbf{C}_j + \mathbf{G}_j\right) + 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YM}, \ \tilde{\mathbf{N}} = [\mathbf{N}_1 \quad \mathbf{N}_2 \quad \cdots \quad \mathbf{N}_m],$$

$$\boldsymbol{\alpha} = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_p\}, \ \boldsymbol{\beta} = \mathrm{diag}\{\beta_1, \beta_2, \ldots, \beta_p\}, \ (i,j = 1, \ldots, m \ ; \ i \neq j).$$

Noting that $\mathbf{Y} > 0$, the sector condition (33) can be written as

$$2y_i[\varphi_i(z_i(t)) - \alpha_i z_i(t)][\varphi_i(z_i(t)) - \beta_i z_i(t)] \leqslant 0 \quad (i = 1, \ldots, p) \tag{38}$$

that is

$$2y_i\varphi_i^2(z_i(t)) - 2y_i(\alpha_i + \beta_i)\varphi_i(z_i(t))z_i(t) + 2y_i\alpha_i\beta_i z_i^2(t) \leqslant 0 \quad (i = 1, \ldots, p) \tag{39}$$

Therefore

$$-2\boldsymbol{\omega}^T(t)\mathbf{Y}\boldsymbol{\omega}(t) - 2\boldsymbol{\omega}^T(t)(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y}\mathbf{z}(t) - 2\mathbf{z}^T(t)\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{z}(t) \geqslant 0 \tag{40}$$

where $\mathbf{z}(t)$ and $\boldsymbol{\omega}(t)$ are defined in equation (1) and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\mathbf{Y}$ are defined in equation (34).

Hence, $\dot{\hat{V}}(\mathbf{x}_t)$ can be expressed as

$$\dot{\hat{V}}(\mathbf{x}_t) = \dot{V}(\mathbf{x}_t) + \dot{\mathbf{x}}^T(t)\left(\sum_{j=1}^{m}\mathbf{T}_j\right)\dot{\mathbf{x}}(t) - \sum_{j=1}^{m}\dot{\mathbf{x}}^T(t-h_j)\mathbf{T}_j\dot{\mathbf{x}}(t-h_j) + 2\boldsymbol{\varphi}^T(\mathbf{z}(t))\mathbf{S}\left(\mathbf{M}\dot{\mathbf{x}}(t) + \sum_{i=1}^{m}\mathbf{N}_i\dot{\mathbf{x}}(t-h_i)\right) \tag{41}$$

where

$$\dot{\mathbf{x}}(t) = \bar{\mathbf{A}}_o\boldsymbol{\Gamma}(t) - \bar{\mathbf{A}}_o\sum_{i=1}^{m}\mathbf{B}_{i1}\int_{t-h_i}^{t}\mathbf{x}(s)ds + \sum_{i=1}^{m}\bar{\mathbf{B}}_i\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\boldsymbol{\omega}(t)$$

Substituting $\dot{\mathbf{x}}(t)$ in equation (41) and considering equations (36) to (41), it is straightforward to show that

$$\dot{\hat{V}}(\mathbf{x}_t) \leqslant \hat{\boldsymbol{\xi}}^T(t)\hat{\boldsymbol{\Xi}}\hat{\boldsymbol{\xi}}(t) \tag{42}$$

