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Robust Controller Design for Takagi–Sugeno Systems with Nonlinear Consequent Part and Time Delay

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Abstract This paper deals with the controller design problem for a class of delayed nonlinear systems by introducing a delayed Takagi–Sugeno system with nonlinear consequent parts. It is assumed that the fuzzy Takagi–Sugeno model contains disturbances or unstructured uncertainties. Depending on whether the system has input delay or not, two kinds of state-feedback controllers are supposed. By the help of Lyapunov–Krasovskii stability theory, some conditions in the form of linear matrix inequalities are presented such that the closed-loop system is asymptotically stable and achieves a prescribed $\mathcal{H}_\infty$ performance level. At the end, three examples are provided to illustrate the effectiveness of the proposed method.

Keywords Nonlinear Takagi–Sugeno model · Time-delay systems · Lyapunov–Krasovskii theory · Robust controller

1 Introduction

Among various kinds of fuzzy control methods, Takagi–Sugeno (T–S) approach is the most popular one for its ability to simplify the design procedure of robust controllers and observers. In fact, linear subsystems of the T–S model let us utilize such a powerful tool like linear matrix inequality (LMI) in the design routine. A mighty review of researches on these models has been made in [10, 22].

One major problem with T–S model is that for a bit more complex systems or for those having a wide range of variations of states, the number of subsystems needed to have an accurate enough model will be increased. As a result, huge number of LMIs will be obtained which cause computational burden. A reasonable solution to face this problem is the introduction of nonlinear terms in the subsystems. This idea has been investigated in two approaches: the first one is using polynomial subsystems suggested in [21, 24] and the second one is linear subsystems plus sector-bounded nonlinearities [5, 19]. The first method results in sum of squares (SOS) instead of LMIs which may be a bit hard to solve when the degree of the polynomial increases. Conversely, the latter still uses the LMIs, which is followed in this paper.

Many practical systems suffer from the existing of delay in state or input of the system. In this case, some special considerations should be made in the design. The T–S-based fuzzy control for nonlinear time-delay systems was firstly introduced by Cao and Frank [2, 3]. Subsequently, many researchers have used this idea and developed it [20, 30]. Chen and Liu [4], applied an $\mathcal{H}_\infty$ control for T–S fuzzy systems with time delay and parametric uncertainty. Hsiao et al. [11], utilized T–S-delayed fuzzy systems for control of nonlinear interconnected systems with multiple time delays. Several articles have also addressed parametric uncertainty in delayed T–S fuzzy systems [7, 15, 17]. Along with these papers for continuous-time systems, the idea of introducing delay in the T–S fuzzy system continued also for discrete-time systems [18, 31, 33]. Recently, new concepts for the development of these systems have been introduced in various applications, including switching T–S fuzzy systems [6] and network control of T–S fuzzy systems

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with time delay [32]. Many remarkable papers have been published newly that try to achieve less conservative results using various methods such as delay partitioning [16], Wirtinger-based double integral inequality [23], and augmented Lyapunov–Krasovskii functional [13]. These papers, by applying their own methods on practical examples such as truck-trailer [25], nonlinear mass–spring–damper [11, 15], and CSTR [2], have shown that these methods are capable of controlling many real systems.

In contrast to the articles reviewed in the previous paragraph that considered the fuzzy system to be linear, some methods have been proposed based on changes on the T–S fuzzy model and have shown that good results can be achieved. Gassara et al. [8, 9] considered T–S with polynomial subsystems and time delay, while Moodi and Farrokhi [19] used nonlinear consequent parts for T–S fuzzy systems. In this regard, very few results have been published with nonlinear consequent parts, all of which do not take into account the time delay. To the best of the authors' knowledge, no results have been reported yet on delayed T–S fuzzy systems with nonlinear consequent part, which is the main contribution of this paper. This type of T–S system can reduce the number of rules in fuzzy modeling while increasing accuracy.

This paper is organized as follows: In Sect. 2, the intended nonlinear T–S model and the main problem is stated. Section 3 discusses the controller design for the proposed system. In Sect. 4, some practical examples are presented and simulated. Section 5 concludes the paper.

Notations Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices. $\mathbf{P}>0$ means that $\mathbf{P}$ is a real positive definite and symmetric matrix. $\mathbf{I}$ is the identity matrix with appropriate dimensions, $\mathbf{A}^{\mathrm{T}}$ denotes the transpose of the real matrix $\mathbf{A}$, and $[\mathbf{A}]_s=\mathbf{A}+\mathbf{A}^{\mathrm{T}}$. Symmetric terms in a symmetric matrix are denoted by $*$.

2 Problem Formulation

Consider a class of nonlinear systems described by

$$\begin{cases}\dot{\mathbf{x}}(t)=\mathbf{f}_a(\mathbf{x}(t),\mathbf{x}(t-\tau))+\mathbf{f}_b(\mathbf{x}(t),\mathbf{x}(t-\tau))\varphi(\mathbf{x}(t))\\ \qquad +\mathbf{f}_c(\mathbf{x}(t),\mathbf{x}(t-\tau))\varphi(\mathbf{x}(t-\tau))\\ \qquad +\mathbf{g}(\mathbf{x}(t),\mathbf{x}(t-\tau))\mathbf{u}(t),\\ \mathbf{y}(t)=\mathbf{f}_{ya}(\mathbf{x}(t),\mathbf{x}(t-\tau))+\mathbf{f}_{yb}(\mathbf{x}(t),\mathbf{x}(t-\tau))\varphi(\mathbf{x}(t))\\ \qquad +\mathbf{f}_{yc}(\mathbf{x}(t),\mathbf{x}(t-\tau))\varphi(\mathbf{x}(t-\tau)),\end{cases} \tag{1}$$

where $\mathbf{x}(t)\in\mathbb{R}^{n_x}$, $\mathbf{u}(t)\in\mathbb{R}^{n_u}$, and $\mathbf{y}(t)\in\mathbb{R}^{n_y}$, denote the state vector, the control input vector, and the measurable output vector, respectively. The $\mathbf{f}_n:\mathbb{R}^{n_x}\times\mathbb{R}^{n_x}\to\mathbb{R}^{n_x}$, $n\in\{a,ya\}$, $\mathbf{f}_m:\mathbb{R}^{n_x}\times\mathbb{R}^{n_x}\to\mathbb{R}^{n_x\times n_\varphi}$, $m\in\{b,c,yb,yc\}$, and $\mathbf{g}:\mathbb{R}^{n_x}\times\mathbb{R}^{n_x}\to\mathbb{R}^{n_x\times n_u}$, are differentiable nonlinear functions, and τ represents a known constant time delay. Furthermore, $\varphi:\mathbb{R}^{n_x}\to\mathbb{R}^{n_\varphi}$ is a vector of sector-bounded continuous nonlinear functions satisfying the following cone condition:

$$\varphi_i(\mathbf{x}(t))\in\mathrm{co}\{0,\mathbf{E}_i\mathbf{x}(t)\},\quad 1\le i\le n_\varphi, \tag{2}$$

where $\mathbf{E}_i$, $i=1,\ldots,n_\varphi$, are known constant matrices. The above condition is equivalent to

$$\varphi_i^{\mathrm{T}}(\mathbf{x}(t))[\varphi_i(\mathbf{x}(t))-\mathbf{E}_i\mathbf{x}(t)]\le 0,\quad 1\le i\le n_\varphi. \tag{3}$$

Defining $\mathbf{E}=[\mathbf{E}_1^{\mathrm{T}},\mathbf{E}_2^{\mathrm{T}},\ldots,\mathbf{E}_{n_\varphi}^{\mathrm{T}}]^{\mathrm{T}}$ and considering a diagonal matrix $\mathbf{\Theta}>0$, it yields that

$$\varphi^{\mathrm{T}}(\mathbf{x}(t))\mathbf{\Theta}\varphi(\mathbf{x}(t))-\varphi^{\mathrm{T}}(\mathbf{x}(t))\mathbf{\Theta}\mathbf{E}\mathbf{x}(t)\le 0. \tag{4}$$

Note that to conclude (4) from (3), without extra properties on φ, the matrix $\mathbf{\Theta}$ has to be diagonal.

The system (1) can be represented by a delayed T–S fuzzy model with nonlinear consequents as follows:

Plant Rule i:

if $z_1(t)$ is $\mu_{i1}(z),\ldots,$ and $z_p(t)$ is $\mu_{ip}(z)$ then:

$$\begin{cases}\dot{\mathbf{x}}(t)=\mathbf{A}_{1i}\mathbf{x}(t)+\mathbf{A}_{2i}\mathbf{x}(t-\tau)+\mathbf{G}_{xi}\varphi(\mathbf{x}(t))\\ \qquad +\mathbf{H}_{xi}\varphi(\mathbf{x}(t-\tau))+\mathbf{B}_i\mathbf{u}(t)+\mathbf{D}_{xi}\mathbf{v}(t),\\ \mathbf{y}(t)=\mathbf{C}_{1i}\mathbf{x}(t)+\mathbf{C}_{2i}\mathbf{x}(t-\tau)+\mathbf{G}_{yi}\varphi(\mathbf{x}(t))\\ \qquad +\mathbf{H}_{yi}\varphi(\mathbf{x}(t-\tau))+\mathbf{D}_{yi}\mathbf{v}(t),\end{cases} \tag{5}$$

where $\mathbf{A}_{ni}\in\mathbb{R}^{n_x\times n_x}$, $\mathbf{B}_i\in\mathbb{R}^{n_x\times n_u}$, $\mathbf{C}_{ni}\in\mathbb{R}^{n_y\times n_x}$, $\mathbf{G}_{xi},\mathbf{H}_{xi}\in\mathbb{R}^{n_x\times n_\varphi}$, $\mathbf{G}_{yi},\mathbf{H}_{yi}\in\mathbb{R}^{n_y\times n_\varphi}$, $\mathbf{D}_{xi}\in\mathbb{R}^{n_x\times n_v}$, and $\mathbf{D}_{yi}\in\mathbb{R}^{n_y\times n_v}$, $n=1,2$, $i=1,\ldots,r$, are constant matrices, r is the number of rules, and $\mathbf{v}(t)\in\mathbb{R}^{n_v}$ is a disturbance vector that belongs to energy-limited signals, i.e., $\mathbf{v}(t)\in\mathcal{L}_2[0,\infty)$. Moreover, $z_j(t)$ and $\mu_{ij}(z)$, $i=1,\ldots,r$, $j=1,\ldots,p$, are the premise variables and the fuzzy sets, respectively.

In this case, the whole fuzzy system (5) can be represented as

$$\begin{cases}\dot{\mathbf{x}}(t)=\mathbf{A}_{1z}\mathbf{x}(t)+\mathbf{A}_{2z}\mathbf{x}(t-\tau)+\mathbf{G}_{xz}\varphi(\mathbf{x}(t))\\ \qquad +\mathbf{H}_{xz}\varphi(\mathbf{x}(t-\tau))+\mathbf{B}_z\mathbf{u}(t)+\mathbf{D}_{xz}\mathbf{v}(t),\\ \mathbf{y}(t)=\mathbf{C}_{1z}\mathbf{x}(t)+\mathbf{C}_{2z}\mathbf{x}(t-\tau)+\mathbf{G}_{yz}\varphi(\mathbf{x}(t))\\ \qquad +\mathbf{H}_{yz}\varphi(\mathbf{x}(t-\tau))+\mathbf{D}_{yz}\mathbf{v}(t),\end{cases} \tag{6}$$

where

$$\mathbf{X}_z = \sum_{i=1}^{r} \omega_i(z)\mathbf{X}_i \tag{7}$$

$\mathbf{X} \in \{\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}, \mathbf{C}_1, \mathbf{C}_2, \mathbf{G}_x, \mathbf{H}_x, \mathbf{G}_y, \mathbf{H}_y, \mathbf{D}_x, \mathbf{D}_y\}$ and

$$\omega_i(z) = \frac{h_i(z)}{\sum_{k=1}^{r} h_k(z)}, \; h_i(z) = \prod_{j=1}^{p} \mu_{ij}(z). \tag{8}$$

The objective is to design a suitable controller for the system (6), such that the resultant closed-loop system is internally asymptotically stable and satisfies

$$\int_0^{\infty} \mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t)\mathrm{d}t \le \gamma^2 \int_0^{\infty} v^{\mathrm{T}}(t)v(t)\mathrm{d}t, \tag{9}$$

for any nonzero $v(t) \in \mathcal{L}_2[0,\infty)$ under zero initial conditions, where $\gamma > 0$ is a prescribed disturbance attenuation level.

In the following section, the conditions for asymptotic stability of the system (6) will be given. The following lemmas are used in this paper.

Lemma 1 [26] *If the following conditions hold:*

$$\begin{aligned} &\mathbf{M}_{ii} < 0, \;\; 1 < i < r, \\ &\frac{1}{r-1}\mathbf{M}_{ii} + \frac{1}{2}(\mathbf{M}_{ij} + \mathbf{M}_{ji}) < 0, \;\; 1 < i \ne j < r, \end{aligned} \tag{10}$$

then, the following inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j\mathbf{M}_{ij} < 0, \tag{11}$$

where $0 \le \alpha_i \le 1$ *and* $\sum_{i=1}^{r} \alpha_i = 1$.

Lemma 2 (Jensen inequality [12]) *Assume that the vector function* $\boldsymbol{\omega} : [0, r] \to \mathbb{R}^n$ *is well defined for the following integrations. For any symmetric matrix* $\mathbf{R} \in \mathbb{R}^{n\times n}$ *and scalar* $r > 0$, *one has*

$$r\int_0^r \boldsymbol{\omega}^{\mathrm{T}}(s)\mathbf{R}\boldsymbol{\omega}(s)\mathrm{d}s \ge \left(\int_0^r \boldsymbol{\omega}(s)\mathrm{d}s\right)^{\mathrm{T}}\mathbf{R}\left(\int_0^r \boldsymbol{\omega}(s)\mathrm{d}s\right).$$

3 Controller Design

In this section, based on whether the input signal comprises time delay or not, two control structures are presented.

3.1 Systems Without Input Delay

Let us construct the fuzzy controller structure as

$$\begin{aligned} \mathbf{u}(t) = {} & \mathbf{K}_{1z}\mathbf{x}(t) + \mathbf{K}_{2z}\mathbf{x}(t-\tau) \\ & + \mathbf{K}_{3z}\varphi(\mathbf{x}(t)) + \mathbf{K}_{4z}\varphi(\mathbf{x}(t-\tau)), \end{aligned} \tag{12}$$

where $\mathbf{K}_{1z}, \mathbf{K}_{2z} \in \mathbb{R}^{n_u\times n_x}$ and $\mathbf{K}_{3z}, \mathbf{K}_{4z} \in \mathbb{R}^{n_u\times n_\varphi}$ are the controller gain matrices to be designed.

By inserting the controller (12) into the system (6), one obtains the closed-loop system as

$$\begin{cases} \dot{\mathbf{x}}(t) = \tilde{\mathbf{A}}_{1zz}\mathbf{x}(t) + \tilde{\mathbf{A}}_{2zz}\mathbf{x}(t-\tau) + \tilde{\mathbf{G}}_{xzz}\varphi(\mathbf{x}(t)) \\ \qquad + \tilde{\mathbf{H}}_{xzz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{D}_{xz}\mathbf{v}(t), \\ \mathbf{y}(t) = \mathbf{C}_{1z}\mathbf{x}(t) + \mathbf{C}_{2z}\mathbf{x}(t-\tau) + \mathbf{G}_{yz}\varphi(\mathbf{x}(t)) \\ \qquad + \mathbf{H}_{yz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{D}_{yz}\mathbf{v}(t), \end{cases} \tag{13}$$

where $\tilde{\mathbf{A}}_{1zz} = \mathbf{A}_{1z} + \mathbf{B}_z\mathbf{K}_{1z}$, $\tilde{\mathbf{A}}_{2zz} = \mathbf{A}_{2z} + \mathbf{B}_z\mathbf{K}_{2z}$, $\tilde{\mathbf{G}}_{xzz} = \mathbf{G}_{xz} + \mathbf{B}_z\mathbf{K}_{3z}$, and $\tilde{\mathbf{H}}_{xzz} = \mathbf{H}_{xz} + \mathbf{B}_z\mathbf{K}_{4z}$.

The following theorem gives the conditions to guarantee the asymptotic stability of the closed-loop system (13).