where

$$\hat{\boldsymbol{\Xi}} = \begin{bmatrix} \hat{\boldsymbol{\eta}} & \bar{\prod}_1 & \bar{\prod}_2 & \cdots & \bar{\prod}_m & \hat{\coprod}_1 & \hat{\coprod}_2 & \cdots & \hat{\coprod}_m & \hat{\boldsymbol{\Psi}} & 0 \\ * & \hat{\boldsymbol{\Sigma}}_{11} & \hat{\boldsymbol{\Sigma}}_{12} & \cdots & \hat{\boldsymbol{\Sigma}}_{1m} & \hat{\Omega}_{11} & \hat{\Omega}_{12} & \cdots & \hat{\Omega}_{1m} & \hat{\boldsymbol{\Psi}}_{11} & 0 \\ * & * & \hat{\boldsymbol{\Sigma}}_{22} & \cdots & \hat{\boldsymbol{\Sigma}}_{2m} & \hat{\Omega}_{21} & \hat{\Omega}_{22} & \cdots & \hat{\Omega}_{2m} & \hat{\boldsymbol{\Psi}}_{12} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & * & * & \hat{\boldsymbol{\Sigma}}_{mm} & \hat{\Omega}_{m1} & \hat{\Omega}_{m2} & \cdots & \hat{\Omega}_{mm} & \hat{\boldsymbol{\Psi}}_{1m} & 0 \\ * & * & * & * & * & \hat{\boldsymbol{\theta}}_{11} & \hat{\boldsymbol{\theta}}_{12} & \cdots & \hat{\boldsymbol{\theta}}_{1m} & \hat{\boldsymbol{\Psi}}_{21} & 0 \\ * & * & * & * & * & * & \hat{\boldsymbol{\theta}}_{22} & \cdots & \hat{\boldsymbol{\theta}}_{2m} & \hat{\boldsymbol{\Psi}}_{22} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & \hat{\boldsymbol{\theta}}_{mm} & \hat{\boldsymbol{\Psi}}_{2m} & 0 \\ * & * & * & * & * & * & * & * & * & \hat{\mho} & -\mathbf{S}\tilde{\mathbf{N}} \\ * & * & * & * & * & * & * & * & * & * & -\tilde{\mathbf{T}} \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{A}}_o^T \\ \bar{\mathbf{B}}_1^T \\ \bar{\mathbf{B}}_2^T \\ \vdots \\ \bar{\mathbf{B}}_m^T \\ -\mathbf{B}_{11}^T\bar{\mathbf{A}}_o^T \\ -\mathbf{B}_{21}^T\bar{\mathbf{A}}_o^T \\ \vdots \\ -\mathbf{B}_{m1}^T\bar{\mathbf{A}}_o^T \\ \bar{\mathbf{D}}^T \\ 0 \end{bmatrix} \boldsymbol{\Phi} \begin{bmatrix} \bar{\mathbf{A}}_o^T \\ \bar{\mathbf{B}}_1^T \\ \bar{\mathbf{B}}_2^T \\ \vdots \\ \bar{\mathbf{B}}_m^T \\ -\mathbf{B}_{11}^T\bar{\mathbf{A}}_o^T \\ -\mathbf{B}_{21}^T\bar{\mathbf{A}}_o^T \\ \vdots \\ -\mathbf{B}_{m1}^T\bar{\mathbf{A}}_o^T \\ \bar{\mathbf{D}}^T \\ 0 \end{bmatrix}^T \tag{43}$$

in which

$$\hat{\boldsymbol{\eta}} = \bar{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\bar{\mathbf{A}} + \textstyle\sum_{i=1}^{m}\left(\mathbf{R}_i + \mathbf{R}_i^T + \mathbf{Q}_i + h_i\mathbf{C}_i - \mathbf{G}_i\right) - 2\mathbf{M}^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{M},$$

$$\bar{\prod}_i = \mathbf{P}\bar{\mathbf{B}}_{i2} - \mathbf{R}_i + \mathbf{G}_i - 2\mathbf{M}^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{N}_1,\ \hat{\coprod}_i = -\mathbf{P}\bar{\mathbf{A}}\mathbf{B}_{i1} + \bar{\mathbf{A}}^T\mathbf{R}_i + \boldsymbol{\mu}\mathbf{B}_{i1} + \mathbf{W}_i,$$

$$\hat{\boldsymbol{\Psi}} = \mathbf{P}\bar{\mathbf{D}} - \bar{\mathbf{A}}_o^T\mathbf{M}^T\mathbf{S} - \mathbf{M}^T(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y},\ \hat{\boldsymbol{\Sigma}}_{ii} = -\mathbf{Q}_i - \mathbf{G}_i - 2\mathbf{N}_i^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{N}_i,$$

$$\hat{\boldsymbol{\Sigma}}_{ij} = -\mathbf{N}_i^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{N},\ \hat{\Omega}_{ii} = \bar{\mathbf{B}}_{i2}^T\mathbf{R}_i - \mathbf{W}_i - \mathbf{G}_i\mathbf{B}_{i1} + 2\mathbf{N}_i^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{M}\mathbf{B}_{i1},$$