Theorem 1 *For any given positive scalar* γ, *and gain matrices* $\mathbf{K}_{1i}$, $\mathbf{K}_{2i}$, $\mathbf{K}_{3i}$, $\mathbf{K}_{4i}$, $i = 1, 2, \ldots, r$, *the closed-loop system (13) is asymptotically stable and satisfies the performance index (9), if there exist* $n_x \times n_x$ *symmetric positive definite matrices* $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{W}$, *and positive definite diagonal matrices* $\boldsymbol{\Theta}_1$, $\boldsymbol{\Theta}_2$, *such that the LMIs in (10) hold with the following definition:*

$$\mathbf{M}_{ij} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_{13} & \boldsymbol{\Sigma}_{14} & \mathbf{W} & \boldsymbol{\Sigma}_{16} & \tau\tilde{\mathbf{A}}_{1ij}^{\mathrm{T}}\mathbf{P} \\ * & \boldsymbol{\Sigma}_{22} & \mathbf{C}_{2i}^{\mathrm{T}}\mathbf{G}_{yj} & \boldsymbol{\Sigma}_{24} & -\mathbf{W} & \mathbf{C}_{2i}^{\mathrm{T}}\mathbf{D}_{yj} & \tau\tilde{\mathbf{A}}_{2ij}^{\mathrm{T}}\mathbf{P} \\ * & * & \boldsymbol{\Sigma}_{33} & \mathbf{G}_{yi}^{\mathrm{T}}\mathbf{H}_{yj} & 0 & \mathbf{G}_{yi}^{\mathrm{T}}\mathbf{D}_{yj} & \tau\tilde{\mathbf{G}}_{xij}^{\mathrm{T}}\mathbf{P} \\ * & * & * & \boldsymbol{\Sigma}_{44} & 0 & \mathbf{H}_{yi}^{\mathrm{T}}\mathbf{D}_{yj} & \tau\tilde{\mathbf{H}}_{xij}^{\mathrm{T}}\mathbf{P} \\ * & * & * & * & -\mathbf{R} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Sigma}_{66} & \tau\mathbf{D}_{xi}^{\mathrm{T}}\mathbf{P} \\ * & * & * & * & * & * & -\mathbf{P} \end{bmatrix}, \tag{14}$$

where

$$\begin{aligned} \boldsymbol{\Sigma}_{11} &= [\mathbf{P}\tilde{\mathbf{A}}_{1ij}]_s + \mathbf{Q} + \tau^2\mathbf{R} + \mathbf{C}_{1i}^{\mathrm{T}}\mathbf{C}_{1j} - \mathbf{P}, \\ \boldsymbol{\Sigma}_{12} &= \mathbf{P}\tilde{\mathbf{A}}_{2ij} + \mathbf{C}_{1i}^{\mathrm{T}}\mathbf{C}_{2j} + \mathbf{P}, \\ \boldsymbol{\Sigma}_{13} &= \mathbf{P}\tilde{\mathbf{G}}_{xij} + \mathbf{C}_{1i}^{\mathrm{T}}\mathbf{G}_{yj} + \mathbf{E}^{\mathrm{T}}\boldsymbol{\Theta}_1, \\ \boldsymbol{\Sigma}_{14} &= \mathbf{P}\tilde{\mathbf{H}}_{xij} + \mathbf{C}_{1i}^{\mathrm{T}}\mathbf{H}_{yj}, \\ \boldsymbol{\Sigma}_{16} &= \mathbf{P}\mathbf{D}_{xi} + \mathbf{C}_{1i}^{\mathrm{T}}\mathbf{D}_{yj}, \\ \boldsymbol{\Sigma}_{22} &= -\mathbf{Q} - \mathbf{P} + \mathbf{C}_{2i}^{\mathrm{T}}\mathbf{C}_{2j}, \\ \boldsymbol{\Sigma}_{24} &= \mathbf{C}_{2i}^{\mathrm{T}}\mathbf{H}_{yj} + \mathbf{E}^{\mathrm{T}}\boldsymbol{\Theta}_2, \\ \boldsymbol{\Sigma}_{33} &= \mathbf{G}_{yi}^{\mathrm{T}}\mathbf{G}_{yj} - 2\boldsymbol{\Theta}_1, \\ \boldsymbol{\Sigma}_{44} &= \mathbf{H}_{yi}^{\mathrm{T}}\mathbf{H}_{yj} - 2\boldsymbol{\Theta}_2, \\ \boldsymbol{\Sigma}_{66} &= -\gamma^2\mathbf{I} + \mathbf{D}_{yi}^{\mathrm{T}}\mathbf{D}_{yj}. \end{aligned}$$

Proof See "Appendix". □

Theorem 1 cannot be used directly for controller design due to the existence of nonlinear terms such as $\mathbf{P}\mathbf{B}_z\mathbf{K}_{1z}$ and

$\mathbf{P}\mathbf{B}_z\mathbf{K}_{2z}$ in (14). Therefore, the following theorem is provided to pave the way for solving the controller design problem.

Theorem 2 *For given positive scalar γ, the closed-loop system (13) is asymptotically stable and satisfies the performance index (9), if there exist $n_x \times n_x$ symmetric positive definite matrices $\hat{\mathbf{P}}$, $\hat{\mathbf{Q}}$, $\hat{\mathbf{R}}$, $\hat{\mathbf{W}}$, general $n_u \times n_x$ matrices $\hat{\mathbf{K}}_{1i}$, $\hat{\mathbf{K}}_{2i}$, general $n_u \times n_\varphi$ matrices $\hat{\mathbf{K}}_{3i}$, $\hat{\mathbf{K}}_{4i}$, and positive definite diagonal matrices $\hat{\boldsymbol{\Theta}}_1$, $\hat{\boldsymbol{\Theta}}_2$, such that the LMIs in (10) hold with the following definition:*

$$\mathbf{M}_{ij} = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{11} & \hat{\boldsymbol{\Sigma}}_{12} & \hat{\boldsymbol{\Sigma}}_{13} & \hat{\boldsymbol{\Sigma}}_{14} & \hat{\mathbf{W}} & \hat{\boldsymbol{\Sigma}}_{16} & \hat{\boldsymbol{\Sigma}}_{17} & \hat{\mathbf{P}}\mathbf{C}_{1i}^{\mathrm{T}} \\ * & \hat{\boldsymbol{\Sigma}}_{22} & 0 & \hat{\mathbf{P}}\mathbf{E}^{\mathrm{T}} & -\hat{\mathbf{W}} & \hat{\mathbf{P}}\mathbf{C}_{2i}^{\mathrm{T}}\mathbf{D}_{yj} & \hat{\boldsymbol{\Sigma}}_{27} & \hat{\mathbf{P}}\mathbf{C}_{2i}^{\mathrm{T}} \\ * & * & -2\hat{\boldsymbol{\Theta}}_1 & 0 & 0 & \hat{\boldsymbol{\Theta}}_1\mathbf{G}_{yi}^{\mathrm{T}}\mathbf{D}_{yj} & \hat{\boldsymbol{\Sigma}}_{37} & \hat{\boldsymbol{\Theta}}_1\mathbf{G}_{yi}^{\mathrm{T}} \\ * & * & * & -2\hat{\boldsymbol{\Theta}}_2 & 0 & \hat{\boldsymbol{\Theta}}_2\mathbf{H}_{yi}^{\mathrm{T}}\mathbf{D}_{yj} & \hat{\boldsymbol{\Sigma}}_{47} & \hat{\boldsymbol{\Theta}}_2\mathbf{H}_{yi}^{\mathrm{T}} \\ * & * & * & * & -\hat{\mathbf{R}} & 0 & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Sigma}_{66} & \tau\mathbf{D}_{xi}^{\mathrm{T}} & 0 \\ * & * & * & * & * & * & -\hat{\mathbf{P}} & 0 \\ * & * & * & * & * & * & * & -\mathbf{I} \end{bmatrix}, \tag{15}$$

where

$$\hat{\boldsymbol{\Sigma}}_{11} = [\mathbf{A}_{1i}\hat{\mathbf{P}} + \mathbf{B}_i\hat{\mathbf{K}}_{1j}]_s + \hat{\mathbf{Q}} + \tau^2\hat{\mathbf{R}} - \hat{\mathbf{P}},$$
$$\hat{\boldsymbol{\Sigma}}_{12} = \mathbf{A}_{2i}\hat{\mathbf{P}} + \mathbf{B}_i\hat{\mathbf{K}}_{2j} + \hat{\mathbf{P}},$$
$$\hat{\boldsymbol{\Sigma}}_{13} = \mathbf{G}_{xi}\hat{\boldsymbol{\Theta}}_1 + \mathbf{B}_i\hat{\mathbf{K}}_{3j} + \hat{\mathbf{P}}\mathbf{E}^{\mathrm{T}},$$
$$\hat{\boldsymbol{\Sigma}}_{14} = \mathbf{H}_{xi}\hat{\boldsymbol{\Theta}}_2 + \mathbf{B}_i\hat{\mathbf{K}}_{4j},$$
$$\hat{\boldsymbol{\Sigma}}_{16} = \mathbf{D}_{xi} + \hat{\mathbf{P}}\mathbf{C}_{1i}^{\mathrm{T}}\mathbf{D}_{yj},$$
$$\hat{\boldsymbol{\Sigma}}_{17} = \tau\left(\hat{\mathbf{P}}\mathbf{A}_{1i}^{\mathrm{T}} + \hat{\mathbf{K}}_{1j}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\right),$$
$$\hat{\boldsymbol{\Sigma}}_{22} = -\hat{\mathbf{Q}} - \hat{\mathbf{P}},$$
$$\hat{\boldsymbol{\Sigma}}_{27} = \tau\left(\hat{\mathbf{P}}\mathbf{A}_{2i}^{\mathrm{T}} + \hat{\mathbf{K}}_{2j}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\right),$$
$$\hat{\boldsymbol{\Sigma}}_{37} = \tau\left(\hat{\boldsymbol{\Theta}}_1\mathbf{G}_{xi}^{\mathrm{T}} + \hat{\mathbf{K}}_{3j}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\right),$$
$$\hat{\boldsymbol{\Sigma}}_{47} = \tau\left(\hat{\boldsymbol{\Theta}}_2\mathbf{H}_{xi}^{\mathrm{T}} + \hat{\mathbf{K}}_{4j}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\right),$$
$$\boldsymbol{\Sigma}_{66} = -\gamma^2\mathbf{I} + \mathbf{D}_{yi}^{\mathrm{T}}\mathbf{D}_{yj}.$$

Moreover, if the above LMI has feasible solution, the gains $\mathbf{K}_{1i}$, $\mathbf{K}_{2i}$, $\mathbf{K}_{3i}$, and $\mathbf{K}_{4i}$ are computed by $\mathbf{K}_{1i} = \hat{\mathbf{K}}_{1i}\hat{\mathbf{P}}^{-1}$, $\mathbf{K}_{2i} = \hat{\mathbf{K}}_{2i}\hat{\mathbf{P}}^{-1}$, $\mathbf{K}_{3i} = \hat{\mathbf{K}}_{3i}\hat{\boldsymbol{\Theta}}_1^{-1}$, and $\mathbf{K}_{4i} = \hat{\mathbf{K}}_{4i}\hat{\boldsymbol{\Theta}}_2^{-1}$, respectively.

Proof See "Appendix". □

3.2 Systems with Input Delay

For the systems with input delay as

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{1z}\mathbf{x}(t) + \mathbf{A}_{2z}\mathbf{x}(t-\tau) + \mathbf{G}_{xz}\varphi(\mathbf{x}(t)) \\ \qquad + \mathbf{H}_{xz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{B}_z\mathbf{u}(t-\tau) + \mathbf{D}_{xz}\mathbf{v}(t), \\ \mathbf{y}(t) = \mathbf{C}_{1z}\mathbf{x}(t) + \mathbf{C}_{2z}\mathbf{x}(t-\tau) + \mathbf{G}_{yz}\varphi(\mathbf{x}(t)) \\ \qquad + \mathbf{H}_{yz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{D}_{yz}\mathbf{v}(t), \end{cases} \tag{16}$$

we shall construct the controller structure as

$$\mathbf{u}(t) = \mathbf{K}_1\mathbf{x}(t) + \mathbf{K}_2\varphi(\mathbf{x}(t)), \tag{17}$$

where $\mathbf{K}_1 \in \mathbb{R}^{n_u \times n_x}$ and $\mathbf{K}_2 \in \mathbb{R}^{n_u \times n_\varphi}$ are the controller gain matrices to be designed. By inserting the controller (17) into the system (16), one obtains the closed-loop system as

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{1z}\mathbf{x}(t) + \tilde{\mathbf{A}}_{2zz}\mathbf{x}(t-\tau) + \mathbf{G}_{xz}\varphi(\mathbf{x}(t)) \\ \qquad + \tilde{\mathbf{H}}_{xzz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{D}_{xz}\mathbf{v}(t), \\ \mathbf{y}(t) = \mathbf{C}_{1z}\mathbf{x}(t) + \mathbf{C}_{2z}\mathbf{x}(t-\tau) + \mathbf{G}_{yz}\varphi(\mathbf{x}(t)) \\ \qquad + \mathbf{H}_{yz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{D}_{yz}\mathbf{v}(t), \end{cases} \tag{18}$$

where $\tilde{\mathbf{A}}_{2zz} = \mathbf{A}_{2z} + \mathbf{B}_z\mathbf{K}_{1z}$ and $\tilde{\mathbf{H}}_{xzz} = \mathbf{H}_{xz} + \mathbf{B}_z\mathbf{K}_{2z}$.

The following corollary gives the conditions to guarantee the asymptotical stability of the closed-loop system (18).

Corollary 1 *For given positive scalar γ, the closed-loop system (13) is asymptotically stable and satisfies (9), if there exist $n_x \times n_x$ symmetric positive definite matrices $\hat{\mathbf{P}}$, $\hat{\mathbf{Q}}$, $\hat{\mathbf{R}}$, $\hat{\mathbf{W}}$, general $n_u \times n_x$ matrices $\tilde{\mathbf{K}}_{1i}$, general $n_u \times n_\varphi$ matrices $\tilde{\mathbf{K}}_{2i}$, and diagonal matrices $\hat{\Theta}_1 > 0$, $\hat{\Theta}_2 > 0$, such that the LMIs in (10) hold with the following definition:*

$$\mathbf{M}_{ij} = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{11} & \hat{\boldsymbol{\Sigma}}_{12} & \hat{\boldsymbol{\Sigma}}_{13} & \hat{\boldsymbol{\Sigma}}_{14} & \hat{\mathbf{W}} & \hat{\boldsymbol{\Sigma}}_{16} & \tau\hat{\mathbf{P}}\mathbf{A}_{1z}^{\mathrm{T}} & \hat{\mathbf{P}}\mathbf{C}_{1i}^{\mathrm{T}} \\ * & \hat{\boldsymbol{\Sigma}}_{22} & 0 & \hat{\mathbf{P}}\mathbf{E}^{\mathrm{T}} & -\hat{\mathbf{W}} & \hat{\mathbf{P}}\mathbf{C}_{2i}^{\mathrm{T}}\mathbf{D}_{yj} & \hat{\boldsymbol{\Sigma}}_{27} & \hat{\mathbf{P}}\mathbf{C}_{2i}^{\mathrm{T}} \\ * & * & -2\hat{\Theta}_1 & 0 & 0 & \hat{\Theta}_1\mathbf{G}_{yi}^{\mathrm{T}}\mathbf{D}_{yj} & \tau\hat{\Theta}_1\mathbf{G}_{xi}^{\mathrm{T}} & \hat{\Theta}_1\mathbf{G}_{yi}^{\mathrm{T}} \\ * & * & * & -2\hat{\Theta}_2 & 0 & \hat{\Theta}_2\mathbf{H}_{yi}^{\mathrm{T}}\mathbf{D}_{yj} & \hat{\boldsymbol{\Sigma}}_{47} & \hat{\Theta}_2\mathbf{H}_{yi}^{\mathrm{T}} \\ * & * & * & * & -\hat{\mathbf{R}} & 0 & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Sigma}_{66} & \tau\mathbf{D}_{xi}^{\mathrm{T}} & 0 \\ * & * & * & * & * & * & -\hat{\mathbf{P}} & 0 \\ * & * & * & * & * & * & * & -\mathbf{I} \end{bmatrix}, \tag{19}$$