$$\hat{\Omega}_{ij} = \bar{\mathbf{B}}_{i2}^T\mathbf{R}_j - \mathbf{G}_i\mathbf{B}_{j1} + 2\mathbf{N}_i^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{M}\mathbf{B}_{j1},\ \hat{\boldsymbol{\Psi}}_{1i} = -\bar{\mathbf{B}}_i^T\mathbf{M}^T\mathbf{S} - \mathbf{N}_i^T(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y},$$

$$\hat{\boldsymbol{\Psi}}_{2i} = \mathbf{R}_i^T\bar{\mathbf{D}} + \mathbf{B}_{i1}^T\bar{\mathbf{A}}_o^T\mathbf{M}^T\mathbf{S} + \mathbf{B}_{i1}^T\mathbf{M}^T(\boldsymbol{\alpha} + \boldsymbol{\beta})\mathbf{Y},\ \hat{\mho} = -2\mathbf{Y} - \mathbf{S}\mathbf{M}\bar{\mathbf{D}} - \bar{\mathbf{D}}^T\mathbf{M}^T\mathbf{S},$$

$$\hat{\boldsymbol{\theta}}_{ii} = 2\mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{i1} - \mathbf{C}_i/h_i - \mathbf{B}_{i1}^T\mathbf{W}_i - \mathbf{W}_i\mathbf{B}_{i1} - \mathbf{B}_{i1}^T\bar{\mathbf{A}}^T\mathbf{R}_i - \mathbf{R}_i^T\bar{\mathbf{A}}\mathbf{B}_{i1} + 2\mathbf{B}_{i1}^T\mathbf{M}^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{M}\mathbf{B}_{i1},$$

$$\hat{\boldsymbol{\theta}}_{ij} = \mathbf{B}_{i1}^T\boldsymbol{\Sigma}\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\mathbf{W}_j - \mathbf{W}_i\mathbf{B}_{j1} - \mathbf{B}_{i1}^T\bar{\mathbf{A}}^T\mathbf{R}_j - \mathbf{R}_i^T\bar{\mathbf{A}}\mathbf{B}_{j1} + 2\mathbf{B}_{i1}^T\mathbf{M}^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{M}\mathbf{B}_{j1},$$

$$\boldsymbol{\xi}(t) = \left[\boldsymbol{\Gamma}^T(t) \quad \boldsymbol{\xi}_1^T(t) \quad \boldsymbol{\xi}_2^T(t) \quad \boldsymbol{\omega}^T(t) \quad \boldsymbol{\xi}_3^T(t)\right]^T,\ \boldsymbol{\xi}_1(t) = \left[\mathbf{x}^T(t-h_1) \quad \cdots \quad \mathbf{x}^T(t-h_m)\right]^T,$$

$$\boldsymbol{\xi}_2(t) = \left[\int_{t-h_1}^{t}\mathbf{x}^T(s)ds \quad \cdots \quad \int_{t-h_m}^{t}\mathbf{x}^T(s)ds\right]^T,\ \boldsymbol{\xi}_3(t) = \left[\dot{\mathbf{x}}^T(t-h_1) \quad \cdots \quad \dot{\mathbf{x}}^T(t-h_m)\right]^T,$$

$$(i,j = 1, \ldots, m\ ;\ i \neq j)$$

and $\mathbf{\Phi}$, $\mathbf{\Sigma}$, $\tilde{\mathbf{T}}$, $\boldsymbol{\mu}$, $\tilde{\mathbf{N}}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ are defined in equation (34).

The rest of the proof is similar to Theorem 1.

Remark 6. Matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ in equation (5) can be full matrices; however, matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are diagonal. In the case of a time-invariant decentralized nonlinear function, $\mathbf{K}_1 = \boldsymbol{\alpha}$ and $\mathbf{K}_2 = \boldsymbol{\beta}$.

Illustrative examples

To demonstrate the applicability of the proposed method and compare them with the previously reported results, three examples are given here. The first example considers a single–time-delay system, while the second and third examples are about multiple time-delay systems.