where

$$\hat{\boldsymbol{\Sigma}}_{11} = [\mathbf{A}_{1i}\hat{\mathbf{P}}]_s + \hat{\mathbf{Q}} + \tau^2\hat{\mathbf{R}} - \hat{\mathbf{P}},$$
$$\hat{\boldsymbol{\Sigma}}_{12} = \mathbf{A}_{2i}\hat{\mathbf{P}} + \mathbf{B}_i\tilde{\mathbf{K}}_{1j} + \hat{\mathbf{P}},$$
$$\hat{\boldsymbol{\Sigma}}_{13} = \mathbf{G}_{xi}\hat{\boldsymbol{\Theta}}_1 + \hat{\mathbf{P}}\mathbf{E}^{\mathrm{T}},$$
$$\hat{\boldsymbol{\Sigma}}_{14} = \mathbf{H}_{xi}\hat{\boldsymbol{\Theta}}_2 + \mathbf{B}_i\tilde{\mathbf{K}}_{2j},$$
$$\hat{\boldsymbol{\Sigma}}_{16} = \mathbf{D}_{xi} + \hat{\mathbf{P}}\mathbf{C}_{1i}^{\mathrm{T}}\mathbf{D}_{yj},$$
$$\hat{\boldsymbol{\Sigma}}_{22} = -\hat{\mathbf{Q}} - \hat{\mathbf{P}},$$
$$\hat{\boldsymbol{\Sigma}}_{27} = \tau\left(\hat{\mathbf{P}}\mathbf{A}_{2i}^{\mathrm{T}} + \tilde{\mathbf{K}}_{1j}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\right),$$
$$\hat{\boldsymbol{\Sigma}}_{47} = \tau\left(\hat{\boldsymbol{\Theta}}_2\mathbf{H}_{xi}^{\mathrm{T}} + \tilde{\mathbf{K}}_{2j}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}\right),$$
$$\boldsymbol{\Sigma}_{66} = -\gamma^2\mathbf{I} + \mathbf{D}_{yi}^{\mathrm{T}}\mathbf{D}_{yj}.$$

Moreover, if the above LMI has feasible solution, the gains $\mathbf{K}_{1i}$ *and* $\mathbf{K}_{2i}$ *are computed by* $\mathbf{K}_{1i} = \tilde{\mathbf{K}}_{1i}\hat{\mathbf{P}}^{-1}$ *and* $\mathbf{K}_{2i} = \tilde{\mathbf{K}}_{2i}\hat{\boldsymbol{\Theta}}_2^{-1}$, *respectively.*

Proof With substituting $\hat{\mathbf{K}}_{1i} = \mathbf{0}$, $\hat{\mathbf{K}}_{3i} = \mathbf{0}$, $\tilde{\mathbf{K}}_{1i} = \hat{\mathbf{K}}_{2i}$, and $\tilde{\mathbf{K}}_{2i} = \hat{\mathbf{K}}_{4i}$ in Theorem (2), the proof is straightforward. □

Remark 1 Although some progress has been made in this article, the established fuzzy control result has not considered any constraint on system transient performance. As stated in [28, 29], finite-time control is also an important challenge in the design. How to establish finite-time fuzzy controller for time-delay systems, will be our future research.

4 Simulation Results

In this section. three illustrative examples are given to show the effectiveness of the proposed design methods. All calculations are performed using Yalmip [14] toolbox.

Example 1 Suppose the problem of backing up the control of a truck-trailer is given in [3]. The following delayed model is supposed:

$$\begin{cases} \dot{x}_1(t) = -a\dfrac{v\bar{t}}{Lt_0}x_1(t) - (1-a)\dfrac{v\bar{t}}{Lt_0}x_1(t-\tau) + \dfrac{v\bar{t}}{Lt_0}u(t), \\ \dot{x}_2(t) = +a\dfrac{v\bar{t}}{Lt_0}x_1(t) + (1-a)\dfrac{v\bar{t}}{Lt_0}x_1(t-\tau), \\ \dot{x}_3(t) = \dfrac{v\bar{t}}{t_0}\sin(x_2(t) + a\dfrac{v\bar{t}}{2L}x_1(t) + (1-a)\dfrac{v\bar{t}}{2L}x_1(t-\tau)), \end{cases}$$

where $l = 2.8$, $L = 5.5$, $v = -1$, $\bar{t} = 2$, $t_0 = 0.5$, $\tau = 1$. The constant $a \in [0,\ 1]$ determines the amount of delay and is set to $a = 0.7$. By defining $\varphi(x) = \sin(x_2) - x_2$, the above system is modeled by fuzzy T–S with nonlinear consequent part as follows:

$$\textit{Rule}\ 1: \text{If } \mathrm{z(t) = x_2(t) + a\frac{vt}{2L}x_1(t) + (1-a)\frac{vt}{2L}x_1(t-\tau)}$$
is about 0 then
$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{11}\mathbf{x}(t) + \mathbf{A}_{21}\mathbf{x}(t-\tau) + \mathbf{B}_1u(t) \\ \qquad + \mathbf{G}_{x1}\varphi(\mathbf{x}(t)) + \mathbf{D}_{x1}v(t), \\ \mathbf{y}(t) = \mathbf{C}_{11}\mathbf{x}(t) + \mathbf{D}_{y1}v(t), \end{cases}$$

$$\textit{Rule}\ 2: \text{If } \mathrm{z(t) = x_2(t) + a\frac{vt}{2L}x_1(t) + (1-a)\frac{vt}{2L}x_1(t-\tau)}$$
is about π or $-\pi$ then
$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{12}\mathbf{x}(t) + \mathbf{A}_{22}\mathbf{x}(t-\tau) + \mathbf{B}_2u(t) \\ \qquad + \mathbf{G}_{x2}\varphi(\mathbf{x}(t)) + \mathbf{D}_{x2}v(t), \\ \mathbf{y}(t) = \mathbf{C}_{12}\mathbf{x}(t) + \mathbf{D}_{y2}v(t), \end{cases}$$

where

$$\mathbf{A}_{11} = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \mathbf{A}_{21} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, \mathbf{G}_{x1} = \begin{bmatrix} 0 \\ 0 \\ \frac{v\bar{t}}{(t_0)} \end{bmatrix}, \mathbf{A}_{12} = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{dv\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v^2\bar{t}^2}{2Lt_0} & d\frac{v\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$\mathbf{A}_{22} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, \mathbf{G}_{x2} = \begin{bmatrix} 0 \\ 0 \\ \frac{dv\bar{t}}{(t_0)} \end{bmatrix},$$

$$\mathbf{C}_{11} = \mathbf{C}_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{D}_{x1} = \mathbf{D}_{x2} = \begin{bmatrix} 0.05 \\ 0 \\ 0.01 \end{bmatrix},$$

$$\mathbf{E} = 2/\pi[0 \quad 1 \quad 0], \mathbf{D}_{y1} = \mathbf{D}_{y2} = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix},$$

$d = 10\ t_0/\pi$, $v(t) = e^{-0.05t}\sin(2t)$, and the membership functions are as follows:

$$h_1(z) = \left(1 - \frac{1}{1+\exp(-3(z-0.5\pi))}\right) \times \left(\frac{1}{1+\exp(-3(z-0.5\pi))}\right),$$
$$h_2(z) = 1 - h_1(z).$$

It can be seen that the open-loop system is unstable. By applying Theorem 2 with $\gamma = 1$, the control gains are obtained as

$$\mathbf{K}_{11} = [1.7348 \quad -1.7725 \quad 0.090802],$$
$$\mathbf{K}_{12} = [1.7627 \quad -1.9538 \quad 0.10806],$$
$$\mathbf{K}_{21} = [0.46767 \quad 0.034505 \quad -0.00029401],$$
$$\mathbf{K}_{22} = [0.49262 \quad 0.0065132 \quad 0.0012818],$$
$$\mathbf{K}_{31} = -0.11919,\ \mathbf{K}_{32} = -0.2275.$$

By using these control gains, the state trajectories of the closed-loop system are shown in Fig. 1 (solid line). For comparison, the results of other published methods are also shown in this figure. It is clear that our result has less fluctuation and has more smooth movements than the methods presented in [3] and [4]. The results of reference [7] have a much bigger settling time than the other methods, including our method, which shows that this method does not work efficiently. The methods of references [3] and [4] have good performance results on controlling the states, but have a large peak value in the control signal, shown in Fig. 2. It can be easily understood that the peak value of the control signal produced by our method is much less than those methods and converges to zero at a faster rate. Therefore, our method requires a smaller actuator and is more practical than those methods.