Example 1. Consider the system described by equation (1) with single time delay and the following parameters:[20,29]

$$\mathbf{A}_o = \begin{bmatrix} -2 & -1 \\ 0.5 & 0.2 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0.5 & 1 \\ -0.1 & -0.8 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix}, \mathbf{M} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{N}_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$\mathbf{K}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{E} = \mathbf{E}_1 = \mathbf{H} = \mathbf{L},$$

$$\mathbf{L} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{B}_{11} = \begin{bmatrix} 0.08 & -0.2 \\ 0.12 & -0.182 \end{bmatrix}$$

A comparison of the delay-dependent conditions between this article and different methods in the studies of Xu and Feng,[20] Kazemy and Farrokhi,[23] Wu et al.,[24] and Han[29] is given in Table 1. It is obvious that the maximum allowable delay $\bar{h}$ calculated by Theorem 1 in this article is larger than other methods. Hence, it can be concluded that Theorem 1 presented in this article yields less conservative results as compared with those in the study of Xu and Feng,[20] Kazemy and Farrokhi,[23] Wu et al.[24] and Han[29]. As mentioned in Remark 3, the matrix decomposition expands the searching area of the feasible solutions for the LMI solver, and hence, it yields better result as compared with the previously reported work of Kazemy and Farrokhi.[23]

Example 2. Consider the system described in Example 1 with multiple time delays and the following parameters

$$\mathbf{A}_o = \begin{bmatrix} -2 & -1 \\ 0.5 & 0.2 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0.5 & 1 \\ -0.1 & -0.8 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} -0.1 & 0.3 \\ -0.1 & -0.1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{N}_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \mathbf{N}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix},$$

Table 1. Comparison of the delay-dependent conditions for Example 1.

Method	Maximum allowable delay $\bar{h}$
Corollary 8 in the study of Han[29]	2.0239
Proposition 11 in the study of Han[29]	2.0263
Theorem 3 in the study of Xu and Fang[20]	2.2103
Theorem 3 in the study of Wu et al.[24] ($n = 2$)	2.3306
Theorem 3 in the study of Wu et al.[24] ($n = 3$)	2.3691
Corollary 1 in the study of Kazemy and Farrokhi[23] ($\alpha = 0.42$)	2.3700
Corollary 2 in the study of Kazemy and Farrokhi[23] ($\alpha = 0.42$)	2.4398
Theorem 1 in this article	2.6112

Table 2. Results of example 2.

Method	$h_1 = 2.5$	$h_1 = 3$	$h_1 = 3.5$
h_2 by Theorem 2 in this article	1.26	1.19	1.16
h_2 by Corollary 2 in the study of Kazemy and Farrokhi[23]	No feasible solution		

$$\mathbf{E} = \mathbf{E}_1 = \mathbf{E}_2 = \mathbf{H} = \mathbf{L}, \mathbf{L} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_{11} = \begin{bmatrix} 0.08 & -0.2 \\ 0.12 & -0.182 \end{bmatrix}, \mathbf{B}_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Maximum allowable delays h_1 and h_2, which are calculated using Theorem 2, are given in Table 2. It should be mentioned that there is no feasible solution using Corollaries 1 and 2 in the study of Kazemy and Farrokhi.[23]

Example 3. Consider a system described by equation (1) with two time delays and the following parameters[23]

$$\mathbf{A}_o = \begin{bmatrix} -1.2 & 0 \\ 0.8 & -1 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} -1 & 0.6 \\ -0.6 & -1 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} -0.5 & 0.4 \\ -1 & -1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{N}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{N}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \boldsymbol{\alpha} = \mathbf{N}_1,$$

$$\boldsymbol{\beta} = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix}, \mathbf{L} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$\mathbf{E}_1 = \mathbf{H} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, \mathbf{E}_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},$$

$$\mathbf{F}(t) = \begin{bmatrix} 0.5\sin(t) & 0 \\ 0 & 0.8\cos(t) \end{bmatrix}, \boldsymbol{\varphi}(\mathbf{z}(t)) = \begin{bmatrix} \tanh(z_1(t)) \\ 1.5\tanh(z_2(t)) \end{bmatrix},$$

$$\mathbf{B}_{11} = \begin{bmatrix} -0.06 & 0.07 \\ -0.05 & -0.1 \end{bmatrix}, \mathbf{B}_{12} = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