Example 2 In this example, the problem of input delay is considered. Supposed a two tank system with the following model [27]:

$$\begin{cases} \dot{x}_1(t) = -\alpha_1 x_1^2(t) + \beta u(t), \\ \dot{x}_2(t) = \alpha_1 x_1^2(t) - \alpha_2 x_2^2(t). \end{cases}$$

where $\alpha_1 = 1$, $\alpha_2 = 1$, and $\beta = 1$. By defining $\varphi(\mathbf{x}(t)) = x_1^2$, the system is modeled by nonlinear T–S fuzzy model as :

Rule 1 : If $z(t) = x_2$ isabout 0.1 then

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{11}\mathbf{x}(t) + \mathbf{B}u(t-\tau) + \mathbf{G}_x\varphi(\mathbf{x}(t)) + \mathbf{D}_{x1}v(t), \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}_{y1}v(t), \end{cases}$$

Rule 2 : If $z(t) = x_2$ isabout 1 then

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{12}\mathbf{x}(t) + \mathbf{B}u(t-\tau) + \mathbf{G}_x\varphi(\mathbf{x}(t)) + \mathbf{D}_{x1}v(t), \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}_{y1}v(t), \end{cases}$$

Rule 3 : If $z(t) = x_2$ isabout 2 then

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_{13}\mathbf{x}(t) + \mathbf{B}u(t-\tau) + \mathbf{G}_x\varphi(\mathbf{x}(t)) + \mathbf{D}_{x1}v(t), \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}_{y1}v(t), \end{cases}$$

where

$$\mathbf{A}_{11} = \begin{bmatrix} 0 & 0 \\ 0 & -1.5811 \end{bmatrix},\ \mathbf{A}_{12} = \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$\mathbf{A}_{13} = \begin{bmatrix} 0 & 0 \\ 0 & -0.3536 \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ \mathbf{G}_x = \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$
$$\mathbf{D}_x = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix},\ \mathbf{D}_y = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ \mathbf{E} = [1 \quad 0].$$

Let $\tau = 1$, $\gamma = 0.01$, $v(t) = e^{-0.05t}\sin(2t)$, and the membership functions are assumed to be triangular. Based on Corollary 1, control gains are obtained as:

$$\mathbf{K}_{11} = [-0.37474 \quad -0.042158],$$
$$\mathbf{K}_{12} = [-0.37078 \quad 0.027274],$$
$$\mathbf{K}_{13} = [-0.35167 \quad 0.050723],$$
$$\mathbf{K}_{21} = 0.0014774,\ \mathbf{K}_{22} = -0.0010483,$$
$$\mathbf{K}_{23} = -0.0020732.$$

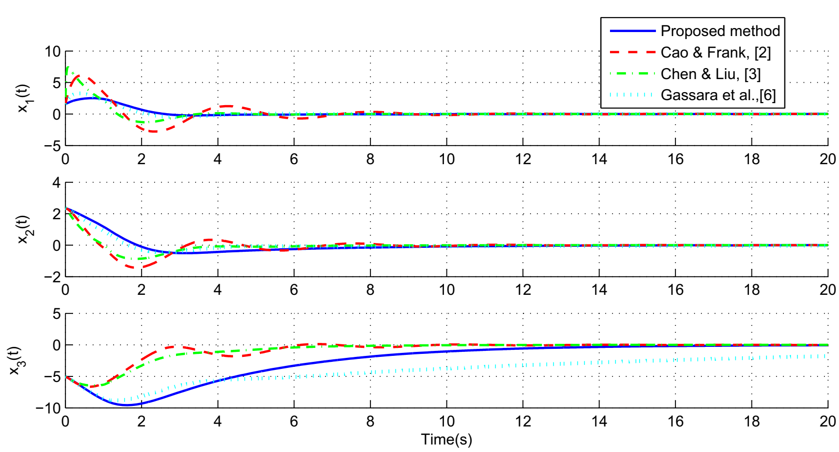

Fig. 1 State trajectories resulted by Theorem 2 (solid line) and the other published methods

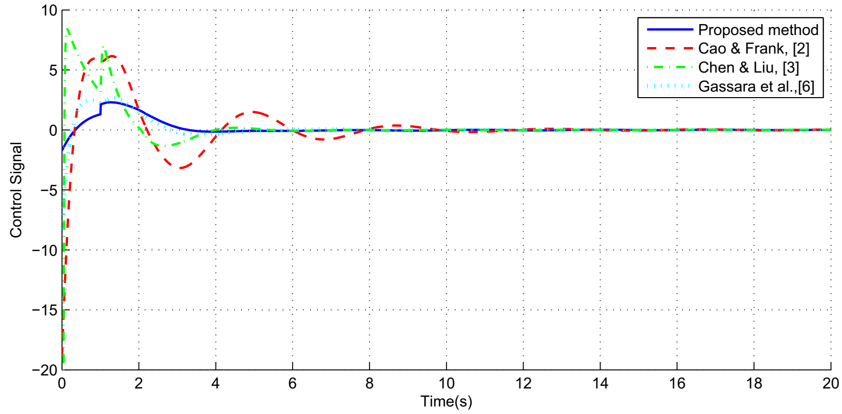

Fig. 2 Control signals produced by Theorem 2 (solid line) and the other published methods

Tracking performance of the closed-loop system is shown in Fig. 3. Control effort is also shown in Fig. 3.

Example 3 Consider a fuzzy system in the form of (5) with the following matrices [1]:

$$\mathbf{A}_{11} = \begin{bmatrix} a & -1 \\ 2 & c \end{bmatrix}, \mathbf{A}_{12} = \begin{bmatrix} a+4b & -1 \\ 2 & c+4d \end{bmatrix},$$

$$\mathbf{A}_{21} = 0.1\begin{bmatrix} a & -1 \\ 2 & c \end{bmatrix}, \mathbf{A}_{22} = 0.1\begin{bmatrix} a+4b & -1 \\ 2 & c+4d \end{bmatrix},$$

$$\mathbf{G}_{x1} = \mathbf{G}_{x2} = \begin{bmatrix} b & 0 \\ 1 & d \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix},$$
$$\mathbf{C}_{11} = \mathbf{C}_{12} = [1 \quad 2],\ \mathbf{C}_{21} = \mathbf{C}_{22} = [0.1 \quad 0.2],$$

$$\mathbf{D}_{x1} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \mathbf{D}_{x2} = \begin{bmatrix} 0.01 \\ 0.05 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$\mathbf{D}_{y1} = 0.02,\ \mathbf{D}_{y2} = 0.01,\ h_2 = x_2^2/4,\ h_1 = 1 - \mu_2,\ \tau = 1.$$

In which $a = 10$, $b = -3$, $c = -0.25$, $d = 0.3427$, $x_1 \in [-1, 1]$, and $x_2 \in [-2, 2]$. The premise variable is x_2^2 and $\varphi(x(t)) = [x_1^3, \sin(x_2)]^T$. Applying Theorem 2 with $\gamma = 0.5$ to this model, the control gains are obtained as

$$\mathbf{K}_{11} = [0.074252 \quad 0.5895],$$
$$\mathbf{K}_{12} = [-0.040227 \quad 0.60678],$$
$$\mathbf{K}_{21} = [0.057353 \quad 0.036026],$$
$$\mathbf{K}_{22} = [0.026825 \quad 0.044516],$$
$$\mathbf{K}_{31} = [0.18521 \quad 0.080762],$$
$$\mathbf{K}_{32} = [0.13467 \quad 0.057182].$$

Utilizing these control gains, the state trajectories of the closed-loop system are shown in Fig. 4 (solid line). For comparison, states of the open-loop system are also shown in this figure.

5 Conclusion

By introducing a delayed Takagi–Sugeno system with nonlinear consequent parts, a robust controller has been proposed for a class of delayed nonlinear systems. It was

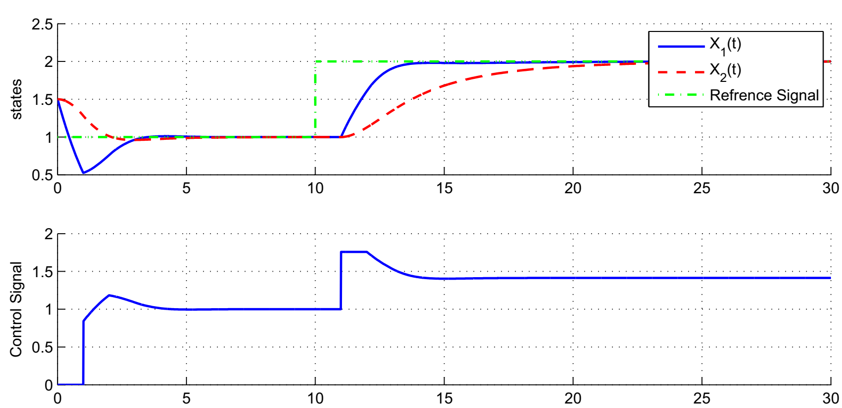

Fig. 3 Up: state trajectories resulted by Corollary 1, x_1 (solid line) and x_2 (dash line). Down: control signal produced by Corollary 1

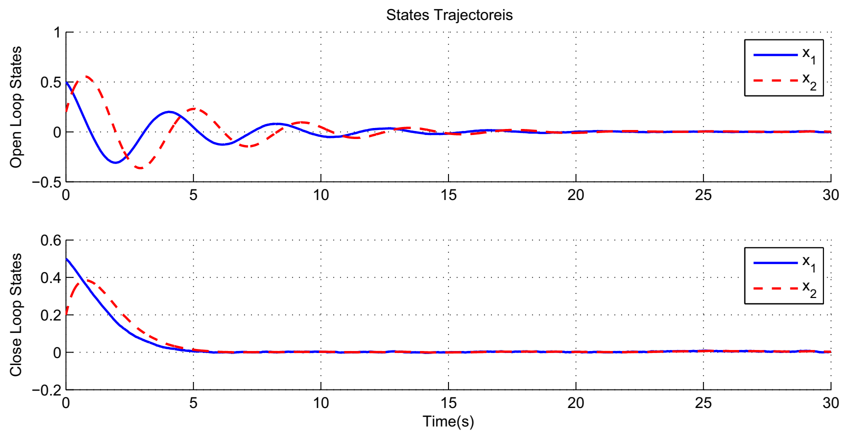

Fig. 4 State trajectories of open-loop system in Example 1 (upper figure) and states of closed-loop system (lower figure)

assumed that the fuzzy T–S model contains unstructured uncertainties. Depending on whether the system has input delay or not, two kinds of state-feedback controller were supposed. By the Lyapunov–Krasovskii stability theory, some conditions in the form of linear matrix inequalities were presented such that the closed-loop system is asymptotically stable and achieves a prescribed $\mathcal{H}_\infty$ performance level. Finally, three examples have been provided to illustrate the effectiveness of the proposed method.

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Appendix

Proof of Theorem 1 Consider a Lyapunov–Krasovskii functional as

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{20}$$

where

$$V_1(t) = \mathbf{x}^{\mathrm{T}}(t)\mathbf{P}\mathbf{x}(t) + \int_{t-\tau}^{t} \mathbf{x}^{\mathrm{T}}(s)\mathbf{Q}\mathbf{x}(s)\mathrm{d}s,$$

$$V_2(t) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\mathbf{x}}^{\mathrm{T}}(s)\mathbf{P}\dot{\mathbf{x}}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_3(t) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \mathbf{x}^{\mathrm{T}}(s)\mathbf{R}\mathbf{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_4(t) = \left(\int_{t-\tau}^{t} \mathbf{x}^{\mathrm{T}}(s)\mathrm{d}s\right)\mathbf{W}\left(\int_{t-\tau}^{t} \mathbf{x}(s)\mathrm{d}s\right).$$

Taking the derivative of $V_1(t)$ and $V_2(t)$ along the solutions of (13) yields

$$\begin{aligned}\dot{V}_1(t) &= 2\mathbf{x}^{\mathrm{T}}(t)\mathbf{P}\dot{\mathbf{x}}(t) + \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) - \mathbf{x}^{\mathrm{T}}(t-\tau)\mathbf{Q}\mathbf{x}(t-\tau)\\ &= 2\mathbf{x}^{\mathrm{T}}(t)\mathbf{P}\big[\tilde{\mathbf{A}}_{1zz}\mathbf{x}(t) + \tilde{\mathbf{A}}_{2zz}\mathbf{x}(t-\tau)\\ &\quad + \tilde{\mathbf{G}}_{xzz}\varphi(\mathbf{x}(t)) + \tilde{\mathbf{H}}_{xzz}\varphi(\mathbf{x}(t-\tau)) + \mathbf{D}_{xz}\mathbf{v}(t)\big]\\ &\quad + \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) - \mathbf{x}^{\mathrm{T}}(t-\tau)\mathbf{Q}\mathbf{x}(t-\tau),\end{aligned} \tag{21}$$

$$\dot{V}_2(t) = \tau^2\dot{\mathbf{x}}^{\mathrm{T}}(t)\mathbf{P}\dot{\mathbf{x}}(t) - \tau\int_{t-\tau}^{t} \dot{\mathbf{x}}^{\mathrm{T}}(s)\mathbf{P}\dot{\mathbf{x}}(s)\mathrm{d}s. \tag{22}$$

According to Lemma 2, one can obtain $\dot{V}_2(t)$ as

$$\begin{aligned}&\dot{V}_2(t) \le \tau^2\dot{\mathbf{x}}^{\mathrm{T}}(t)\mathbf{P}\dot{\mathbf{x}}(t)\\ &-\left(\int_{t-\tau}^{t} \dot{\mathbf{x}}^{\mathrm{T}}(s)\right)\mathbf{P}\left(\int_{t-\tau}^{t} \dot{\mathbf{x}}(s)\right) = \tau^2\dot{\mathbf{x}}^{\mathrm{T}}(t)\mathbf{P}\dot{\mathbf{x}}(t) - (\mathbf{x}^{\mathrm{T}}(t)\\ &- \mathbf{x}^{\mathrm{T}}(t-\tau))\mathbf{P}(\mathbf{x}(t) - \mathbf{x}(t-\tau)).\end{aligned} \tag{23}$$

Taking the derivative of $V_3(t)$ respect to t yields

$$\dot{V}_3(t) = \tau^2\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) - \tau\int_{t-\tau}^{t} \mathbf{x}^{\mathrm{T}}(s)\mathbf{R}\mathbf{x}(s)\mathrm{d}s.$$

Based on Lemma 2, the above equation can be written as

$$\dot{V}_3(t) \le \tau^2\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) - \left(\int_{t-\tau}^{t} \mathbf{x}^{\mathrm{T}}(s)\right)\mathbf{R}\left(\int_{t-\tau}^{t} \mathbf{x}(s)\right). \tag{24}$$

Calculating $\dot{V}_4(t)$ yields

$$\dot{V}_4(t) = 2\left(\mathbf{x}^{\mathrm{T}}(t) - \mathbf{x}^{\mathrm{T}}(t-\tau)\right)\mathbf{W}\left(\int_{t-\tau}^{t} \mathbf{x}(s)\right). \tag{25}$$

According to (4), the following inequalities are true:

$$-2\varphi^{\mathrm{T}}(\mathbf{x}(t))\mathbf{\Theta}_1\varphi(\mathbf{x}(t)) + 2\varphi^{\mathrm{T}}(\mathbf{x}(t))\mathbf{\Theta}_1\mathbf{E}\mathbf{x}(t) \ge 0, \tag{26}$$

$$
\begin{aligned}
&-2\varphi^{\mathrm{T}}(\mathbf{x}(t-\tau))\boldsymbol{\Theta}_2\varphi(\mathbf{x}(t-\tau)) \\
&\quad + 2\varphi^{\mathrm{T}}(\mathbf{x}(t-\tau))\boldsymbol{\Theta}_2\mathbf{E}\mathbf{x}(t-\tau) \geq 0.
\end{aligned} \tag{27}
$$

From Eqs. (21)–(27), one has

$$
\begin{aligned}
&\dot{V}(t) + \mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t) - \gamma^2\mathbf{v}^{\mathrm{T}}(t)\mathbf{v}(t) \leq \sum_{i=1}^{4}\dot{V}_i(t) + \mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t) \\
&\quad - \gamma^2\mathbf{v}^{\mathrm{T}}(t)\mathbf{v}(t) - 2\varphi^{\mathrm{T}}(\mathbf{x}(t))\boldsymbol{\Theta}_1\varphi(\mathbf{x}(t)) \\
&\quad + 2\varphi^{\mathrm{T}}(\mathbf{x}(t))\boldsymbol{\Theta}_1\mathbf{E}\mathbf{x}(t) \\
&\quad - 2\varphi^{\mathrm{T}}(\mathbf{x}(t-\tau))\boldsymbol{\Theta}_2\varphi(\mathbf{x}(t-\tau)) \\
&\quad + 2\varphi^{\mathrm{T}}(\mathbf{x}(t-\tau))\boldsymbol{\Theta}_2\mathbf{E}\mathbf{x}(t-\tau) = \zeta^{\mathrm{T}}(t)\Xi\zeta(t),
\end{aligned} \tag{28}
$$