Figure 1 shows the stability regions defined by Theorems 1 and 2. In the case of $\mathbf{B}_{11} = \mathbf{B}_{21} = \mathbf{0}$,

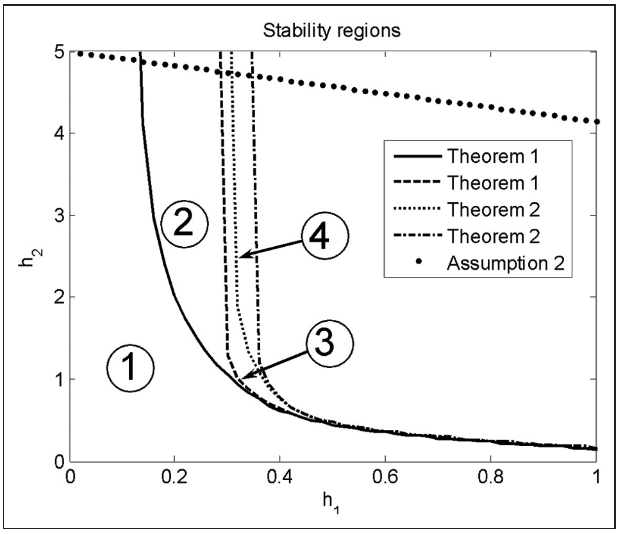

Figure 1. Stability regions for Example 3.

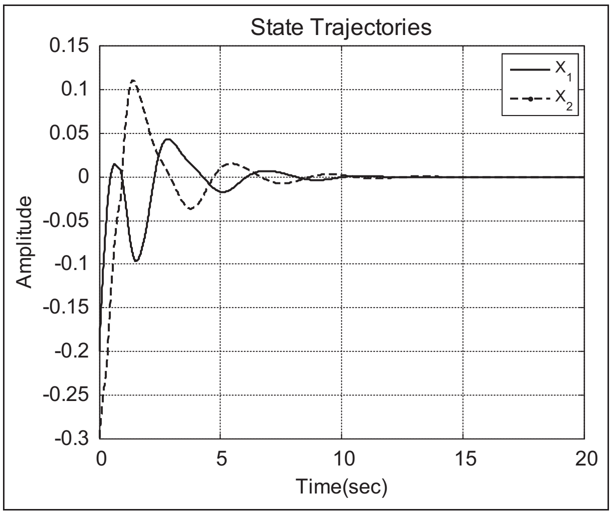

Figure 2. State trajectories of the system introduced in Example 3.

Theorem 1 guarantees the system stability if the time delays h_1 and h_2 fall in Region 1 (Figure 1). However, using the decomposition method proposed in this article and selecting suitable values for $\mathbf{B}_{11}$ and $\mathbf{B}_{21}$, the stability region is expanded to Region 1 + 2. This shows that the decomposition method can improve the results and make them less conservative. Similarly, Theorem 2 without the decomposition guarantees the system stability in Region 1 + 2 + 3. By using the decomposition method, the stability region extends to 1 + 2 + 3 + 4. The sector bound (5) is marked with heavy-dot line in Figure 1. Time delays should be selected in the regions below this line to satisfy Assumption 2. The results show that the theorems introduced in this article are less conservative as compared with those presented in the study of Kazemy and Farrokhi.[23] State trajectories of the system are shown in Figure 2 with $h_1 = 0.33$ and $h_2 = 0.9$.

Conclusion

This article provided some conditions for delay-dependent robust absolute stability for uncertain Lur'e systems with multiple time delays and sector-bounded nonlinearity. The nonlinearity was assumed to be both time invariant and time varying. Based on the Lyapunov–Krasovskii stability theory and matrix decomposition methods, some sufficient conditions for the robust absolute stability of the Lur'e system were derived and expressed in the form of LMIs. Numerical examples showed that the proposed stability criteria are less conservative as compared with the results in recently published literatures.

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