where

$$
\Xi = \begin{bmatrix}
\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_{13} & \boldsymbol{\Sigma}_{14} & \mathbf{W} & \boldsymbol{\Sigma}_{16} \\
* & \boldsymbol{\Sigma}_{22} & \mathbf{C}_{2z}^{\mathrm{T}}\mathbf{G}_{yz} & \boldsymbol{\Sigma}_{24} & -\mathbf{W} & \mathbf{C}_{2z}^{\mathrm{T}}\mathbf{D}_{yz} \\
* & * & \boldsymbol{\Sigma}_{33} & \mathbf{G}_{yz}^{\mathrm{T}}\mathbf{H}_{yz} & 0 & \mathbf{G}_{yz}^{\mathrm{T}}\mathbf{D}_{yz} \\
* & * & * & \boldsymbol{\Sigma}_{44} & 0 & \mathbf{H}_{yz}^{\mathrm{T}}\mathbf{D}_{yz} \\
* & * & * & * & -\mathbf{R} & 0 \\
* & * & * & * & * & \boldsymbol{\Sigma}_{66}
\end{bmatrix}
+ \begin{bmatrix}
\tau\tilde{\mathbf{A}}_{1zz}^{\mathrm{T}} \\ \tau\tilde{\mathbf{A}}_{2zz}^{\mathrm{T}} \\ \tau\tilde{\mathbf{G}}_{xzz}^{\mathrm{T}} \\ \tau\tilde{\mathbf{H}}_{xzz}^{\mathrm{T}} \\ 0 \\ \tau\mathbf{D}_{xz}^{\mathrm{T}}
\end{bmatrix}
\mathbf{P}
\begin{bmatrix}
\tau\tilde{\mathbf{A}}_{1zz}^{\mathrm{T}} \\ \tau\tilde{\mathbf{A}}_{2zz}^{\mathrm{T}} \\ \tau\tilde{\mathbf{G}}_{xzz}^{\mathrm{T}} \\ \tau\tilde{\mathbf{H}}_{xzz}^{\mathrm{T}} \\ 0 \\ \tau\mathbf{D}_{xz}^{\mathrm{T}}
\end{bmatrix}^{\mathrm{T}},
$$

$$
\zeta(t) = \left[\mathbf{x}^{\mathrm{T}}(t), \mathbf{x}^{\mathrm{T}}(t-\tau), \varphi^{\mathrm{T}}(\mathbf{x}(t)), \varphi^{\mathrm{T}}(\mathbf{x}(t-\tau)), \int_{t-\tau}^{t}\mathbf{x}^{\mathrm{T}}(s)\mathrm{d}s, \mathbf{v}^{\mathrm{T}}(t)\right]^{\mathrm{T}}.
$$

and the other parameters are given in (14). If $\Xi < 0$, which is the equivalent to the condition (14), based on the Schur complement and Lemma 1, then (28) can be written as

$$
\dot{V}(t) + \mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t) - \gamma^2\mathbf{v}^{\mathrm{T}}(t)\mathbf{v}(t) \leq 0.
$$

Integrating both sides of the above inequality from 0 to ∞ with zero initial condition gives

$$
\int_0^{\infty}\left[\mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t) - \gamma^2\mathbf{v}^{\mathrm{T}}(t)\mathbf{v}(t)\right]\mathrm{d}t \leq 0,
$$

which means that the inequality (9) is satisfied. □

Proof of Theorem 2 By substituting $\tilde{\mathbf{A}}_{1zz} = \mathbf{A}_{1z} + \mathbf{B}_z\mathbf{K}_{1z}$, $\tilde{\mathbf{A}}_{2zz} = \mathbf{A}_{2z} + \mathbf{B}_z\mathbf{K}_{2z}$, $\tilde{\mathbf{G}}_{xzz} = \mathbf{G}_{xz} + \mathbf{B}_z\mathbf{K}_{3z}$, and $\tilde{\mathbf{H}}_{xzz} = \mathbf{H}_{xz} + \mathbf{B}_z\mathbf{K}_{4z}$ from (13) into (14), pre- and post-multiply it by $\mathrm{diag}\{\mathbf{P}^{-1}, \mathbf{P}^{-1}, \boldsymbol{\Theta}_1^{-1}, \boldsymbol{\Theta}_2^{-1}, \mathbf{P}^{-1}, \mathbf{I}, \mathbf{P}^{-1}\}$, and considering $\hat{\mathbf{P}} = \mathbf{P}^{-1}$, $\hat{\boldsymbol{\Theta}}_1 = \boldsymbol{\Theta}_1^{-1}$, $\hat{\boldsymbol{\Theta}}_2 = \boldsymbol{\Theta}_2^{-1}$, $\hat{\mathbf{Q}} = \mathbf{P}^{-1}\mathbf{Q}\mathbf{P}^{-1}$, $\hat{\mathbf{R}} = \mathbf{P}^{-1}\mathbf{R}\mathbf{P}^{-1}$, $\hat{\mathbf{W}} = \mathbf{P}^{-1}\mathbf{W}\mathbf{P}^{-1}$, $\hat{\mathbf{K}}_{1i} = \mathbf{K}_{1i}\mathbf{P}^{-1}$, $\hat{\mathbf{K}}_{2i} = \mathbf{K}_{2i}\mathbf{P}^{-1}$, $\hat{\mathbf{K}}_{3i} = \mathbf{K}_{3i}\boldsymbol{\Theta}_1^{-1}$, and $\hat{\mathbf{K}}_{4i} = \mathbf{K}_{4i}\boldsymbol{\Theta}_2^{-1}$, the condition (14) can be written as

$$
\begin{bmatrix}
\hat{\boldsymbol{\Sigma}}_{11} & \hat{\boldsymbol{\Sigma}}_{12} & \hat{\boldsymbol{\Sigma}}_{13} & \hat{\boldsymbol{\Sigma}}_{14} & \hat{\mathbf{W}} & \hat{\boldsymbol{\Sigma}}_{16} & \hat{\boldsymbol{\Sigma}}_{17} \\
* & -\hat{\mathbf{Q}}-\hat{\mathbf{P}} & 0 & \hat{\mathbf{P}}\mathbf{E}^{\mathrm{T}} & -\hat{\mathbf{W}} & \hat{\mathbf{P}}\mathbf{C}_{2z}^{\mathrm{T}}\mathbf{D}_{yz} & \hat{\boldsymbol{\Sigma}}_{27} \\
* & * & -2\hat{\boldsymbol{\Theta}}_1 & 0 & 0 & \hat{\boldsymbol{\Theta}}_1\mathbf{G}_{yz}^{\mathrm{T}}\mathbf{D}_{yz} & \hat{\boldsymbol{\Sigma}}_{37} \\
* & * & * & -2\hat{\boldsymbol{\Theta}}_2 & 0 & \hat{\boldsymbol{\Theta}}_2\mathbf{H}_{yz}^{\mathrm{T}}\mathbf{D}_{yz} & \hat{\boldsymbol{\Sigma}}_{47} \\
* & * & * & * & -\hat{\mathbf{R}} & 0 & 0 \\
* & * & * & * & * & \boldsymbol{\Sigma}_{66} & \tau\mathbf{D}_{xz}^{\mathrm{T}} \\
* & * & * & * & * & * & -\hat{\mathbf{P}}
\end{bmatrix}
+ \begin{bmatrix}
\hat{\mathbf{P}}\mathbf{C}_{1z}^{\mathrm{T}} \\ \hat{\mathbf{P}}\mathbf{C}_{2z}^{\mathrm{T}} \\ \hat{\boldsymbol{\Theta}}_1\mathbf{G}_{yz}^{\mathrm{T}} \\ \hat{\boldsymbol{\Theta}}_2\mathbf{H}_{yz}^{\mathrm{T}} \\ 0 \\ 0 \\ 0
\end{bmatrix}
\begin{bmatrix}
\hat{\mathbf{P}}\mathbf{C}_{1z}^{\mathrm{T}} \\ \hat{\mathbf{P}}\mathbf{C}_{2z}^{\mathrm{T}} \\ \hat{\boldsymbol{\Theta}}_1\mathbf{G}_{yz}^{\mathrm{T}} \\ \hat{\boldsymbol{\Theta}}_2\mathbf{H}_{yz}^{\mathrm{T}} \\ 0 \\ 0 \\ 0
\end{bmatrix}^{\mathrm{T}} < 0,
$$

where the parameters are given in (15). The Schur complement follows that the above inequality is equivalent to (15). □

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